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Coherent and incoherent beam combination using thick holographic substrates

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Abstract

We present a mathematical model of coherent and incoherent beam combination in a thick hologram. We also derive the formulae relating the read and write angles to the read and write wavelengths for the combiner. Furthermore, we present a new technique for determining the $M_{\#}$, and establish that the $M_{\#}$ required for a coherent combiner is substantially less than that needed for an incoherent one.

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1. Introduction

Lasers have rapidly found use in many industrial and consumer settings. A majority of the applications make use of small, reliable, inexpensive diode lasers. The cost of low power lasers has been drastically reduced, thereby increasing demand and applications. High power lasers, however, have not experienced the same cost reduction. Many new applications and improvements to existing technologies would be possible with less expensive higher power lasers. One way to deal with this shortage is to combine multiple low power diode lasers into a single beam [1–6]. Gratings in a thick hologram [7–10] can be of help in this regard. In particular, an incoherent holographic beam combiner (HBC) can be used to combine the output from many lasers into a singe-aperture, diffraction-limited beam. The beams can be combined coherently as well. A coherent HBC can be used as a multiport splitter/combiner in architectures where a master oscillator is first split into many copies, and recombined after amplification, for example. A coherent HBC is also a critical element in a super-parallel holographic

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optical correlator [11]. The principle behind the HBC is based on the storage of multiple holographic gratings in the same spatial location. Incoherent beam combination is particularly suited to thick holograms due to the narrow spectral selectivity of the gratings. In order to limit cross talk between an incident beam and other gratings, a minimum wavelength separation $\Delta\lambda$ is required. Other diffractive beam combiners, such as blazed gratings can typically achieve a $\Delta\lambda$ greater than 1.5 nm [1]. In contrast, a thick holographic plate can easily achieve $\Delta\lambda$ on the order of 0.05 nm [12]. The maximum number of lasers that can be combined for a given application is therefore potentially much larger.

The paper is organized as follows. In Section 2, we present an analysis of the coherent HBC via multi-wave mixing in a thick hologram. Here, we follow the model used by Kogelnik [13], although the results presented here represent a non-trivial extension of this work as well as other analyses of this type [14–17]. In Section 3, we extend this analysis to model the incoherent HBC. In Section 4, we formulate explicitly the constraints imposed on the read and write angles for the situation where the read and write wavelengths differ. In Section 5, we establish the values of the so-called $M_{\#}$ required for both type of combiners, and present a new technique for determining the value of the $M_{\#}$ [18–21].

2. Coherent beam combination

Fig. 1 illustrates the basic model used in this analysis. For simplicity we consider first the combination of two read beams incident on a hologram with two gratings. We will then extrapolate the result for N beams and N gratings. A further simplification is the assumption that the index of refraction is the same throughout the system. This model is used merely to simplify the notation. In reality the input and output angles would have to take into account refraction at the entrance and exit surfaces.

The scalar wave equation for this system can be written as

$$\nabla^2 E(x,z) + k^2 E(x,z) = 0, \tag{1}$$

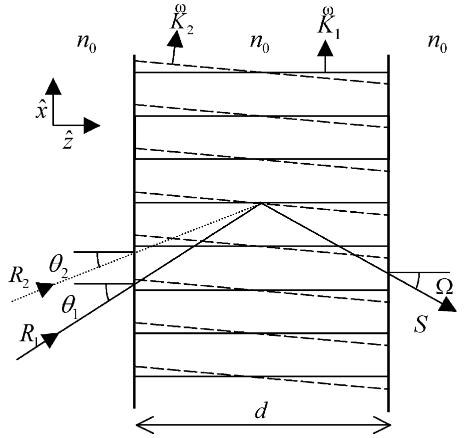

Fig. 1. Schematic illustration of the combination of two mutually coherent input beams into a single output beam.

where k is the wavenumber and E is the electric field describing all the oscillatory fields at the degenerate frequency, ω. Here, we assume the field to be TE polarized. An extension to TM or arbitrary polarization can be made in the same manner as illustrated in [13]. We also assume that the field does not vary in the y direction, and express the field as the sum of the input (R_1 and R_2) and the output (S) waves

$$E(x,z) = R_1(z)\mathrm{e}^{-\mathrm{i}\vec{\rho}_1\cdot\vec{x}} + R_2(z)\mathrm{e}^{-\mathrm{i}\vec{\rho}_2\cdot\vec{x}} + S(z)\mathrm{e}^{-\mathrm{i}\vec{\sigma}\cdot\vec{x}}. \tag{2}$$

The wavevectors for each of these beams are assumed to be of the same magnitude

$$\rho_1^2 = \rho_2^2 \equiv \beta^2, \tag{3}$$

$$\sigma^2 = \beta^2. \tag{4}$$

We also assume Bragg matching for each input beam

$$\vec{\sigma} = \vec{\rho}_1 - \vec{K}_1 = \vec{\rho}_2 - \vec{K}_2, \tag{5}$$

where $\vec{K}_1$ and $\vec{K}_2$ are the grating vectors.

Using phase matching terms, in the slowly varying envelope approximation (SVEA) [22], the wave equation reduces to

$$C_{R1}\frac{\partial R_1}{\partial z} = -\mathrm{i}\kappa_1 S, \tag{6}$$

$$C_{R2}\frac{\partial R_2}{\partial z} = -\mathrm{i}\kappa_2 S, \tag{7}$$

$$C_S\frac{\partial S}{\partial z} = -\mathrm{i}\kappa_1 R_1 - \mathrm{i}\kappa_2 R_2,$$

where

$$C_S = \cos\Omega, \tag{8}$$

$$C_{Rj} = \cos\theta_j, \tag{9}$$

$$\kappa_j = \frac{\pi n_j}{\lambda}, \quad j = 1,2. \tag{10}$$

Here, n_1 and n_2 are the amplitudes of the index modulations, defined by

$$n(x,z) = n_0 + n_1 \cos\vec{K}_1\cdot\vec{x} + n_2\cos\vec{K}_2\cdot\vec{x}. \tag{11}$$

Using boundary conditions that $R_1(0) = r_1$ (complex), $R_2(0) = r_2$ (complex), $S(0) = 0$, the solutions to these equations, evaluated at the exit surface, become

$$R_1(d) = \kappa_2 A + \kappa_1 C_{R2} B\cdot\cos(\alpha_0 d), \tag{12}$$

$$R_2(d) = \kappa_1 A + \kappa_2 C_{R1} B\cdot\cos(\alpha_0 d), \tag{13}$$

$$S(d) = -\mathrm{i}C\cdot\sin(\alpha_0 d), \tag{14}$$

where

$$A = \frac{C_{R1}\kappa_2 r_1 - C_{R2}\kappa_1 r_2}{C_{R1}\kappa_2^2 + C_{R2}\kappa_1^2}, \tag{15}$$

$$B = \frac{\kappa_1 r_1 + \kappa_2 r_2}{C_{R1}\kappa_2^2 + C_{R2}\kappa_1^2}, \tag{16}$$

$$C = (\kappa_1 r_1 + \kappa_2 r_2)\cdot\left(\frac{C_{R1}C_{R2}}{C_S(C_{R1}\kappa_2^2 + C_{R2}\kappa_1^2)}\right)^{1/2}, \tag{17}$$

$$\alpha_0 = \left(\frac{C_{R1}\kappa_2^2 + C_{R2}\kappa_1^2}{C_{R1}C_{R2}C_S}\right)^{1/2}. \tag{18}$$

These equations are formulated in a way such that the energy flow is conserved in the z-direction [13]

$$C_{R1}|R_1|^2 + C_{R2}|R_2|^2 = C_S|S|^2. \tag{19}$$

The intensity of the diffracted beam is $I_\mathrm{d} = F|S|^2$ where the obliquity factor, F, is given by

$$F = \frac{r_1^2 + r_2^2 - |R_1(d)|^2 - |R_2(d)|^2}{|S(d)|^2}. \tag{20}$$

The diffraction efficiency, η, is given by $\eta \equiv I_\mathrm{d}/(r_1^2 + r_2^2)$, where $\eta = 1$ corresponds to the situation where the intensity of R_1 and R_2 fall to zero at the output, indicating that all of the incident power has been transferred to the diffracted beams.

One possible condition for achieving $\eta = 1$ is $d = \pi/2\alpha_0$ and

$$\frac{r_1}{r_2} = \frac{\kappa_1}{\kappa_2}\cdot\frac{C_{R2}}{C_{R1}}. \tag{21}$$

For symmetry, one can then infer that the general condition for $\eta = 1$ with n input beams is

$$\frac{C_{R1}\cdot r_1}{\kappa_1} = \frac{C_{R2}\cdot r_2}{\kappa_2} = \frac{C_{R3}\cdot r_3}{\kappa_3} = \cdots = \frac{C_{Rn}\cdot r_n}{\kappa_n} \tag{22}$$

and

$$d = \frac{\pi}{2}\cdot\sqrt{C_S}\cdot\left[\sum_{j=1}^{n}\frac{\kappa_j^2}{C_{Rj}}\right]^{-1/2} \tag{23}$$

The results show that it is possible to combine N mutually coherent beams with 100% efficiency, provided some constraints are met. The most important requirement is that all of the input beams be in phase at the interface. We also note that the individual index modulation necessary for achieving 100% diffraction for N beams is less than the modulation necessary to produce 100% diffraction of a single beam, as illustrated in Section 5. In the next section we investigate how this requirement differs for combining mutually incoherent beams [23].

3. Combining mutually incoherent beams

Fig. 2 illustrates the case for combining mutually incoherent beams. The equations here are considerably simpler, so that we can deal with the N beams explicitly. As before, we begin with the wave equation

$$\nabla^2 E(x,z) + k^2 E(x,z) = 0, \tag{24}$$

$$E(x,z) = \sum_{j=1}^{N}\left(R_j(z)\mathrm{e}^{-\mathrm{i}\vec{\rho}_j\cdot\vec{x}} + S_j(z)\mathrm{e}^{-\mathrm{i}\vec{\sigma}_j\cdot\vec{x}}\right). \tag{25}$$

We assume Bragg matching for each beam, so that $\vec{\sigma}_1 = \vec{\sigma}_2 = \cdots = \vec{\sigma}_j \equiv \vec{\sigma}$, $\rho_j^2 = \sigma^2 = \beta^2$, and $\vec{\sigma} = \vec{\rho}_j - \vec{K}_j$. In general, the wave mixing equations are *not* mutually independent. This can be seen easily by considering the following physical situation. Consider, for example, the diffraction of input beam R_1. It scatters off grating K_1, producing an output beam in the direction of the bold arrow. As

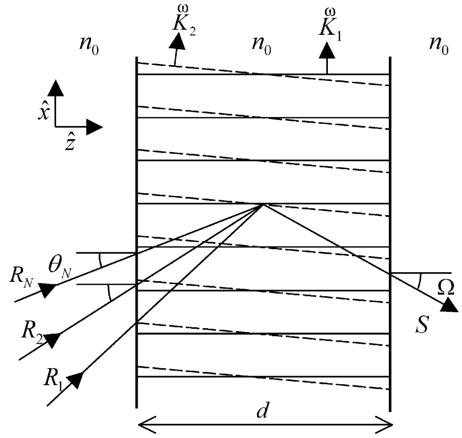

Fig. 2. Schematic illustration of the combination of N mutually incoherent input beams into a single output beam.

soon as this beam acquires a non-vanishing amplitude, it will then diffract off gratings $K_2, K_3, \ldots, K_N$, producing additional beams in the direction parallel to beams $R_2, R_3, \ldots, R_N$, respectively. This difficulty can be solved by requiring that each beam differs in wavelength from its neighbors by at least an amount $\Delta\lambda_{\mathrm{CS}}$ big enough to suppress this undesired coupling. In order to determine the minimum value of the channel spacing, $\Delta\lambda_{\mathrm{CS}}$, we recall from Kogelnik [13] that the full-width half-maximum (FWHM) wavelength selectivity of transmission holograms is given by

$$\Delta\lambda_{\mathrm{FWHM}} \approx \mathrm{Cot}(\theta) * (\Lambda/d) * \lambda, \tag{26}$$

where θ is the Bragg angle, Λ is the grating periodicity, d is the sample thickness, and λ is the read wavelength. (This equation is true for the case where the input and output beams are symmetric. In Fig. 2, this applies strictly to beam 1 only; however, if the other beams are close to this beam, then the deviation from this result is negligible.) Consider, for example, the case where the first grating is written at $\pm 30°$, using a 514 nm source. The value of $\Delta\lambda_{\mathrm{FWHM}}$ for a read wavelength of 980 nm, for example, is then 0.05 nm for $d = 2$ mm. In the rest of this discussion, we assume that $\Delta\lambda_{\mathrm{CS}} > \Delta\lambda_{\mathrm{FWHM}}$ [24].

As such, we can now assume that the wave equations are mutually independent, so that for each input beam ($j = 1, 2, \ldots, N$) we can write $C_{Rj}\partial R_j/\partial z = -\mathrm{i}\kappa_j S_j$, $C_S \partial S_j/\partial z = -\mathrm{i}\kappa_j R_j$ where

$$C_{Rj} = \cos\theta_j, \tag{27}$$

$$C_S = \cos\Omega, \tag{28}$$

$$\kappa_j = \frac{\pi n_j}{\lambda}, \tag{29}$$

$$n(x,z) = n_0 + \sum_{j=1}^{N} n_j \cdot \cos\vec{K}_j \cdot \vec{x}, \tag{30}$$

Using the boundary conditions $R_j(0) = r_j$, $S_j(0) = 0$ we determine the amplitudes of the fields at the output

$$R_j(d) = r_j \cos(\alpha_j d), \tag{31}$$

$$S_j(d) = -\mathrm{i} r_j \sqrt{\frac{C_{Rj}}{C_S}} \cdot \sin(\alpha_j d), \tag{32}$$

$$\alpha_j(d) = \kappa_j \cdot \sqrt{\frac{1}{C_{Rj} C_S}}. \tag{33}$$

The intensity in the jth diffracted beam is $I_{\mathrm{d}j} = F_j |S_j \cdot (d)|^2$ where the obliquity factor F_j, is given by

$$F_j = \frac{r_j^2 - |R_j(d)|^2}{|S_j(d)|^2} = \left|\frac{C_S}{C_{Rj}}\right|. \tag{34}$$

The diffraction efficiency, η_j, is given by $\eta_j = I_{\mathrm{d}}/|r_j|^2 = \sin^2(\alpha_j \cdot d)$ The total diffraction efficiency is given by

$$\eta = \sum_{j=1}^{N} \frac{\eta_j}{N}. \tag{35}$$

The condition for achieving a 100% efficient beam combination is $\eta = N$ which requires $\eta_j = 1$ for each j. This means we must have $\alpha_j = (\pi/2) \cdot (1/d)$ for each j, which in turn requires

$$\frac{\kappa_j^2}{C_{Rj}} = \frac{\pi^2 \cdot C_S}{4d^2}. \tag{36}$$

The value of C_S is determined from the Bragg matching condition

$$C_S = \cos(\theta_j) - \frac{\vec{K}_j \cdot \vec{z}}{\beta}, \tag{37}$$

which, by construction, is the same for each j. The condition for 100% diffraction efficiency for N beams is therefore the same as that for 100% diffraction efficiency for each beam individually.

4. Differing write and read wavelengths

Many holographic materials, though efficient in reading at many wavelengths, are particularly sensitive to a narrow band of wavelengths during writing. Therefore one may need to write and read holograms at different wavelengths. The following parameters are assumed to be given as inputs: the values of the read wavelengths λ_{R1} and λ_{R2}, the write wavelength λ_W, the mean index of the holographic substrate, the thickness thereof, and the amplitude of the index modulation for each grating. The equations are used to determine the following output parameters: the angles θ_{W1} and θ_{W2} (for both reference and object beams) used for writing the holograms, and the angles at which the lasers at λ_{R1} and λ_{R2} have to be applied to the beam combiner (i.e., the Bragg angles). The desired output angle θ_S of the combined beam can be treated as a given input or as an output to be determined depending on the desired degree of cross-talk suppression. This is because the value of $\Delta\lambda_{FWHM}$ depends on this angle, in addition to other parameters that are assumed to be given [13,25]. In the following discussion we show the relationship between the write angles and read angles when different wavelength lasers are employed for each step.

Fig. 3 illustrates the basic writing geometry. Consider the process for writing the first grating, using beams W_1 (reference) and W_2 (object) with laser beams of wavelength λ_W (e.g. 514.5 nm). We choose the first read wavelength λ_{R1} (e.g. 980 nm) and the desired angle of diffraction θ_S. These two choices plus the chosen writing wavelength will determine the first two writing angles θ_{W1} and θ_{W2}. If read by a laser beam also at λ_W, the read beam will diffract efficiently only if it is Bragg matched, i.e., incident at the same angle as W_2, and will produce a diffracted beam on the other side parallel to the reference beam W_1. When read by laser beam O_1 at λ_{R1} the Bragg incidence angle as well as the diffracted angle θ_S is larger.

Consider next the process for writing the second grating, using a new pair of beams with λ_W (W'_1 and W'_2 from Fig. 3). Our goal is to choose the incident angles for these two beams such that when this hologram is read by a laser beam O_2 at the second read wavelength of λ_{R2} the diffracted beam will emerge at the *same* angle θ_S. We choose the angular distance between the first and the second read beams δ (see Fig. 4), as well as the wavelength of the second read beam, O_2. These constraints yield a new pair of writing angles, θ'_{W1} and θ'_{W2}, for the beams W'_1 and W'_2, respectively, in Fig. 3. Explicit analysis shows that these two angles are given by

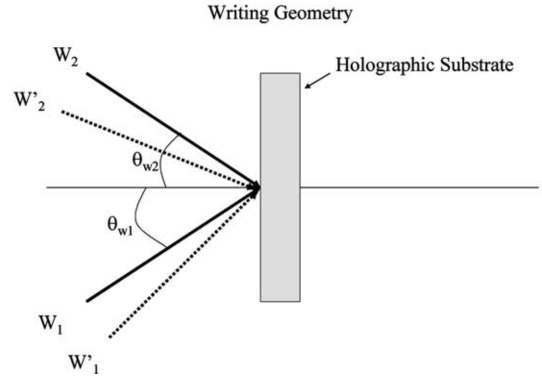

Fig. 3. Schematic illustration of the geometry for writing two holograms at 514.5 nm. The angles of the writing beams are chosen to ensure that when the holograms are read by lasers at 980 nm, the output beams overlap.

$$\theta'_{W1} = \sin^{-1}\left[n_W \cdot \sin\left\{\sin^{-1}\left[\frac{n_R}{n_W}\cdot\frac{\lambda_W}{\lambda_R} \cdot \sin\left(\tilde{\theta}_S ... + \tilde{\delta}/2\right)\right] - \tilde{\delta}/2\right\}\right], \quad (38)$$

$$\theta'_{W2} = \sin^{-1}\left[n_W \cdot \sin\left\{\sin^{-1}\left[\frac{n_R}{n_W}\cdot\frac{\lambda_W}{\lambda_R} \cdot \sin\left(\tilde{\theta}_S ... + \tilde{\delta}/2\right)\right] + \tilde{\delta}/2\right\}\right], \quad (39)$$

where we have defined

$$\tilde{\theta}_S = \sin^{-1}\left(\frac{\sin\theta_S}{n_R}\right),$$

$$\tilde{\delta} = \sin^{-1}\left(\frac{\sin(\theta_S+\delta)}{n_R}\right) - \sin^{-1}\left(\frac{\sin\theta_S}{n_R}\right),$$

$$\delta = \theta_{Oj} - \theta_{O(j+1)}$$

$n_W \equiv$ index at the writing wavelength,

$n_R \equiv$ index at reading wavelength,

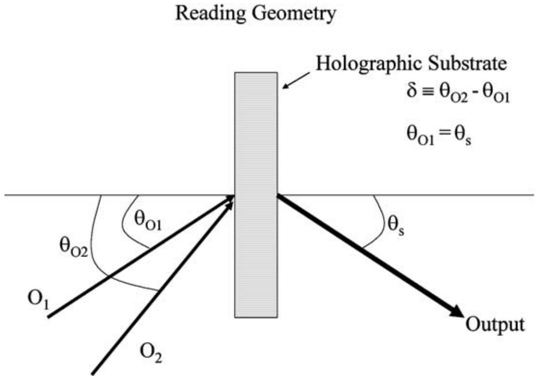

Fig. 4. Illustration of the geometry for reading the two holograms, the first one (O_1) at 980 nm and the second (O_2) at 980 nm + $\Delta\lambda$. Both beams will be diffracted at the same exit angle, θ_S.

$\lambda_W \equiv$ the writing wavelength,

$\lambda_R \equiv$ the reading wavelength.

We point out that these equations take into account the effect of holographic magnification when the read wavelength is longer than the writing wavelength and the effect of potentially different indices of refraction at the read and write wavelengths.

One of the key issues for an HBC is the maximum number of beams that can be combined efficiently. This is determined by the so-called $M_\#$ of the medium. In the next section, we discuss the actual constraint for both coherent and incoherent combining.

5. $M_\#$ for coherent and incoherent combining

Consider a situation where a single holographic substrate has an average index of n_0, before any grating is written. The dynamic range of such a medium is characterized by the so-called modulation depth, defined as the amplitude n' of a single sinusoidal variation (e.g., in the x-direction) of this index. The maximum achievable amplitude of n' is denoted n'_{sat}. After a single grating is written, the index becomes a function of x, and can be written as

$$n(x) = n_0 + n' \sin(kx); n' \leqslant n'_{sat}, \tag{40}$$

where k represents the wavenumber of the grating. When this grating is illuminated by a laser beam of wavelength λ, the diffraction efficiency (ignoring Fresnel reflection, which can be eliminated by antireflection coating, for example, or can be taken into account in the definition) is defined to be

$$\eta \equiv \frac{I_d}{I_0}, \quad \eta \leqslant 1, \tag{41}$$

where I_0 is the input intensity, and I_d is the diffracted intensity. The value of η is maximized (at η_m) when the angle of incidence and the input wavelength satisfy the Bragg matching condition. According to the wave-mixing (i.e., non-perturbative) theory of such a grating, the value of η_m (bounded by unity by definition) varies sinusoidally in the following manner: $\eta_m = \sin^2(n'\xi\lambda d)$, where d is the thickness of the substrate and ξ is a constant determined by the orientation of the grating [13]. This expression can be rewritten as

$$\eta_m = \sin^2\left(\frac{\pi n'}{2n_c}\right), \quad n_c \equiv \frac{\pi}{2\xi\lambda d}, \tag{42}$$

where n_c represents a characteristic scale for the index modulation amplitude. The diffraction efficiency reaches the largest possible value – unity – when the amplitude of the index modulation is an odd-integer multiple of n_c:

$$\eta_m = 1 \text{ when } n' = (2M+1)n_c, \quad M = 1, 2, 3, \ldots \tag{43}$$

This is illustrated in Fig. 5. It is important to note here that η_m does not increase monotonically with the grating amplitude, n'.

In a given substrate, the deepest grating that can be formed is determined by the material parameters. In general, for most material, the maximum achievable density, ρ_m, of the photo-sensitive ingredient limits the value of n'_{sat}. The parameter $M_\#$ can be defined simply as

$$M_\# \equiv \frac{\pi}{2}\frac{n'_{sat}}{n_c}. \tag{44}$$

This definition is also illustrated in Fig. 5, which shows the value n'_{sat} indicated by the dotted vertical line. This case corresponds to $M_\# \approx (\pi/2)$ 6.75. Thus in general, $M_\#$ represents the double of the number of full oscillations achievable on the

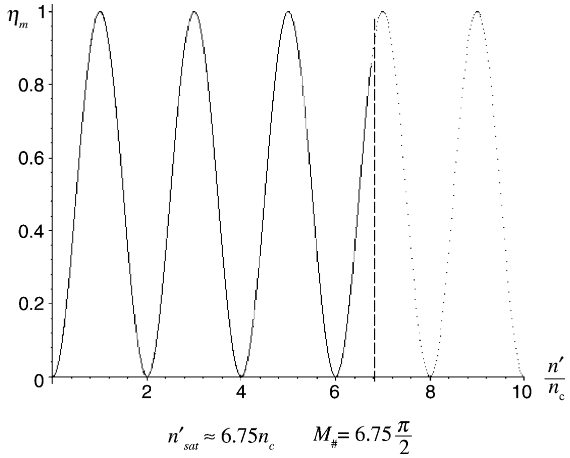

Fig. 5. Schematic illustration of the functional dependence of the optimal (i.e., Bragg-matched diffraction efficiency) as a function of the index modulation amplitude and $M_{\#}$. See text for details.

η_m vs. n'/n_c curve. According to this definition, it follows that the value of $M_{\#}$ is linearly proportional to the density ρ of the photo-sensitive agent, and the thickness d of the substrate: $M_{\#} \propto \rho d$. In practice, both ρ and d are limited by practical considerations, such as the requirement of volume homogeneity and low scattering, thus limiting the maximum value of $M_{\#}$ for a given type of material.

In order to illustrate further the meaning of this definition, consider for example a situation where $M_{\#} = (\pi/2)N$, so that for a single grating one could achieve $n' = Nn_c$ (yielding a null value of the diffraction efficiency if N is even, and a unity value if N is odd). This would use up all the active photo-sensitive elements, by definition. However, one could choose to write N independent grating (superimposed, but with orthogonal Bragg conditions), each using only a $(1/N)$th of the active elements. Therefore, for each grating we would have $n' = n_c$, and the diffraction efficiency of each grating would be unity. As such, the $M_{\#}$ can be viewed as the maximum number of unity efficiency, orthogonal, superimposed gratings one can write in a single substrate.

The plot shown in Fig. 5 is useful for illustrating the concept of the $M_{\#}$. However, since it is difficult to measure n' directly, such a plot can not be generated easily. Instead, one can determine $M_{\#}$ indirectly by writing many weak holograms [19,20]. Here, we first review (for comparison) this approach, and then offer a new method which is potentially simpler under certain circumstances.

Consider a situation where N identical gratings are written in a single substrate. The gratings are superimposed on one another, but are also orthogonal to one another. The orthogonality means that if we illuminate the grating at a given fixed angle and tune the input frequency, only one grating will diffract at a particular frequency. That is, there will be a unique frequency corresponding to each grating. Assuming further that each grating is very weak, one can express the diffraction efficiency of each grating as $\eta_{m,0} \approx (\pi^2/4)(n'_0/n_c)^2$ where n'_0 is the amplitude of *each* grating. Let us now define

$$M_{\#\text{used}} \equiv (Nn'/n_c). \tag{45}$$

Note that $M_{\#\text{used}}$ represents the fact that the N gratings have potentially *used* up only a part of the total amount of active elements in the medium: $Nn'_0 \leqslant n'_m$. This in turn implies that $M_{\#\text{used}} \leqslant M_{\#}$. The efficiency of each grating can now be expressed as $\eta_{m,0} \approx (\pi^2/4)(M_{\#\text{used}}/N)^2$ so that

$$M_{\#\text{used}} \approx N * (\pi/2)\sqrt{\eta_{m,0}}. \tag{46}$$

More generally, even if the diffraction efficiency of each grating is not the same as the others, but is small, we can show that

$$M_{\#\text{used}} \approx \frac{\pi}{2}\sum_{i=1}^{N}\sqrt{\eta_{m,i}}. \tag{47}$$

Experimentally, one can make N very large, so that the very last grating written has a vanishing diffraction efficiency. Under this condition, we get

$$M_{\#} \approx M_{\#\text{used}} \approx \frac{\pi}{2}\sum_{i=1}^{N}\sqrt{\eta_{m,i}}. \tag{48}$$

One of the problems with this approach is that one has to write many holograms on the same spot, and assume that there is no grating washout. This assumption may not always be valid. We now discuss a new approach that does not depend on this assumption.

Consider the process of writing a grating, with a light exposure W. In general, the amplitude of the resulting grating, n', may be modeled to depend on W in the following manner [26]:

$$n' = n'_{sat}[1 - \exp(-\beta W)], \quad (49)$$

where β is a constant determined by the sensitivity of the medium. This model is based on the physical picture that the hologram writing in most cases can be viewed as an optical pumping process whereby the active elements are transformed from one stable configuration to another stable one. In the limit of small $W\beta$, one can express this relation as

$$n' \approx n'_{sat}(\beta W - \beta^2 W^2). \quad (50)$$

Normalizing both sides by n_c, and using Eq. (44), we get

$$\frac{n'}{n_c} \approx \frac{M_\#}{\pi/2}(\beta W - \beta^2 W^2). \quad (51)$$

In this limit, one can also approximate the Bragg-matched diffraction efficiency of Eq. (42) by $\eta_m \approx (\pi^2/4)(n'/n_c)^2$. Combining, we get

$$\eta_m \approx M_\#^2(\beta^2 W^2 - 2\beta^3 W^3). \quad (52)$$

Consider next the following quantities:

$$S_1 \equiv \lim_{W\to 0}\left(\frac{\partial^2 \eta_m}{\partial W^2}\right), \quad S_2 \equiv \lim_{W\to 0}\left(\frac{\partial^3 \eta_m}{\partial W^3}\right). \quad (53)$$

It is easy to show that $\beta = -(S_2/6S_1)$ and

$$M_\# = \sqrt{\frac{18S_1^3}{S_2^2}}. \quad (54)$$

Consider now a situation where we have a large set of identically prepared substrates (or, equivalently, a large set of areas on a single substrate, each of which can be used for writing a different grating). We now write a single grating in each substrate, with increasing duration of exposure, W, to the writing beams. The diffraction efficiency of each of these gratings can be measured experimentally, generating the function $\eta_m(W)$. The parameters in Eq. (53) can now be determined directly, which in turn can be used to compute the $M_\#$ by using Eq. (54).

As mentioned above, an incoherent $1 \times N$ HBC corresponds to writing N superimposed gratings such that: (i) for a fixed input angle, there is a unique frequency that will diffract efficiently from a given grating, and (ii) the diffraction efficiency of each grating is close to unity. In this case, it then follows immediately that the material must satisfy the following requirement:

$$M_\# \geqslant N. \quad (55)$$

For the coherent HBC, the gratings are not orthogonal in the same sense as in the section above. Rather, in this case, the input beam diffracts efficiently from each of the N gratings simultaneously. As such, the diffraction efficiency of each grating only has to have a maximum value of $1/N$, which is much less than unity for a large N. As shown above, Eq. (48) still applies, so that the required constraint is simply

$$M_\# \geqslant \frac{\pi}{2}\sqrt{N}. \quad (56)$$

Therefore, the requirement of a high $M_\#$ is substantially relaxed for coherent combining. One can also call upon reciprocity to see this result. When used in reverse, this same volume hologram should take a single coherent input beam and split it equally into N components, each with $1/N$th the power and thus with an individual diffraction efficiency of $1/N$.

6. Conclusion

We have presented a mathematical model that reveals the critical parameters for high efficiency beam combining volume holograms for coherent and incoherent source beams. We have also derived the explicit equations relating read and write angles for these combiners. In addition, we have shown that the $M_\#$ requirement for an N beam coherent combiner is substantially less than that needed for an N beam incoherent combiner.

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[23] Absorption at the readout wavelength has not been considered here or in Section 3, and could potentially be an important issue. The current analysis is valid for many of the materials that are currently used for writing thick holograms, provided that the users restrict the sample thickness so that absorption remains negligible.
[24] In general, $\Delta\lambda_{CS}$ must be such that the peak of one channel coincides with the null of the sinc-like diffraction efficiency of neighboring channels (see [13]). The sinc-square-like nature of the diffraction efficiency is attributable to the fact that it is related to the Fourier transform of the index profile (this is the reason why the Bragg selectivity is inversely proportional to the thickness). In principle, the sinc-like behavior can be eliminated by a well-known process called apodization. In this process, the dye concentration is rendered smooth, for example, via pre-exposure to incoherent radiation, so that the concentration is at a maximum at the center, but vanishes at the two surfaces. The Fourier transform of this smooth profile is then another smooth – apodized – profile. In general, it is necessary to have a $\Delta\lambda_{CS}$ larger than Q times $\Delta\lambda_{FWHM}$, where the value of Q depends on the degree to which one wants to avoid potential cross talks. For an apodized medium with a Gaussian profile for diffraction efficiency, for example, the degree of cross-talk falls of approximately as $\exp(-Q^2)$, so that a value of Q greater than 2 would reduce the cross talk to about 1%, and bigger value of Q can be used for greater suppression. For an unapodized system, the degree of cross talk falls off as at least Q^{-2}. The best suppression is achieved by a more careful choice of Q, so that the peak of one channel coincides with a null of the sinc-squared-like diffraction efficiency of the nearest channels.
[25] As discussed in detail in [24], the degree of this suppression is determined by the factor we have called Q, which is the ratio of $\Delta\lambda_{FWHM}$ and $\Delta\lambda_{CS}$. (note that $\Delta\lambda_{CS}$ is the difference between λ_{R1} and λ_{R2}, and is therefore to be treated as an input parameter). There are two distinct regimes: (I) Q has to match or exceed a threshold value, or (II) Q has to have a precise value. For regime I, this means that $\Delta\lambda_{FWHM}$ has to be smaller than a threshold value, which in turn means that the output angle has an allowed range. If the user has chosen a value for the output angle, than he/she must check to see if the value is within the allowed range. If not, the user then either chooses a different value of Q, or a different value of the output angle. For regime II, Q has a discreet value, which in turn means that $\Delta\lambda_{FWHM}$ has to have a distinct value. This in turn implies that the output angle has to have a distinct value, making it an output parameter.
[26] For holographic materials with a recording curve that does not fit the model used in the paper, one can determine the necessary derivatives used in Eqs. (52)–(54) by experimentally determining the functional dependence of the recording curve on the exposure intensity and duration.