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Optical force on the Raman dark state in two standing waves

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We use the normal modes of atom–field coupling to explain the force on a three-level (Λ) atom excited by two standing wave optical fields. An expression for the force in the weak field, small detuning limit is derived by inspection, and shown to agree with the exact solution to the optical Bloch equations. We describe the physical origin of the completely rectified force components as well as components which vary on scales much narrower than the optical wavelength. We emphasize the importance of the force component associated with the "dark state" and discuss the influence of the phase difference between the fields.

1. Introduction

Recently there has been considerable interest in the optical force on three-level atoms [1–4]. Research has been focused on the Λ system, which undergoes coherent population trapping. The force associated with coherent population trapping due to counterpropagating traveling wave fields has cooled atoms below the single photon limit [1]. There have been disparate calculations of the force on an atom in two standing wave excitation fields [2,3,5], which have not discussed the underlying physical processes in any detail. In this paper we describe the origin of the force on stationary atoms [#1] in terms of the evolutions of the normal modes of the atom field system. We show that most of the qualitative features of the force can be derived by inspection without actually solving the optical Bloch equations (OBE's).

Fig. 1a illustrates schematically the three-level system in the Λ configuration. The levels $|a\rangle$ and $|b\rangle$ are long-lived, and the level $|e\rangle$ is short lived. We assume that the field at frequency ω_1 only couples

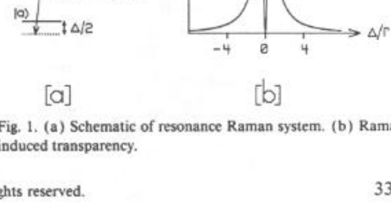

Fig. 1. (a) Schematic of resonance Raman system. (b) Raman induced transparency.

[#1] The physical process behind force on a moving atom in such a system will be treated in an upcoming publication.

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$|a\rangle$ to $|e\rangle$, and the one at ω_2 only couples $|b\rangle$ to $|e\rangle$. Here, we denote the common mode detuning by δ and the difference detuning by Δ. It is well known [6,7] that when the difference detuning vanishes ($\Delta=0$), the system gets optically pumped into a non-absorbing or dark state. Experimentally, this produces a dip in absorption or fluorescence as the laser difference frequency is scanned, as illustrated in fig. 1b. This transparency always appears at $\Delta=0$ independent [8] of the value of the common detuning, δ.

2. Force expression in the dark state basis

For this paper we will restrict discussions to calculation of the force when both laser fields are pure standing waves (SW). An important parameter will be the spatial phase difference, χ, between the standing wave patterns. In particular, we consider the case when the two laser fields have nearly the same wavelengths so that χ can be treated as constant on the optical wavelength scale.

In one dimension, the Lorentz force is written as $F=\mathrm{Tr}(\rho f)$. Here, ρ is the density operator and f is the force operator given by the product of the dipole operator and the gradient of the electric field, $f=-\mu\nabla E$. For the system of fig. 1a, the nonzero elements of f are given, in the rotating wave approximation, by the gradients of the Rabi frequencies

$$f_{ea}=f^*_{ae}=\nabla g_1\,,\quad f_{eb}=f^*_{be}=\nabla g_2\,,\quad (\hbar=1)\,. \tag{1}$$

The force on the three-level atom can thus be trivially computed by solving the optical Bloch equations (OBE) in the atomic states basis. However, the behavior of the force is easier to understand if the force is computed in a basis that is more natural for this system. To see what this is, recall what happens when $\Delta=0$. The atom then gets optically pumped into the dark state. Thus, the dark state is a natural choice for one of the basis states of the system.

The dark state and its orthogonal states are given by the following expressions [6]

$$|-\rangle\equiv(g_2\,|\bar{a}\rangle-g_1\,|\bar{b}\rangle)/g\,,$$
$$|+\rangle\equiv(g_1\,|\bar{a}\rangle+g_2\,|\bar{b}\rangle)/g\,,\quad |\bar{e}\rangle\,, \tag{2}$$

where the $|-\rangle$ state is the dark state, and we have defined $g^2\equiv g_1^2+g_2^2$. Here we have used the atom-field composite states

$$|\bar{a}\rangle\equiv|a\rangle\,|\omega_1\rangle\,,\quad |\bar{b}\rangle\equiv|b\rangle\,|\omega_2\rangle\,,\quad |\bar{e}\rangle\equiv|e\rangle\,, \tag{3}$$

where $|\omega_1\rangle$ and $|\omega_2\rangle$ are semi-classical photon states [9].

To see how this dark state basis simplifies the interaction, consider its hamiltonian. To obtain this, we first write down the hamiltonian in the $|\bar{a}\rangle$, $|\bar{b}\rangle$ and $|\bar{e}\rangle$ basis:

$$\mathbf{H}=\frac{1}{2}\begin{bmatrix}\Delta & 0 & -g_1\\ 0 & -\Delta & -g_2\\ -g_1 & -g_2 & -2\delta\end{bmatrix}, \tag{4}$$

where the zero of energy is chosen such that $\omega_e+\delta=0$. This hamiltonian can be obtained by inspection from fig. 1a. Next, we note that the $|-\rangle$ and $|+\rangle$ states can be expressed as a rotation of the $|\bar{a}\rangle$ and $|\bar{b}\rangle$ states, where the rotation angle θ is defined by $\tan\theta=g_1/g_2$. Making the appropriate transformations gives the hamiltonian in the $|-\rangle$, $|+\rangle$ and $|\bar{e}\rangle$ basis

$$\mathbf{H}=\frac{1}{2}\begin{bmatrix}\Delta\cos(2\theta) & \Delta\sin(2\theta) & 0\\ \Delta\sin(2\theta) & -\Delta\cos(2\theta) & -g\\ 0 & -g & -2\delta\end{bmatrix}. \tag{5}$$

To get a physical feel for this hamiltonian, consider again the case of $\Delta=0$. In this case, the $|-\rangle$ state is not coupled to the rest of the system by the applied fields, as expected. However, the $|+\rangle$ and $|\bar{e}\rangle$ states are coupled together by an effective Rabi frequency, g. This is illustrated by the energy level diagram in fig. 2a. As shown in the figure, spontaneous decay is allowed to both the $|-\rangle$ and the $|+\rangle$ state, so that all the atoms get optically pumped into the $|-\rangle$ state.

Next, consider the case of $\Delta\neq0$. When $g_1=g_2$, the hamiltonian shows that the $|-\rangle$ state is coupled to the $|+\rangle$ state by an effective Rabi frequency, Δ (since $\sin(2\theta)=1$). Physically, this can be explained by considering the following wave function which contains only the ground state contributions

$$|\Psi(t)\rangle=[\,|\bar{a}\rangle\exp(-\mathrm{i}\Delta t/2)-|\bar{b}\rangle\exp(\mathrm{i}\Delta t/2)\,]\,. \tag{6}$$

This wave function is a pure $|-\rangle$ state at $t=0$, but

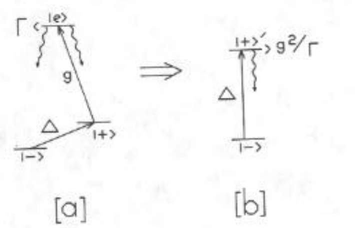

Fig. 2. (a) Resonance Raman system in dark state basis, for $g_1=g_2$. (b) Raman system in the dark and damped state basis.

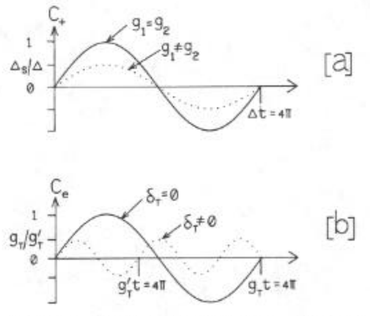

Fig. 3. (a) Plot of the $|+\rangle$ state amplitude, C_+, versus time, t, neglecting interaction with the excited state $|\bar{e}\rangle$. The solid curve is for equal Rabi frequencies $g_1=g_2$ and the dotted curve is for unequal Rabi frequencies $g_1\neq g_2$. (b) Analogous plot of the excited state amplitude, C_e, versus time, t, for an undamped two-level system. Solid curve is for zero detuning $\delta_T=0$. Dotted curve is for nonzero detuning $\delta_T\neq 0$.

evolves into a pure $|+\rangle$ state after a time $t=\pi/\Delta$ because of the time dependent coefficients. This is illustrated by the solid curve in fig. 3a. These time dependent coefficients in turn arise because the atom field composite states have different energies when $\Delta\neq 0$. However, the $|-\rangle$ and $|+\rangle$ states are still degenerate when $g_1=g_2$, since their energies are split by $\Delta\cos(2\theta)$ and $\cos(2\theta)=0$.

In the more general case, when the Rabi frequencies are not equal ($g_1\neq g_2$), the effective Rabi coupling is $\Delta\sin(2\theta)=2\Delta g_1 g_2/g^2$. In this case, the $|-\rangle$ and $|+\rangle$ states are no longer degenerate, and they differ in energy by $\Delta\cos(2\theta)=(g_2^2-g_1^2)/g^2\neq 0$. Physically, the energy difference arises from the fact that when $g_1\neq g_2$ the $|-\rangle$ and $|+\rangle$ states have unequal contributions from the composite states $|\bar{a}\rangle$ and $|\bar{b}\rangle$, which in turn have different energies for $\Delta\neq 0$. For example, at the $g_2=0$ node, the $|-\rangle$ state is the pure composite state $|\bar{b}\rangle$ which has energy $-\Delta/2$ and the $|+\rangle$ state is the state $|\bar{a}\rangle$ which has energy $+\Delta/2$. More generally, the $|-\rangle$ state can be viewed as having a state $|\bar{a}\rangle$ population of g_2^2/g^2 (energy $\Delta/2$) and a state $|\bar{b}\rangle$ population of g_1^2/g^2 (energy $-\Delta/2$). Combining these gives an effective $|-\rangle$ state energy of $(\Delta/2)\,(g_2^2-g_1^2)/g^2$ which agrees with the hamiltonian. A similar argument for the $|+\rangle$ state also shows agreement.

Finally, to see why the $|-\rangle$ to $|+\rangle$ state coupling is reduced when $g_1\neq g_2$, recall that this coupling represents the tendency of the $|-\rangle$ state to evolve into the $|+\rangle$ state with time. However, for unequal Rabi frequencies, the $|-\rangle$ state cannot evolve completely into the $|+\rangle$ state since these two states contain unequal contributions of $|\bar{a}\rangle$ and $|\bar{b}\rangle$. For example, when $g_2=0$ the $|-\rangle=|\bar{b}\rangle$ state never evolves into the $|+\rangle=|\bar{a}\rangle$ state for any value of the difference detuning. More generally, what starts as

$$|\Psi(0)\rangle=|-\rangle=(g_2|\bar{a}\rangle-g_1|\bar{b}\rangle)/g\,,$$

turns into

$$|\Psi(\pi/\Delta)\rangle=-\mathrm{i}(g_2|\bar{a}\rangle+g_1|\bar{b}\rangle)/g\,,$$

after a time $t=\pi/\Delta$. This is illustrated by the dotted curve in fig. 3a. The result is that the effective coupling is reduced by a factor of

$$|\langle+|\Psi(\pi/\Delta)\rangle|=2g_1g_2/g^2=\sin(2\theta)\,,$$

in agreement with the hamiltonian.

At this point it is useful to draw an analogy between the $|-\rangle$ and $|+\rangle$ state system and the familiar two-level system. As is well known, when a two-level system is driven on resonance, $\delta_T=0$, the ground state evolves completely into the excited state at a rate determined by Rabi coupling q_T (solid curve of fig. 3b). Off resonance ($\delta_T\neq 0$), the ground state no longer evolves completely to the excited state. Rather, the maximum excited state amplitude is given by g_T/g'_T (dotted curve in fig. 3b), where $g'_T=\sqrt{g_T^2+\delta_T^2}$ is the new amplitude oscillation frequency. Replacing g_T by $\Delta\sin(2\theta)$ and δ_T by $\Delta\cos(2\theta)$ gives the $|-\rangle$ and $|+\rangle$ state results of fig. 3a. The correspondence between the undamped two-level system and the $|-\rangle$ and $|+\rangle$ state system are summarized in table 1. Note that for SW excitation of the $|-\rangle$ and $|+\rangle$ state system it is the population flopping frequency, Δ, rather than the effective detuning, that is independent of position.

Now consider the force operator f in the dark state basis. The nonzero elements are

$$f_{\tilde{e}-}=f^*_{-\tilde{e}}=(g_2\nabla g_1-g_1\nabla g_2)/g=g\nabla\theta\,, \qquad (7a)$$

$$f_{\tilde{e}+}=f^*_{+\tilde{e}}=(g_1\nabla g_1+g_2\nabla g_2)/g=\nabla g\,, \qquad (7b)$$

where $f_{\tilde{e}-}$ and $f_{\tilde{e}+}$ are real for pure standing wave excitation. These elements can be derived by inspection by noting, for example, that the force operator corresponding to the $|-\rangle$ state is simply a weighted sum of the $|\tilde{a}\rangle$ and $|\tilde{b}\rangle$ state force operators where the weights are the same as in the definition of the $|-\rangle$ state.

The $|+\rangle$ state force operator $f_{\tilde{e}+}$ is the same as would be expected if the $|+\rangle$ and $|\tilde{e}\rangle$ states were a simple two-level system coupled by an effective Rabi frequency g. In contrast, the $|-\rangle$ state force operator $f_{-\tilde{e}}$ does not have a simple two-level analogy. In fact, a two level argument would lead to zero force on the $|-\rangle$ state, since it is transparent to the optical fields, i.e. $H_{\tilde{e}-}=0$. To see why the $|-\rangle$ state force operator is not zero, recall that the dark state is defined using Rabi frequencies which are constant locally. Thus, for phase shifted SW's, the dark state at position z, denoted by $|-\rangle_z$, is not a pure dark state at a position $\mathrm{d}z$ away, but contains a local $|+\rangle_{z+\mathrm{d}z}$ state contribution. It is easy to show that the amplitude of this contribution is given by $\nabla\theta\,\mathrm{d}z$. Moreover, since the $|+\rangle_{z+\mathrm{d}z}$ state has an interaction energy $H^{z+\mathrm{d}z}_{\tilde{e}+}=g$, it follows that the $|-\rangle_z$ state acquires an interaction energy of $H^{z+\mathrm{d}z}_{\tilde{e}-}=g\nabla\theta\,\mathrm{d}z$ at position $z+\mathrm{d}z$. This change in interaction energy with position can be viewed as arising from an effective force of $g\nabla\theta$ on the $|-\rangle_z$ state, which agrees with the force operator, $f_{\tilde{e}-}$.

Using the results so far gives the following explicit form for the force on a three level atom in standing wave fields

$$F=f_{\tilde{e}-}\,\mathrm{Re}(\rho_{-\tilde{e}})+f_{\tilde{e}+}\,\mathrm{Re}(\rho_{+\tilde{e}})\,. \qquad (8)$$

3. Perturbative estimate of force

We now compute the optical force (eq. (8)) by estimating the steady-state values of the density matrix elements in several limiting cases. Consider first the case of $\Delta=0$, wherein all the atoms are optically pumped into the $|-\rangle$ state. Clearly, in this case the only nonzero element is $\rho_{--}=1$, so that the off-diagonal elements are zero. Thus, the force is zero everywhere, independent of all other parameters.

Next, consider the case when $\Delta\neq 0$ but $\Delta\sin(2\theta)=0$. This occurs at the nodes of either of the two standing waves. Once again, the atoms are optically pumped into the $|-\rangle$ state (which is now a pure composite state) so that the off-diagonal elements are again zero. Therefore, the steady state force always goes to zero at the field nodes even if $\Delta\neq 0$.

Now consider the case when $\Delta\sin(2\theta)$ is non-zero but small compared to the optical pumping rate, g^2/Γ. In this limit, the nonzero density matrix elements can be obtained by perturbing around the $\rho_{--}=1$ solution. To simplify the analysis further we make the additional approximation that $g\ll\Gamma$, and ignore [#2] the influx of atoms into the $|+\rangle$ and $|-\rangle$

For footnote see next page.

Table 1

Similarities between two-level system and dark state basis. (The variable z in parenthesis implies that the quantity is a function of position.)

Description	Two-level system	Dark state basis
Effective Rabi frequency	$g_\mathrm{T}(z)$	$-\Delta_\mathrm{s}(z)$
Effective detuning	δ_T	$\Delta_\mathrm{c}(z)$
Population flopping rate	$\sqrt{g_\mathrm{T}^2+\delta_\mathrm{T}^2}=g'_\mathrm{T}(z)$	$\sqrt{\Delta_\mathrm{s}^2+\Delta_\mathrm{c}^2}=\Delta$
Maximum mixing amplitude	$\left[\mathrm{i}\frac{g_\mathrm{T}}{g'_\mathrm{T}}\right](z)$	$-\mathrm{i}\frac{2g_1g_2}{g^2}=-\left[\mathrm{i}\frac{\Delta_\mathrm{s}}{\Delta}\right](z)$

states from the $|\bar{e}\rangle$ state. This allows us to avoid solving the complex density matrix equations (OBE's) entirely, by instead representing the system with a wave function of the form $\Psi = A_-|-\rangle + A_+|+\rangle + A_{\bar{e}}|\bar{e}\rangle$.

The dynamics of the system is now contained in the $|-\rangle$, $|+\rangle$, $|\bar{e}\rangle$ state amplitudes, whose equations of motion are given by the Schrödinger equation:

$$\frac{\partial}{\partial t}\begin{bmatrix} A_- \\ A_+ \\ A_{\bar{e}} \end{bmatrix} = (-i/2)\begin{bmatrix} \Delta_c & \Delta_s & 0 \\ \Delta_s & -\Delta_c & -g \\ 0 & -g & -i(\Gamma - 2i\delta) \end{bmatrix}\begin{bmatrix} A_- \\ A_+ \\ A_{\bar{e}} \end{bmatrix}, \quad (9)$$

where $\Delta_c \equiv \Delta\cos(2\theta)$, $\Delta_s \equiv \Delta\sin(2\theta)$, and the phenomenological damping Γ is added to the hamiltonian.

Since $\Gamma \gg g$, we can use the adiabatic following approximation [10], to solve the third equation (of eq. (9)) for $A_{\bar{e}}$. The result is $A_{\bar{e}} = i(g/\Gamma_\delta)A_+$, where $\Gamma_\delta = \Gamma - 2i\delta$. Using this result to eliminate $A_{\bar{e}}$, the wave function now becomes, $\Psi = A_-|-\rangle + A_+|+\rangle_d$. The new basis state $|+\rangle_d = |+\rangle + i(g/\Gamma_\delta)|\bar{e}\rangle$ is called the damped state. In the adiabatic limit, the dark and damped states form a closed two level system, as illustrated in fig. 2b. Here, it should be noted [#3] that the damped state amplitude is still A_+.

It is easy to show that the amplitude equations for the dark and damped states are simply:

$$\frac{\partial}{\partial t}\begin{bmatrix} A_- \\ A_+ \end{bmatrix} = (-i/2)\begin{bmatrix} \Delta_c & \Delta_s \\ \Delta_s & -i(\Gamma_+ - i\Delta_c) \end{bmatrix}\begin{bmatrix} A_- \\ A_+ \end{bmatrix}, \quad (10)$$

where $\Gamma_+ = g^2/\Gamma_\delta$. Thus, the decay rate of the $|+\rangle_d$ state is the optical pumping rate g^2/Γ_δ, which is proportional to total laser intensity. Physically, this decay rate is analogous to the lowest order scattering rate for a two level system, which is also proportional to intensity.

To solve the remaining amplitude equations in the limit of $\Delta \ll \Gamma_+$ we can again make the adiabatic approximation to solve the second equation (of eq. (10) for A_+. The result is

$$A_+ = -i[\Delta_s/(\Gamma_+ - i\Delta_c)]A_- .$$

From these results, we can generate the approximate density matrix elements

$$\rho_{-\bar{e}} = \left(\frac{g}{\Gamma_\delta}\right)\left(\frac{\Delta_s}{\Gamma_+ - i\delta_c}\right), \quad \rho_{+\bar{e}} = -i\left(\frac{g}{\Gamma_\delta}\right)\rho_{++},$$

$$\rho_{++} = \left|\frac{\Delta_s}{\Gamma_+ - i\Delta_c}\right|^2, \quad (11)$$

where again $\rho_{--} \simeq 1$ has been assumed. Substituting these into the explicit force expression (eq. (8)) and remembering the approximations $\Delta \ll \Gamma_+ \ll \Gamma$ gives

$$F = f_{\bar{e}-}\,\mathrm{Re}(\rho_{-\bar{e}}) = 2\Delta g_1 g_2(g_2\nabla g_1 - g_1\nabla g_2)/g^4, \quad (12)$$

where the additional approximation of $\delta \ll \Gamma$ has been used. It should be noted that the $|+\rangle$ state contribution to the force is negligible for these approximations. Thus, the force on a three level atom, in this limit, is due entirely to the force on the $|-\rangle$ or dark state [#4]. Also note that the force is independent of δ (for $\delta \ll \Gamma$). This is related to the fact that the Raman induced transparency is independent of δ.

This force expression is plotted and compared to the OBE solution in fig. 4b, for the case of two standing waves with equal amplitudes ($\equiv g_0$), i.e., $g_1 = g_0\sin(kz)$ and $g_2 = g_0\sin(ikz + \chi)$, where $k = k_1 \approx k_2$. The phase difference is chosen to be $\chi = \pi/4$. Specifically, fig. 4a shows a plot of the two SW field amplitudes. The solid curve in fig. 4b is a plot of the above force expression (eq. (12)) and the circles correspond to the OBE solution. As can be seen, the lowest order force estimate agrees well with the OBE result.

The force plot in fig. 4 shows that the force can vary by a large amount over a distance much shorter than the optical wavelength. Since the force crosses zero in these regions of rapid variations (see, for example, points q and s in fig. 4), very narrow potentials can result. To understand the origins of these

[#2] This approximation is often used in first order perturbation, and will be justified later by excellent agreement with the OBE result.

[#3] Note that the coefficient for the damped state is the same as that for the $|+\rangle$ state, according to the way the damped state is defined.

[#4] An exact solution [5] of the OBE's shows that the $|+\rangle$ state contribution to the force is always zero when $\delta = 0$.

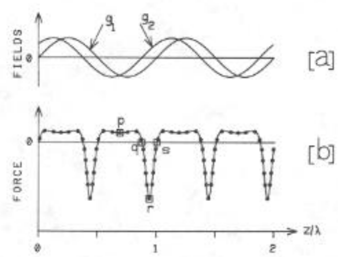

Fig. 4. Optical force a on three level atom versus position. (a) Standing wave field amplitudes. (b) Force on optical wavelength scale. Solid curve is lowest order estimate. Dotted curve is OBE solution. For each plot, $g_0=1000\Delta=0.1\Gamma$.

rapid zero crossings, recall that the force must be zero at the nodes of either g_1 or g_2 because the $|-\rangle$ state becomes a pure composite state at these positions, and can no longer mix with the other states to generate nonzero coherences. However, between each of the nodes, the force must reach a maximum (unless it is zero everywhere). Thus, it follows that the force can have structures on a distance scale determined by the node spacing, which can easily be much smaller than the optical wavelength.

Of course, to obtain the sharp structures in the force plot of fig. 4 the magnitude of the force must somehow be enhanced in the region between the closely spaced nodes (see for example, point r in fig. 4). To see why this occurs, it is necessary to separately consider the coherence $\mathrm{Re}(\rho_{-\tilde{e}})$ and the force operator $f_{\tilde{e}-}$.

Consider first, the force operator $f_{\tilde{e}-}=g\nabla\theta$. In view of earlier discussions, this can be seen as a measure of the energy gradient of the locally defined state $|-\rangle_z$ as a function of position. Between the closely spaced field nodes this energy gradient would tend to be large because the $|-\rangle$ (and $|+\rangle$) state must rotate from one pure composite state $|\tilde{a}\rangle$ or $|\tilde{b}\rangle$ to the other in a distance much less than the optical wavelength. For example, the composite state $|\tilde{b}\rangle$ is the dark state $|-\rangle_q$ (interaction energy 0) at point q (fig. 4b), but the strongly interacting $|+\rangle_s$ (interaction energy g) at the nearby point s. To see explicitly how much the force operator can be enhanced, consider the points p and r in fig. 4. At point p, the force operator is

$$f_{\tilde{e}-}=kg_0^2\sin\chi/g(z)=kg_0\sqrt{2}\sin(\chi/2)\ ,$$

and at point r the force operator is $f_{\tilde{e}-}=kg_0\sqrt{2}\cos(\chi/2)$. Thus, compared to point p, the force operator at point r is enhanced by a factor of $\mathrm{cotan}(\chi/2)$. In the limit when $\chi\to 0$, $f_{\tilde{e}-}\to\sqrt{2}g_0k$ at point r and $f_{\tilde{e}-}\to 0$ at point p.

Now consider the coherence. There are three factors which must be considered: the $|-\rangle\to|+\rangle_d$ coupling strength Δ_s, the contribution of $|\tilde{e}\rangle$ to the $|+\rangle_d$ state (proportional to g/Γ), and the $|+\rangle_d$ state decay rate Γ_+. For simplicity, consider only the equal Rabi frequency points, p and r in fig. 4b. For a given $|+\rangle_d$ state decay rate, the $\rho_{-\tilde{e}}$ coherence tends to increase with increasing g, because the $|+\rangle_d$ state contains a larger component of the $|\tilde{e}\rangle$ state, and the $|-\rangle\to|+\rangle$ coupling, $|\Delta_s|=|\Delta\sin(2\theta)|=|\Delta|$, is independent of g when $|g_1|=|g_2|$. However, the $|+\rangle_d$ state decay rate is not fixed, but depends on g^2/Γ. This position dependent decay rate has no two level analog and its net effect is to give a $\rho_{-\tilde{e}}$ coherence that depends inversely on g. This can be understood by noting that a smaller $|+\rangle_d$ state decay rate leads to a larger steady state mixing of the $|-\rangle$ state with the $|+\rangle_d$ state. As a result the $\rho_{-\tilde{e}}$ coherence at point r in fig. 4b tends to be large since $g(z)$ is a minimum there. Thus, both the force operator and the coherence contribute to the sharp, sub-optical-wavelength structures in fig. 4b.

4. Force rectification

Not only does the force in the Λ system show sub-optical-wavelength behavior, but it can show strong rectification (i.e., non-zero value when averaged over an optical wavelength). One case where the origin of this rectification can be clearly illustrated is when the difference detuning is large enough to saturate the $|-\rangle\to|+\rangle_d$ transition.

For simplicity, we set $\delta=0$ in this discussion, so that $\Gamma_\delta=\Gamma$ and $\Gamma_+=g^2/\Gamma$ is purely real. When the $|-\rangle\to|+\rangle_d$ transition is saturated, the second adiabatic following approximation is no longer valid. However, the $|-\rangle$ and $|+\rangle_d$ system is effectively a closed two-level system, so that its steady state den-

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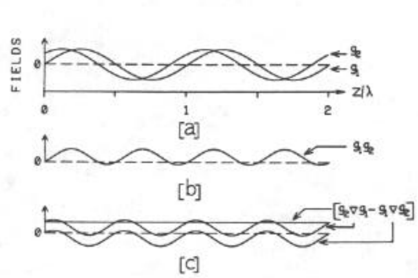

Fig. 5. Force contributions on optical wavelength scale when $|\Delta| \gg |\Gamma_+|$ and $\Delta| \gg g$. (a) Standing wave fields. (b) $g_1 g_2$ term. (c) $(g_2\nabla g_1 - g_1\nabla g_2)$ term.

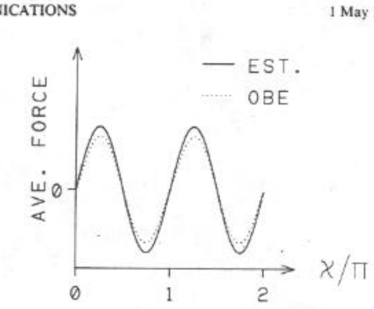

Fig. 6. Rectified optical force versus phase χ. Solid curve is estimate when $|\Delta| \gg |\Gamma_+|$ and $|\Delta| \ll g$. Dotted curve is OBE solution. For each plot, $g_0 = 3.3\Delta = 0.1\Gamma$.

sity matrix elements can be written,

$$\rho_{-+} = -\mathrm{i}\left(\frac{\Delta_s}{\Gamma_+ - 2\mathrm{i}\Delta_c}\right)(\rho_{--} - \rho_{++}) , \quad (13a)$$

$$(\rho_{--} - \rho_{++}) = \frac{\Gamma_+^2 + 4\Delta_c^2}{\Gamma_+^2 + 4\Delta_s^2 + 4\Delta_c^2} . \quad (13b)$$

Remembering the first adiabatic approximation then gives

$$\mathrm{Re}(\rho_{-\bar{e}}) = \mathrm{Re}\left[\left(\frac{\Delta_e}{\Delta_+}\right)^* \rho_{-+}\right] = \left(\frac{g}{\Gamma}\right)\frac{\Delta_s \Gamma_+}{\Gamma_+^2 + 4\Delta^2} , \quad (13c)$$

where use has been made of the identity $\Delta_s^2 + \Delta_c^2 = \Delta^2$.

In the limit of $|\Delta| \ll \Gamma_+$ this result reduces to that obtained using the second adiabatic following approximation, as expected. However, in the opposite limit of $|\Delta| \gg |\Gamma_+|$ (but $|\Delta| \ll g$), the force becomes

$$F = (g_1 g_2)(g_2 \nabla g_1 - g_1 \nabla g_2) 2/\Delta\Gamma^2$$
$$= k g_0^4 [\cos\chi - \cos(2kz + \chi)](\sin\chi)/4\Delta\Gamma^2 . \quad (14)$$

Here, the $(g_2\nabla g_1 - g_1\nabla g_2)$ term arises from the force operator. This term equals $g_0^2 \sin\chi$, which is effectively constant over an optical wavelength. In contrast, the $g_1 g_2 = (g_0^2/2)[\cos\chi - \cos(2kz+\chi)]$ term, which comes from the $\rho_{-\bar{e}}$ coherence, is not a constant over an optical wavelength. However, it has a component $(\cos\chi)$ which is a constant. The other component of $\rho_{-\bar{e}}$, $[\cos(2kz+\chi)]$ is periodic over an optical wavelength. Therefore, when averaged over an optical wavelength, the force is proportional to $\sin(2\chi)$. This rectified force as a function of χ is shown by the solid line in fig. 6. The dotted curve in fig. 6 is the rectified force computed using the exact solution of the OBE's. Again, reasonably good agreement is seen.

If $k_1 = k_2$ then χ is independent of position. Thus, the rectified force is spatially invariant. The amplitude and direction of this constant force (for a given g_0 and Δ) can then be controlled by simply varying the spatially invariant phase difference between the two standing waves. In contrast, if $k_1 \neq k_2$ then $\chi = \chi_0 + (k_1 - k_2)z$ where χ_0 is a constraint phase difference. The rectified force is now no longer spatially invariant, but is periodic over a long distance given by the wave length of the difference frequency (e.g., 17 cm for the ground state hyperfine splitting in sodium). Thus, in the limit of $k_1 \to k_2$, the scale over which this force is periodic approaches infinity, i.e., it becomes spatially invariant.

5. Conclusion

We have used the normal modes of the atom–field interaction to physically model the origins of novel structures that appear in the force on a Λ system atom under standing wave excitation. In particular, we have identified a situation where the force is only on the dark state, thus leading to simple, closed form expressions for the force. Finally, our estimated results agree well with solutions of the OBE's in the regions where the approximations used are valid. Future work will involve using the normal modes of the atom-field system to model phase (χ) dependent cooling.

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