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Cavity dark states for quantum computing

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Abstract

We show that two multilevel atoms can perform quantum communication with each other via interaction with an enclosing cavity containing no photons. The physical mechanism is analogous to the way populations can be exchanged between the extremal states in a three level system via adiabatic following, without populating the intermediate states. The combined system of the two atoms, the cavity, and two laser beams contains a dark state corresponding to the cavity in its ground state. Using a counter-intuitive pulse sequence, quantum information can be transferred adiabatically from one atom to the other via this *cavity dark state*. This process can be used to circumvent the effect of cavity decay in a quantum computer formed by cavity interconnected qubits. © 2001 Elsevier Science B.V. All rights reserved.

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In recent years, there have been a wide range of activities aimed at quantum computing. A quantum computer with a large number of bits may help solve certain problems much more efficiently than its classical counterpart [1–3]. While the theoretical work [4–13] has progressed rapidly, the experimental realization [14–17] of a many bit quantum computer remains to be a daunting challenge [14–17]. It is not clear, for example, whether NMR or trapped-ion based quantum computing can be scaled to a large number of qubits. As such, novel approaches are being proposed and pursued by many groups. Some of these proposals involve a collection of distinct quantum systems (such as atoms, molecules, or quantum dots) that are not directly coupled to one another. Instead, an effective coupling is induced via interaction of these quantum systems to an optical cavity.

Pellizari et al. proposed a scheme where each atom has a pair of identical Λ-system transitions [7]. To summarize this scheme briefly, consider the case where two spatially separated but spectrally identical atoms are coupled using a cavity. This is illustrated in Fig. 1, where we have shown only one of the two Λ transitions in each atom. Here, one leg of the Λ transition in each atom is simultaneously excited by the photons of the cavity mode, while the remaining legs are excited by

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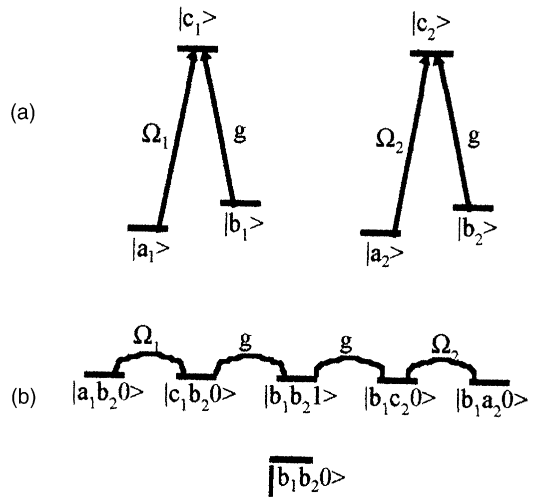

Fig. 1. (a) Illustration of the coupling of two atoms using the dark resonance inside a cavity. Here, g is the vacuum Rabi frequency of the cavity, $\Omega_1(\Omega_2)$ is the Rabi frequency of the first(second) laser beam. (b) The atom cavity composite states (rotating wave frame) corresponding to two of the closed transition manifolds having a maximum of 1 or 0 cavity photons. The ket notation for the composite states is indexed by the internal states of the first and second atoms followed by the photon number in the cavity mode.

classical laser beams, applied externally, as shown in Fig. 1a. In the limit where the cavity mode has only zero or one photon, the atoms-field coupled states are shown in Fig. 1b. This system has a non-trivial dark state, which can be written as a superposition of the three states that do not contain any component of the atomic excited state, as well as a trivial dark state (shown uncoupled at the bottom of the figure). Such a system can be used to transfer quantum information between the two atoms, using adiabatic following, and also to perform quantum logic when each atom has a pair of identical Λ transitions [7,18–20].

During the transfer, the system evolves adiabatically, while in a superposition of these dark states. As such, this process is impervious to any decoherence caused by spontaneous emission from atomic excited states. However, the non-trivial dark state contains a component corresponding to one photon in the cavity. As such, any cavity decay causes the system to decohere. Since the cavity lifetime is often at least as short as the atomic excited state lifetime, the potential benefit of using the dark state is mitigated substantially. Furthermore, there are situations where the cavity decay rate is orders of magnitude bigger than the line width of the atomic system, so that the benefit of using the dark state is minimal. For example, we have recently proposed a scheme where this approach can be used to couple spectrally distinct atoms in a spectral hole burning crystal for quantum computing [21–23]. One candidate system for implementing such a scheme is a cryogenically cooled, thin layer of Pr:YSO, embedded in a cavity. In this case, the atomic excited state is very longed lived (160 μs), compared to typical cavity lifetimes (tens of nanoseconds) [17,24]. As such, avoiding the atomic excited state at the cost of populating the photon mode is counter productive.

In this article, we present a solution to this problem, by using a scheme where the information exchange takes place through a *dark state of the cavity*, which contains no cavity photons, while a finite population of the atomic excited state is allowed for a short time. To see how such a state might be formed, consider the level diagram of Fig. 2. [1] The objective here is to find a dark state that does not contain the middle state (with one photon in the cavity), and contains as small a fraction as possible of the states with components of the atomic excited state. By detuning the classical fields, while keeping the cavity resonant, we find that we can produce a state which has no photons in the cavity mode: a cavity dark state. This state does not contain any significant component of $|b_1b_21\rangle$, and has a small component (proportional to $|\Omega/\delta|^2 \ll 1$) of states containing the atomic excited states. This state is produced by combining the strong-field seeking dressed states corresponding to the two-level transition in each atom, in the limit where $|\Omega/\delta|^2 \ll 1$. Explicitly, the cavity dark state is given by

[1] For simplicity, we have used the original model of Ref. [7] where the two atoms are spectrally identical; it can be generalized easily to the scheme for coupling spectrally adjacent atoms.

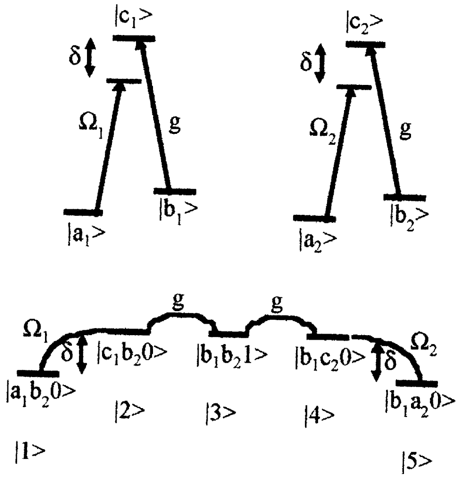

Fig. 2. Illustration of the excitation scheme needed to implement a cavity dark state. As shown in the top diagram, the classical laser beams are detuned, while the cavity is kept on resonance. The bottom diagram shows the same situation in the rotating wave frame. The state $|b_1b_20\rangle$ (not shown) is still the trivial dark state.

$$|DC\rangle \propto \Omega_2\left(|a_1b_20\rangle + \frac{\Omega_1}{2\delta}|c_1b_20\rangle\right) - \Omega_1\left(|b_1a_20\rangle + \frac{\Omega_2}{2\delta}|b_1c_20\rangle\right) \quad (1)$$

Since this state does not contain any photons, it is impervious to the cavity decay, in the same manner that a conventional dark state is unaffected by atomic decay. Of course, this state is not completely dark with respect to the atomic decay. However, the effect of atomic decay is reduced by a factor of $|\Omega/\delta|^2$, which can be made small by increasing the detuning. Moreover, since the atomic decay rate in Pr:YSO is much smaller than the decay rate of the cavity, this state is particularly suited for our scheme.

The potential success of this model depends strongly on the details of the adiabatic following. It is necessary to determine the conditions under which the system can be made to evolve in this state during the counter intuitive pulse sequence used for the transfer. We have looked at this issue in detail, and have identified conditions under which the transfer takes place in a state that is very close to this cavity dark state.

In general, during the adiabatic passage, the system is susceptible to decoherence from several sources. To minimize decoherence effects, it is desirable to complete the adiabatic passage as quickly as possible. But, as the passage time becomes shorter, non-adiabatic effects are introduced. While non-adiabaticity is not a decoherence effect, it can of course cause the coherent transfer to fail, and it can cause the system to become more susceptible to decay as a result of populating unstable states. To use adiabatic passage for coherent transfer, the actual passage time must be carefully optimized: slow enough to be adiabatic, but fast enough to avoid significant decay.

The defining parameters of the system are the vacuum Rabi frequency g, the cavity decay rate κ, and the spontaneous emission rate γ. The vacuum Rabi frequency g is determined by the cavity geometry and the strength of the atomic dipole moment; κ depends on the cavity geometry, the reflectivity of the cavity mirrors, and the presence of scattering centers within the cavity; γ is determined from the atomic dipole moment (we will assume that the decay rate inside the cavity does not differ significantly from the free space rate). Here, we assume that these parameters are fixed, and determine how a variation of the control parameters can be used to improve the quality of adiabatic passage.

For notational simplicity, we rename the five basis states of Fig. 3 as follows: $|1\rangle = |ab0\rangle$, $|2\rangle = |cb0\rangle$, $|3\rangle = |bb1\rangle$, $|4\rangle = |bc0\rangle$, $|5\rangle = |ba0\rangle$, as shown. Consider a situation where the system is in the state $|1\rangle$ at $t = 0$. The counter-intuitive pulse sequence is applied as follows: Ω_1 is kept zero and Ω_2 is turned on for a duration T_1. At $t = T_1$, Ω_1 is also turned on over a duration T while Ω_2 is turned off. At $t = T_1 + T$, Ω_1 is also turned off over a duration T_2, and the operation is complete at $t = T_1 + T + T_2$. Obviously, the transfer has to take place during the time when both fields are non-zero, i.e., during the interval T. Fig. 3a shows the energies of the five eigenstates of the system during this interval. Here, the unit of energy is chosen to be $\hbar g = 1$, the peak value of Ω_1 and Ω_2 is $\Omega_0 = 10g$, and the detuning is $\delta = -100g$. Note that the states $|2\rangle, |3\rangle$ and $|4\rangle$ are degenerate (in the rotating wave frame) in the absence of interactions. We

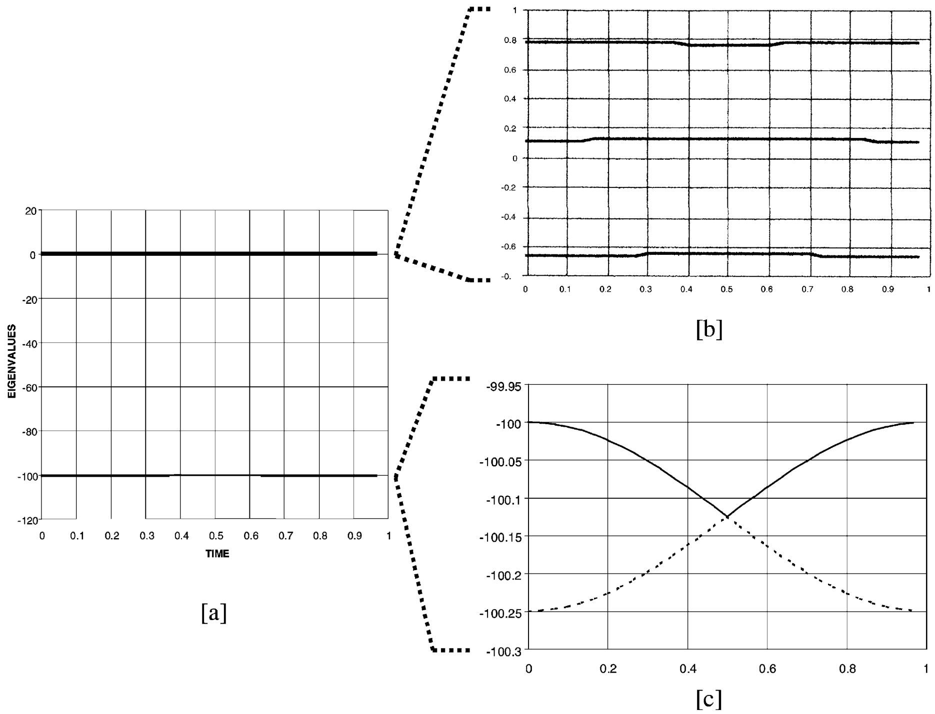

Fig. 3. Illustration of the dressed states corresponding to the system of two atoms coupled to the cavity, as functions of the interaction interval during which both laser pulses are present. (a) All five dressed states on the same scale; (b) expanded view of the dressed states that evolve adiabatically from/to the states $|2\rangle$, $|3\rangle$ and $|4\rangle$; (c) expanded view of the dressed states that evolve adiabatically from/to the states $|1\rangle$ and $|5\rangle$.

have chosen this to be the zero of energy in this plot. As the laser beams are turned on, these three states evolve into a band of three dressed states, which are shown on an expanded scale in Fig. 3b. Similarly, the states $|1\rangle$ and $|5\rangle$ are degenerate in the absence of interactions, with a energy equaling δ. As the laser beams are turned on, these two states evolve into another band of two dressed states, which are shown on an expanded scale in Fig. 3c.

It is difficult to express the eigenvectors corresponding to these levels in exact analytic form. However, one can easily derive the approximate form of these states by using well-known expressions for two-level dressed states [25]. Furthermore, this approach allows us to derive approximate analytic expressions for the energy levels as well. In the case considered here, the system starts out in the state denoted by $|\alpha\rangle$ (the solid line in Fig. 3c). What we need to determine are the conditions under which the system will stay in this state. The state that it can couple to via non-adiabaticity is the one denoted by $|\beta\rangle$ (the dotted line in Fig. 3c), since it appears to become degenerate with the desired state at the middle of the interaction time.

To interpret the eigenstates and the eigenenergies, consider first the state $|5\rangle$ interacting with state $|4\rangle$. Assuming that the detuning is much greater than the Rabi frequency, we get the light shifted state:

$$|5'\rangle = |5\rangle + \frac{\Omega_2}{2\delta}|4\rangle \tag{2}$$

where the normalization is omitted since it is approximately unity. At $t \leqslant T_1$, we thus have $|\alpha\rangle = |1\rangle$ and $|\beta\rangle = |5'\rangle$ with an energy difference given by: $\varepsilon \equiv \langle\alpha|H|\alpha\rangle - \langle\beta|H|\beta\rangle = -\Omega_2^2/4\delta$.

As Ω_1 is turned on, the state $|1\rangle$ is also light shifted, via its interaction with state $|2\rangle$, producing the state:

$$|1'\rangle = |1\rangle + \frac{\Omega_1}{2\delta}|2\rangle \quad (3)$$

However, the states $|1'\rangle$ and $|5'\rangle$ are not fully decoupled from each other. We proceed in steps to determine the eigenstates $|\alpha\rangle$ and $|\beta\rangle$ when both laser fields are non-zero.

Consider the coupling of the light shifted state $|1'\rangle$ to the intermediate state $|3\rangle$, mediated by the vacuum Rabi frequency g. The coupling rate is:

$$g_1 = 2\langle 1'|H|3\rangle = \frac{\Omega_1}{2\delta}2\langle 2|H|3\rangle = \frac{\Omega_1 g}{2\delta} \quad (4)$$

with a detuning (i.e., the energy difference between $|1'\rangle$ and $|3\rangle$, under the rotating wave transformation) given approximately by δ. Since this detuning is much larger than the coupling strength g_1 the state $|1'\rangle$ is further light shifted by this interaction, producing the state:

$$|1''\rangle = |1'\rangle + \frac{g_1}{2\delta}|3\rangle \quad (5)$$

Similarly, the state $|5'\rangle$ interacts with state $|3\rangle$ to produce the light-shifted state:

$$|5''\rangle = |5'\rangle + \frac{g_2}{2\delta}|3\rangle \quad (6)$$

where $g_2 = 2\langle 5'|H|3\rangle = \Omega_2 g/2\delta$. The energy difference between $|1''\rangle$ and $|5''\rangle$ is now given by:

$$\Delta = (\Omega_2^2 - \Omega_1^2)\left(\frac{1}{4\delta} + \frac{g^2}{16\delta^3}\right) \approx (\Omega_2^2 - \Omega_1^2)/4\delta \quad (7)$$

The states $|1''\rangle$ and $|5''\rangle$ couple to each other as well, since each contains a component of state $|3\rangle$. The coupling rate is:

$$G = 2\langle 1''|H|5''\rangle = \frac{g_1^2}{2\delta} + \frac{g_2^2}{2\delta} = \frac{g^2}{8\delta^3}2\Omega_1\Omega_2 \quad (8)$$

Diagonalizing this interaction G in the presence of the detuning Δ yields the eigenstates:

$$\begin{aligned} |\alpha\rangle &= \cos\theta|1''\rangle - \sin\theta|5''\rangle \\ |\beta\rangle &= \sin\theta|1''\rangle + \cos\theta|5''\rangle \end{aligned} \quad (9)$$

where $\tan 2\theta = G/\Delta$, and the energy separation is given by $\varepsilon = \sqrt{\Delta^2 + G^2}$.

Before proceeding further, it instructive to consider this result in the limits. Just at the onset of the active period T, we have $\Omega_1 = 0$, so that $G = 0$ and $\Delta = \Omega_2^2/4\delta$, yielding $\varepsilon = \Omega_2^2/4\delta$, as determined before. Since $\theta = 0$, the eigenstates are given by $|\alpha\rangle = |1''\rangle = |1'\rangle = |1\rangle$ and $|\beta\rangle = |5''\rangle = |5'\rangle$, again as determined before. At the end of the period T, we have $\Omega_2 = 0$, so that $G = 0$ and $\Delta = \Omega_1^2/4\delta$, yielding $\varepsilon = \Omega_1^2/4\delta$, as expected. Now $\theta = \pi/2$ and the eigenstates are given by $-|\alpha\rangle = |5''\rangle = |5'\rangle = |5\rangle$ and $|\beta\rangle = |1''\rangle = |1'\rangle = |1\rangle$, as wanted. Finally, at the cross-over point, $\Omega_1 = \Omega_2 = \Omega_0/\sqrt{2}$, so that $\Delta = 0$ and $G = g^2\Omega_0^2/8\delta^3$, yielding $\varepsilon = g^2\Omega_0^2/8\delta^3$, in close agreement with the energy separation shown in Fig. 4, which is an expanded view of the anti-crossing region of Fig. 3c. Here $\theta = \pi/4$ and the eigenstates are:

$$\begin{aligned} |\alpha\rangle &= \frac{1}{\sqrt{2}}(|1''\rangle - |5''\rangle) = \frac{1}{\sqrt{2}}(|1'\rangle - |5'\rangle) \\ |\beta\rangle &= \frac{1}{\sqrt{2}}(|1''\rangle + |5''\rangle) = \frac{1}{\sqrt{2}}\left(|1'\rangle + |5'\rangle + \frac{g\Omega_0}{2\delta^2}|3\rangle\right) \end{aligned} \quad (10)$$

Thus, the state $|\alpha\rangle$ is exactly dark with respect to the cavity mode at this point.

During adiabatic following, the system parameters must change slowly compared to the energy separation between these two eigenstates. More quantitatively, we can say that the rate of mixing between these two states, Q_{NA}, must be constrained by:

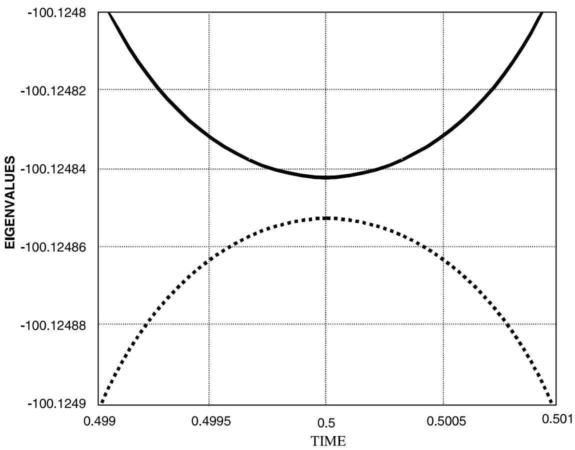

Fig. 4. Expanded view of the anti-crossing of the dressed states at the center. The separation at the center is about 1.2×10^{-5}, in close agreement with the analytical result of 1.25×10^{-5}, both expressed in units of $\hbar g$.

$$Q_{\rm NA} \equiv \left\langle \alpha \left| \frac{\partial}{\partial t} \right| \beta \right\rangle \leqslant \frac{\varepsilon}{\xi} \quad (11)$$

where ξ is the adiabaticity parameter. Typically, a value of $\xi = 10$ or greater ensures that the system will stay primarily in the state $|\alpha\rangle$ during the evolution. For example, in the case where ε is kept constant during the evolution, the adiabaticity constraint can be interpreted simply as the transit time broadening of the energy levels due to the finite time of interaction. The inverse of the transit time (which is characteristic of the rate of mixing between the dressed states) then must be less than the energy separation ε by the factor of ξ in order to assure that the levels do not get too close to each other. In the case at hand, however, ε varies with time. In order to minimize the time necessary for the adiabatic transfer, we adopt the method where the rate of change of the two Rabi frequencies are varied dynamically as the value of ε changes.

In order to constrain time variations of the two Rabi frequencies (for computational simplicity), we consider an equivalent model where the two laser beam profiles are fixed in time, and vary in space sinusoidally (or cosinusoidally), extending over a distance L: $\Omega_1(x) = \Omega_0 \sin(x\pi/2L)$, $\Omega_2(x) = \Omega_0 \cos(x\pi/2L)$. The atom plus cavity is then assumed to travel through the field profile, at a speed $v(t)$ that varies with time. Once the exact functional form of this time varying speed is determined, the total travel time T is found by inverting the relation:

$$L = \int_0^T \mathrm{d}t\, v(t) \quad (12)$$

In order to determine $v(t)$, we estimate first the non-adiabatic coupling rate, $Q_{\rm NA}$, using the explicit expressions for the eigenstates determined above. The resulting expression is quite cumbersome. In order to simplify further, we note first that:

$$\tan(2\theta) = \frac{G}{\Delta} = \eta \tan(\pi x/L); \quad \eta \equiv \frac{g^2}{2\delta^2} \quad (13)$$

Given that η is very small, we can identify two distinct zones during the adiabatic transfer. For a very small zone $L/2 - d/10 < x < L/2 + d/10$ (where $d = \eta L/\pi$) around the center ($x = L/2$), we have $G \gg \Delta \approx 0$, so that $\theta \approx \pi/4$ and $\varepsilon \approx G$. Once we get away from the center by a distance of more than $\pm 10d$ we have $\Delta \gg G$, so that $\varepsilon \approx \Delta$, $\cos(\theta) \approx 1$, and $\sin(\theta) \approx \theta \approx G/2\Delta$. The velocity in the intermediate zone can be estimated via interpolation. The resulting total time for adiabatic transfer is given by $T \approx \xi/9\varepsilon_{\rm min}$, where $\varepsilon_{\rm min}$ is the minimum separation between the energies of the eigenstates, given by $g^2\Omega_0^2/4\delta^3$, as determined before. For $\xi = 10$, we have verified via numerical methods that this value of T results in nearly perfect adiabatic transfer.

As an explicit example, consider the case where $\Omega_0 = g = \delta/3$, so that all our approximations remain valid. We then have $\varepsilon_{\rm min} \approx 10^{-2}$ g, and the time for adiabatic transfer is $T \approx 12\xi g^{-1}$. This is about an order of magnitude slower than the time needed for the Pellizari scheme. However, the effect of cavity decay, integrated over the transfer time, is now much smaller. Explicitly, the maximum population of the state $|3\rangle$ is about 0.4×10^{-4}, as compared to $1/3$ for the Pellizari case. The effective rate of decoherence due to cavity photon decay is thus reduced by nearly three orders of magnitude. Thus, the cavity dark state described here achieves the desired transfer of quantum information without being affected significantly by the cavity decay, and yet does not take much longer than the original Pellizari scheme.

To summarize, we have shown that two multilevel atoms can perform quantum communication with each other via interaction with an enclosing cavity containing virtually no photons at all times. The physical mechanism is analogous to the way populations can be exchanged between the extremal states in a three level system via adiabatic following, without populating the intermediate states. The combined system of the two atoms, the cavity, and two laser beams contains a dark state corresponding to the cavity in its ground state. Using a counter-intuitive pulse sequence, quantum information can be transferred adiabatically from one atom to the other via this cavity dark state. This process can be used to circumvent the effect of cavity decay in a quantum computer formed by interconnected qubits. Finally, it should be possible to generalize this model to other situations where a damped channel is used to couple stable systems.

Acknowledgements

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