# Demonstration of a simple technique for determining the $M/\#$ of a holographic substrate by use of a single exposure

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We propose and demonstrate a simple technique for determining the $M/\#$ parameter of a holographic recording material. In this method, divergent object and reference beams are used to produce a spatially varying index modulation. One can analyze the resultant diffraction pattern to find $M/\#$ by using only a single grating; existing techniques require many gratings. © 2004 Optical Society of America

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The dynamic range of a holographic medium is an important parameter for determining storage density and diffraction efficiency of holographic memory systems and holographic beam combiners.[1–4] In these applications, many holographic gratings are multiplexed in the medium at the same spatial location. $M/\#$ is a parameter that defines the dynamic range of the holographic medium; it is essentially $\pi/2$ times the ratio of the maximum achievable index modulation and the index modulation that corresponds to a unity diffraction efficiency grating. The existing techniques[4–8] for measuring $M/\#$ require one to write many holograms on the material. Here we discuss a technique to determine $M/\#$ for a holographic recording material that is potentially simpler, and we present simulated and experimental results for a photopolymer-based holographic recording medium.

Typically, illumination of a holographic substrate with a spatially periodic, sinusoidal intensity pattern produces a periodic index modulation $n(x) \equiv n_0 + n' \cos(Kx)$, where $n_0$ is the spatially averaged index of refraction of the medium, $n'$ is the index modulation depth, and $K$ represents the wave number of the grating. When a laser beam of wavelength $\lambda$ illuminates this grating at the Bragg angle, diffraction efficiency $\eta$ is given by[9]

$$\eta = \frac{I_d}{I_0} = \sin^2\left(\frac{\pi n' \alpha d}{\lambda}\right), \tag{1}$$

where $I_0$ is the input intensity, $I_d$ is the diffracted intensity, $d$ is the thickness of the substrate, and $\alpha$ is the obliquity factor determined by the orientation of the grating. A characteristic scale for the index modulation is $n_c \equiv \lambda/(2\alpha d)$, so $\eta$ becomes

$$\eta = \sin^2\left(\frac{\pi}{2}\frac{n'}{n_c}\right), \qquad \eta = 1\,, \quad n' = n_c. \tag{2}$$

In many situations the modulation depth[4] can be modeled as

$$n' = n_m\left[1 - \exp\left(-\frac{t}{\tilde{\tau}}\right)\right], \tag{3a}$$

where $t$ is the exposure time, $\tilde{\tau}$ is a time constant that depends on the material sensitivity and the intensity of the writing laser beams, and $n_m$ is the maximum index modulation. A convenient way to quantify the value of $n_m$ is through the use of $M/\#$, which can be defined as $M/\# = (\pi n_m)/(2n_c)$. For notational convenience we define a scaled version of this expression: $Q \equiv n_m/n_c$, such that $M/\# = (\pi/2)Q$. When $Q$ is an integer, it represents essentially the maximum number of orthogonal, unit diffraction efficiency gratings that can ideally be written in a given spatial location.

Consider a situation in which $N$ equalized diffraction efficiency gratings are multiplexed on a single substrate by use of the Bragg (angle or wavelength) orthogonality condition. For $N \gg Q$ the diffraction efficiency for each grating can be approximated by $\eta \cong (\pi^2/4)(Q/N)^2$.[4,5] More generally, if the diffraction efficiencies of the gratings are not identical, it is possible to define and measure $M/\#$ of the material from the relation[4,5]

$$Q \approx \sum_{i=1}^{N} \sqrt{\eta_i}\,, \qquad \eta_i \ll 1\,. \tag{4}$$

Although one can measure $M/\#$ by using one exposure in certain cases,[5,7] in general to measure $M/\#$ by this approach may require one to write many holograms. As an alternative method, one can also use the fact that the diffraction efficiency of a single grating in the small index-modulation limit is a quadratic function (to first order) of the exposure time, described by

$$\eta(t) \approx Q^2\left(\frac{t^2}{\tilde{\tau}^2}\right)\left(\frac{\pi^2}{4}\right), \tag{5}$$

which follows directly from Eqs. (2), (3a), and (3b). $M/\#$ can thus be determined from the curve that defines the diffraction efficiency of a single hologram as a function of exposure time. This method also requires recording many successive holograms with different exposures on the holographic substrate.[4]

In this Letter we offer a potentially simpler approach to determining $M/\#$ from a single recording on the holographic medium. To illustrate this method we first combine Eqs. (2) and (3a) to express the diffraction efficiency as a function of time:

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$$\eta(t) = \sin^2\left\{\frac{\pi}{2}Q\left[1 - \exp\left(-\frac{t}{\tilde{\tau}}\right)\right]\right\}. \tag{6a}$$

Now, according to the generalized optical pumping model,[4] the saturation rate ($\tilde{\tau}^{-1}$) depends linearly on the intensity of the radiation for writing the grating: $\tilde{\tau}^{-1} = \beta\tilde{I}$, where $\beta$ is the sensitivity of the medium and $\tilde{I}$ is the amplitude of the intensity modulation, defined as

$$I \equiv \tilde{I}[1 + \cos(K_G x)], \tag{6b}$$

where $K_G$ is the grating vector. For typical values of $\tilde{I}$ used, the value of $\beta$ can be assumed to be a constant. If the value of $\tilde{I}$ depends on position $\mathbf{r}$ as well, we can write

$$\eta(t, \mathbf{r}) = \sin^2\left(\frac{\pi}{2}Q\{1 - \exp[-\beta\tilde{I}(\mathbf{r})t]\}\right). \tag{7}$$

Specifically, let us consider a situation when two equal-intensity, coherent Gaussian beams write a grating in a holographic medium. The intensity distribution will be

$$I(\mathbf{r}) = 2I_0 \exp(-2r^2/\omega_0{}^2)[1 + \cos(\mathbf{K}_G \cdot \mathbf{r})],$$
$$\mathbf{K}_G = \mathbf{K}_1 - \mathbf{K}_2, \tag{8}$$

where $\mathbf{K}_1$ and $\mathbf{K}_2$ are the propagation wave vectors, $I_0$ is the intensity at the center of each beam, and $\omega_0$ is the Gaussian beam radius of each writing beam. Comparing Eq. (8) with Eq. (6b), we find that $\tilde{I} = 2I_0 \exp(-2r^2/\omega_0{}^2)$. When this expression is used in Eq. (7), the resultant diffraction efficiency is given by

$$\eta(t, \mathbf{r}) = \sin^2\left(\frac{\pi}{2}Q\left\{1 - \exp\left[-f(r)\frac{t}{\tau}\right]\right\}\right),$$
$$0 \le f(r) \le 1, \tag{9}$$

where $f(r) = \exp(-2r^2/\omega_0{}^2)$ and $\tau = 1/2\beta I_0$. Across the spatial profile of the writing beams, the value of $f(r)$ varies from 1 in the center for $r = 0$ to a value of 0 for $r \gg \omega_0$.

Now, if $t/\tau \approx 5$, for example, then, at $r = 0$, $\exp[-f(r)t/\tau]$ approaches zero. However, for $r >> \omega_0$, $f(r) << 1/5$ and $1 - \exp[-f(r)t/\tau]$ approaches unity. This argument holds for larger values of $t/\tau$ as well. Thus, for $t/\tau \ge 5$, the quantity $1 - \exp[-f(r)t/\tau]$ varies monotonically from one to zero. Therefore the total number of circular fringes is of the order of $Q/2$ for $t/\tau \ge 5$. Accordingly, we note that for the proper exposure time one can be sure to observe the full number of fringes. To be more precise, let us express $Q$ as follows:

$$Q = 2m + n + \alpha, \qquad \alpha < 1, \quad n = 0, 1. \tag{10}$$

In this notation $\alpha$ is the fractional part of $Q$ and $n$ determines whether $Q$ is odd or even. Consider first the case $n = 0$ and $\alpha = 0$. In this case, for $t/\tau \ge 5$ the number of full circular fringes equals $m$ with a null at the center. Consider next the situation in which $n = 1$ and $\alpha = 0$. In this case the number of full circular fringes will still be $m$, but there will be a peak at the center. Finally, for $\alpha \ne 0$ the efficiency at the center will have a dip if $n = 1$ and a peak if $n = 0$. The actual value of the efficiency at the center reveals the value of $\alpha$.

We studied the phenomenon by using simulations. Figure 1 shows the result for dependence of diffraction efficiency on exposure time for an even-$Q$ value material with a plane-wave readout. This result shows

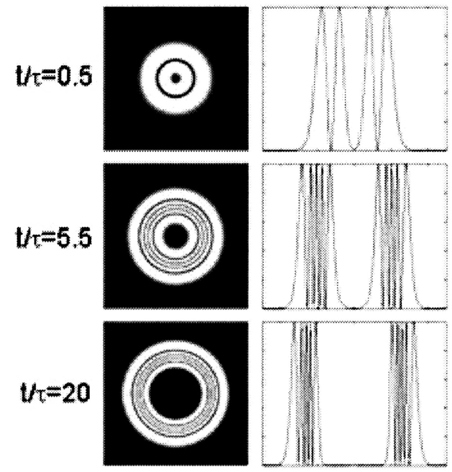


Fig. 1. Result of simulation showing the evolution of the diffracted pattern as a function of holographic exposure for an even-$Q$ [$m = 5$, $n = 0$, $\alpha = 0$, in Eq. (10)] value material with a plane-wave readout beam. Normalized diffraction efficiency is plotted versus radial distance.

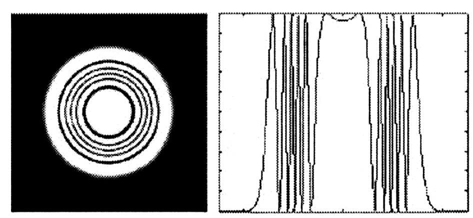

Fig. 2. Result of simulation for the diffraction pattern for fractional $Q$ with a plane-wave readout beam [$m = 5$, $n = 0$, $\alpha = 0.2$ in Eq. (10)]. Normalized diffraction efficiency is plotted versus radial distance.

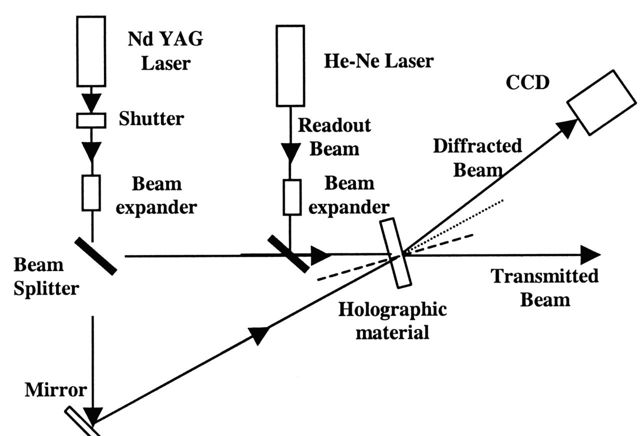


Fig. 3. Hologram writing and readout geometry.

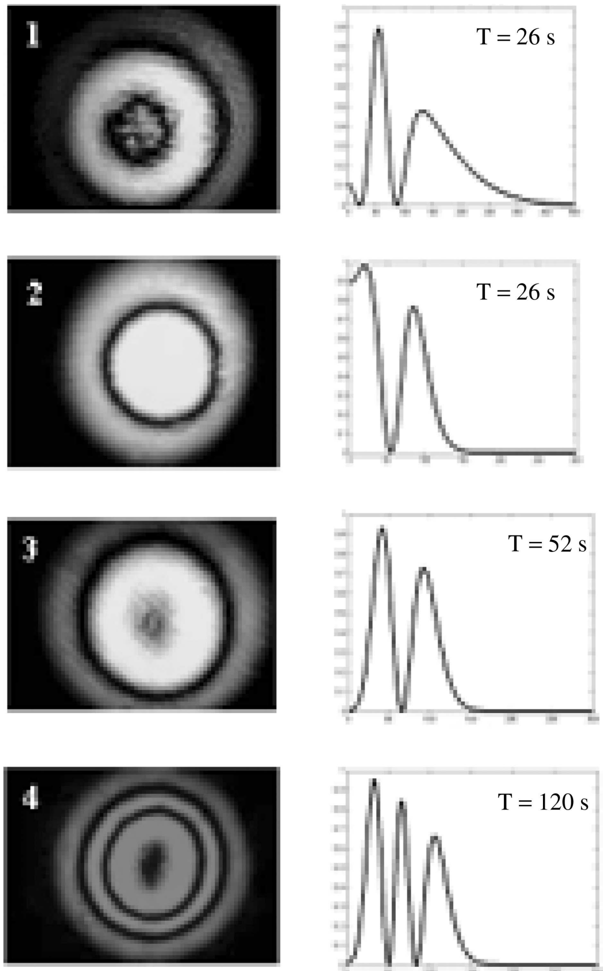


Fig. 4. Experimentally observed diffraction patterns. As one reaches the optimum limit for holographic exposure, the number of fringes that are visible in the diffracted beam reaches a maximum. Exposure time $T$ is labeled in the top right corner of eadch graph. All images are diffracted beams except for image 1, which is the transmitted beam for $T = 26$ s.

the expected dark center for an even-$Q$ material. Figure 2 shows the simulation result for diffraction for a material with an odd-$Q$ value plus a fractional part. As expected, there is an intensity peak at the center of the diffraction pattern and a dip that is due to the fractional part. The value at the center yields the value of fractional $\alpha$ as 0.2, resulting in a $Q$ value of 11.2, which corresponds to an $M/\#$ value of 17.584.

To show the principle of operation experimentally, we used a dye-doped polymer material called Memplex.[10] This material has a $Q$ value of 6, as claimed by the manufacturer. Figure 3 shows the combined setup for hologram writing and readout. Writing was done with a frequency-doubled Nd:YAG laser ($\lambda = 532$ nm), and readout was performed with a He–Ne laser operating at 632.8 nm. This material required baking after holographic exposure. During this experiment, the exposure times were gradually increased. After exposure, the material was baked until the number of observable interference fringes reached maximum. Figure 4 shows the results for a series of exposures for the holographic substrate. It shows that, as we reach the optimum limit for holographic exposure, the number of interference fringes visible in the diffracted beam reaches a maximum (three in this case). Thus the $Q$ for our material is $\sim 6$. This value of $Q$ yields an $M/\#$ of 9.42 for the material.

We have proposed and demonstrated a simple approach to determining the parameter $M/\#$ for any holographic recording material. This easy-to-use technique will be attractive for holographic data storage when *a priori* knowledge about the storage material is valuable in determining the storage density and the recording schedule for the holograms.

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