Polarization of holographic grating diffraction. II. Experiment

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Received July 16, 2003; revised manuscript received October 21, 2003; accepted November 20, 2003

The transmittance, ellipsometric parameters, and depolarization of transmission, diffraction, and reflection of two volume holographic gratings (VHGs) are measured at a wavelength of 632.8 nm. The measured data are in good agreement with the theoretical simulated results, which demonstrated the correlation between the diffraction strength and the polarization properties of a VHG. Vector electromagnetic theory and polarization characterization are necessary for complete interpretation of the diffraction property of a VHG. The diffraction efficiency is measured at 532 nm in a polarization-sensing experiment. The measured data and theoretical simulation have demonstrated the potential application of the holographic beam splitter for polarization-sensor technology. © 2004 Optical Society of America


1. INTRODUCTION

The holographic multibeam device is a holographic plate that works as a holographic laser beam combiner/splitter (HBCS). It is a device with multiple diffraction gratings. The full polarization property of diffraction by a volume holographic grating (VHG) was investigated theoretically in the preceding paper in this issue.1 By using a simple volume grating model, we derived the diffracted fields and the Mueller matrices from Maxwell’s equations, using Green’s function algorithms. We investigated and formulated the polarization property of VHG diffraction with weak and strong coupling between the transmitted and the diffracted beams. We then derived and simulated the Mueller matrices of the diffraction and the transmitted beams by choosing appropriate parameters. In this paper we measure the polarization properties of two VHG samples and compare them with those predicted by the theory developed in Ref. 1. The measured optical properties of the quinine-doped polymethyl methacrylate (PMMA) substrate are reported in Section 2. The measured data of single- and three-hologram VHG samples (A and B), respectively, are reported in Section 3. Theoretical simulation results are presented in Section 4 and compared with the experimental data. An experiment demonstrating the polarization sensing of the three-hologram grating sample (B) and a theoretical justification are reported in Section 5. A discussion and conclusion are presented in Section 6.

2. OPTICAL PROPERTIES OF SUBSTRATE MATERIAL

The substrate material is made of phenanthraquinone-doped PMMA. As part of the holographic development procedure, the substrate was subjected to a ~60 °C bake for 12 h and then exposed to a mercury lamp (with its UV radiation blocked) for 2 h. The spectral index of refraction \( n \) and the extinction coefficient \( k \) of the substrate were measured for wavelengths between 500 and 1600 nm. A halogen lamp and lasers at wavelengths 514, 544, and 633 nm were used as light sources for the measurements. \( n \) was determined by measuring the ellipsometer parameter \( \psi \) of reflection near the Brewster angle, and \( k \) was obtained by fitting the ratio of the measured transmittance at normal incidence to the transmittance calculated from the measured \( n \). The measured \( n \) and \( k \) are shown in Fig. 1. The \( k \) curve is consistent with the previously reported absorption spectrum of PMMA. Birefringence was observed in this substrate. The birefringence \( \Delta n \) was obtained from the transmission retardation \( \Delta \) measured at different incidence angles. \( \Delta \) was determined by null ellipsometry. With a He–Ne laser source, \( \Delta n \) was measured as \( 3.42 \times 10^{-5} \) at a wavelength of 632.8 nm.
3. OPTICAL PROPERTIES OF VOLUME HOLOGRAPHIC GRATINGS: EXPERIMENTS

A. Samples
Two uncoated VHG samples were investigated. Sample A is a 2-mm-thick uncoated holographic beam combiner (HBC) substrate containing a single VHG. Sample B is 1.71 mm thick and contains three VHGs. All hologram writings were performed on a floating optical table. The gratings were written in the substrate by using a frequency-doubled Nd–YVO4 laser operating at 532 nm. Sample A was exposed for 90 s at a laser output power of 2.0 W, which was evenly divided into two Gaussian beams, each with a diameter of \( \frac{5}{2} \) cm. For sample B, each grating was exposed for 70 s. The three gratings of sample B were designed to have the same incidence angles at a wavelength of 532 nm. However, the optical properties of both VHG samples were measured at a wavelength of 632.8 nm.

B. Diffraction Angles
The angles used in this paper are defined in Fig. 2. The z-axis is chosen along the normal of the sample plane. The x–z plane is chosen as the plane of incidence. \( \theta_i \), \( \theta_t \), and \( \theta_r \) (all equal to \( \theta \)) represent the angles of incidence, specular transmission, and reflection, respectively. \( \phi \) and \( \phi_t \) represent, respectively, the polar angles of the transmissive diffraction and the reflective diffraction. \( \Phi \) denotes the azimuthal angle of the diffraction beams. The direction of grating vector \( \mathbf{K} (\theta_K, \phi_K) \) is perpendicular to the planes of the gratings. The three gratings of sample B were designed such that their corresponding \( \mathbf{K} \) vectors fall on the same plane \( (\phi_K = 0^\circ) \). Each sample was aligned such that \( \Phi \) is also in the plane of incidence. The diffracted beams of both sample A and sample B are all in the plane of incidence so that \( \Phi = 180^\circ \) for all measurements. Sample A has maximum diffraction at \( \theta = 29.55^\circ \), \( \Theta = 15.97^\circ \). From these data, \( \theta_K = 4.3444^\circ \) and the grating spacing \( x_0 = 821.3 \) nm are determined by the photon-momentum relation [Eqs. (15) and (24b) of Ref. 1]. The results for both samples are listed in the top part of Table 1. Note that the incidence angles for the three gratings of sample B are different at a wavelength of 632.8 nm. The diffraction angles of sample B that have the same incidence angle \( \theta = 2.17^\circ \) at a wavelength of 532 nm are listed in the bottom part of Table 1.

C. Polarization
The principal Mueller matrix for reflected, transmitted, or diffracted beam in the plane of incidence can be expressed as:

\[
M = R \begin{bmatrix}
1 & P_x & 0 & 0 \\
P_x & 1 - 2P_y & 0 & 0 \\
0 & 0 & P_y & P_z \\
0 & 0 & -P_z & P_y
\end{bmatrix},
\]

Table 1. Volume Gratings’ Diffraction Properties

<table>
<thead>
<tr>
<th>Sample</th>
<th>Grating</th>
<th>A</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample A(^a)</td>
<td>( \theta ) (deg)</td>
<td>29.55</td>
<td>4.77</td>
<td>5.32</td>
<td>5.85</td>
</tr>
<tr>
<td></td>
<td>( \Theta ) (deg)</td>
<td>15.97</td>
<td>27.80</td>
<td>34.32</td>
<td>40.91</td>
</tr>
<tr>
<td></td>
<td>( \theta_K ) (deg)</td>
<td>4.3444</td>
<td>-7.52</td>
<td>-9.33</td>
<td>-11.08</td>
</tr>
<tr>
<td></td>
<td>( x_0 ) (nm)</td>
<td>821.25</td>
<td>1141.6</td>
<td>951.09</td>
<td>820.58</td>
</tr>
<tr>
<td>Sample B(^b)</td>
<td>( \theta ) (deg)</td>
<td>2.17</td>
<td>2.17</td>
<td>2.17</td>
<td>2.17</td>
</tr>
<tr>
<td></td>
<td>( \Theta ) (deg)</td>
<td>25.10</td>
<td>30.93</td>
<td>36.75</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\lambda = 632.8 \text{ nm}, n = 1.4899, m = -1, \Phi = 180^\circ, \phi_K = 0^\circ.\)
\(^b\lambda = 532 \text{ nm}, n = 1.4942, m = -1, \Phi = 180^\circ.\)
It is interesting to see that the properties of the reflective diffractometer were calculated by using Eqs. (2) and (4). Table 2 lists the measured and simulated properties for sample A at wavelength 632.8 nm:

<table>
<thead>
<tr>
<th>Transmission ($T_i$)</th>
<th>Diffraction ($T_d$)</th>
<th>Reflection ($R_r$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td>Calculated</td>
</tr>
<tr>
<td>$R$</td>
<td>0.703</td>
<td>0.864</td>
</tr>
<tr>
<td>$\psi$</td>
<td>48.08</td>
<td>46.50°</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>29.21</td>
<td>29.20°</td>
</tr>
<tr>
<td>$D$</td>
<td>0.015</td>
<td>0.000</td>
</tr>
<tr>
<td>$D_v$</td>
<td>0.007</td>
<td>0.000</td>
</tr>
<tr>
<td>$\eta = T_d/(T_i + T_d)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Calculation parameters: $n = 1.4899$, $A_{pol} = 1.183 \times 10^{-4}$, $Cdn = -4.4$, $\Delta \delta_a = 4.10^\circ$, $\Delta \delta_b = 33.22^\circ$, $dn/n = -3.49 \times 10^{-4}$. 

where $R$ is either transmittance or reflectance and $\mathcal{P}_x$, $\mathcal{P}_y$, and $\mathcal{P}_z$ are, respectively, the linear, preserved, and circular polarizations. Polarization $\mathcal{P}$ and depolarization $D$ are defined by

$$\mathcal{P} = (P_x^2 + P_y^2 + P_z^2)^{1/2},$$  \hfill (2)

and

$$D = 1 - \mathcal{P} = D_u + D_v, $$  \hfill (3)

where $D_u$ and $D_v$ are, respectively, the co- and cross-polarized parts of depolarization. $\mathcal{P}_x$, $\mathcal{P}_y$, and $\mathcal{P}_z$ are related to the ellipsometric parameters $\psi$ and $\Delta$ by

$$\mathcal{P}_x = -\mathcal{P} \cos 2\psi, $$ \hfill (4a)

$$\mathcal{P}_y = \mathcal{P} \sin 2\psi \cos \Delta, $$ \hfill (4b)

$$\mathcal{P}_z = \mathcal{P} \sin 2\psi \sin \Delta, $$ \hfill (4c)

where $\psi$, $\Delta$, $D$, and $D_v$ are four independent parameters in addition to $R$. They were measured with null ellipsometry.\(^5\)\(^7\) The precise values of $\psi$ and $\Delta$ for our null ellipsometers are 0.01° and 0.02°, respectively. The precise values of $D$ and $D_v$ are both 0.001. $\mathcal{P}_x$, $\mathcal{P}_y$, and $\mathcal{P}_z$ were calculated by using Eqs. (2) and (4).

The measured and simulated properties for sample A are listed in Table 2. The properties of the reflective diffraction $R_d (m = -1)$ were not measured because of the limitation of the instrument. Nevertheless, $R_d$ is small. The transmittances for specular transmission $T_i (m = 0)$ and transmissive diffraction $T_d (m = -1)$ are measured. The transmission diffraction efficiency $\eta = T_d/(T_i + T_d)$ were calculated and are shown in the last row of this table.

$\psi$ and $\Delta$ are two independent parameters representing pure polarization. In Table 2 the $\psi$ of diffraction is different from those of specular transmission and reflection. In general, $\psi > 45^\circ$ for specular transmission and $\psi < 45^\circ$ for specular reflection, which agree with the measurements. It is interesting to see that $\psi < 45^\circ$ for transmissive diffraction, in contrast to the specular transmission. Because the substrate is birefringent, the effect of the gratings on the measured retardation $\Delta$ is mixed with the retardation of the substrate. If the detected light contains only one kind of polarization, such as that of specular transmission or of pure direct diffraction, depolarization should be zero. If the detected light also includes other kinds of light, such as incoherent multiple reflection, depolarization will be appreciable. The notsmall measured nonvanishing depolarization (0.128) of the diffracted beam indicates that the detected diffraction is not purely a direct diffraction. We may ignore the measured cross depolarization, because $D_v$ is easily subject to the misalignment of the system with respect to the sample.

4. OPTICAL PROPERTIES OF VOLUME HOLOGRAPHIC GRATINGS: THEORETICAL INTERPRETATION

For sample A, the theoretically simulated results of transmission $T_s$, diffraction $T_d$, and $\eta$ are calculated for $A_{pol} = 0$ to $1.5 \times 10^{-4}$ and shown in Fig. 3.

$$A_{pol} = \alpha/(A_{\mu}x_0) $$ \hfill (5)

is a dimensionless diffraction-strength parameter.\(^1\) $A_{\mu}$ is a constant with the dimension of area, and $\alpha$ (unit of volume) is the effective electric polarizability of the holo-
Figure 4. Calculated \( \psi \) and \( \Delta \) of sample A. Horizontal lines are the measured data; the fitted points are marked by \( \times \). Parameters are the same as in Fig. 3.

Since the holographic grating is fabricated by laser interference, as shown in Fig. 5 of Ref. 1, \( u_0 = 0.2821 \mu m \) is an acceptable choice (Fig. 5 of Ref. 1). The diffraction-strength parameter \( A_{pol} \) is determined by the measured data. As shown in Fig. 3, \( A_{pol}(\eta) \) is not a unique-valued function of \( \eta \). It is not appropriate to use only the measured \( \eta \) for determining the value of \( A_{pol} \).

Since the ellipsometrically measured \( \psi \) has higher accuracy than other parameters, we would use \( \psi(A_{pol}) \) to determine the fitted parameter \( A_{pol} \). In Fig. 4, the measured \( \psi \) shown in Table 2 are plotted as the horizontal lines. The solid and dashed curves are the diffraacted and transmitted beams, respectively. \( A_{pol} = 1.1832 \times 10^{-4} \) is a best-fit value fitting the \( \psi \) data of both the diffraacted and the transmitted beams. The fitted \( \psi \) of the transmitted and diffraacted beams are marked as "x." The corresponding \( T_t \), \( T_d \), and \( \eta \) are calculated from this fitted \( A_{pol} \). They are shown as the marked points in Fig. 3. The measured intensity for the diffracted beam may also contain multiple reflected lights because the measured depolarization is large. Therefore the measured \( \eta = 0.243 \) is considerable larger than the calculated value (=0.059) when the \( \psi \)-fitted \( A_{pol} \) is used.

The one-dimensional volume grating would create additional birefringence, even though the substrate material does not have birefringence. Therefore there is an appreciable effect on the ellipsometric parameter \( \Delta \). To fit the measured data of \( \Delta \), we use an empirical birefringence model in which the average index of refraction of the beam with \( (\sigma, \phi) \)-polarization-vector direction for the \( s \) or \( p \) wave is

\[
f(u) = \frac{1}{u_0 \sqrt{\pi}} \exp \left( -\frac{u^2}{u_0^2} \right). \tag{6}
\]

This function is similar to the Gaussian form of the normalized one-dimensional grating profile function \( f(u) \):

\[
f(u) = \frac{1}{u_0 \sqrt{\pi}} \exp \left( -\frac{u^2}{u_0^2} \right).
\]

### Table 3. Mueller Matrix Properties at Wavelength 632.8 nm for Sample B

<table>
<thead>
<tr>
<th>Grating #1*</th>
<th>Transmission ((T_t))</th>
<th>Diffraction ((T_d))</th>
<th>(\eta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>Calculated</td>
<td>Measured</td>
<td>Calculated</td>
</tr>
<tr>
<td>(R(T_1, T_d))</td>
<td>0.8661</td>
<td>0.9089</td>
<td>0.0199</td>
</tr>
<tr>
<td>(\psi)</td>
<td>45.04°</td>
<td>45.03°</td>
<td>44.98°</td>
</tr>
<tr>
<td>(\Delta)</td>
<td>-7.54°</td>
<td>-7.56°</td>
<td>5.541</td>
</tr>
<tr>
<td>(D)</td>
<td>3.27 \times 10^{-5}</td>
<td>0.000</td>
<td>3.29 \times 10^{-3}</td>
</tr>
<tr>
<td>(\Delta\delta) (deg)</td>
<td>-49.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grating #2*</td>
<td>Transmission ((T_t))</td>
<td>Diffraction ((T_d))</td>
<td>(\eta)</td>
</tr>
<tr>
<td>Measured</td>
<td>Calculated</td>
<td>Measured</td>
<td>Calculated</td>
</tr>
<tr>
<td>(R(T_1, T_d))</td>
<td>0.8666</td>
<td>0.8905</td>
<td>0.0114</td>
</tr>
<tr>
<td>(\psi)</td>
<td>45.05°</td>
<td>45.11°</td>
<td>43.56°</td>
</tr>
<tr>
<td>(\Delta)</td>
<td>-7.34°</td>
<td>-7.32°</td>
<td>12.77°</td>
</tr>
<tr>
<td>(D)</td>
<td>2.99 \times 10^{-5}</td>
<td>0.000</td>
<td>8.15 \times 10^{-3}</td>
</tr>
<tr>
<td>(\Delta\delta)</td>
<td>33.85°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grating #3*</td>
<td>Transmission ((T_t))</td>
<td>Diffraction ((T_d))</td>
<td>(\eta)</td>
</tr>
<tr>
<td>Measured</td>
<td>Calculated</td>
<td>Measured</td>
<td>Calculated</td>
</tr>
<tr>
<td>(R(T_1, T_d))</td>
<td>0.8653</td>
<td>0.8588</td>
<td>0.0071</td>
</tr>
<tr>
<td>(\psi)</td>
<td>45.04°</td>
<td>45.09°</td>
<td>45.34°</td>
</tr>
<tr>
<td>(\Delta)</td>
<td>-7.27°</td>
<td>-7.29°</td>
<td>20.08°</td>
</tr>
<tr>
<td>(D)</td>
<td>3.88 \times 10^{-5}</td>
<td>0.000</td>
<td>2.22 \times 10^{-2}</td>
</tr>
<tr>
<td>(\Delta\delta)</td>
<td>-5.75°</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*\( (\sigma, \phi, \Phi) = (4.77°, 27.80°, 180°) \), \( Cdn = -4.4, A_{pol} = 8.007 \times 10^{-5} \), \( n = 1.4899, D_x = D = 0, dn/n = -2.36 \times 10^{-4} \).

*\( (\sigma, \phi, \Phi) = (5.32°, 34.32°, 180°) \), \( Cdn = -4.4, A_{pol} = 1.0411 \times 10^{-4} \), \( n = 1.4899, D_x = D = 0, dn/n = -3.07 \times 10^{-4} \).

*\( (\sigma, \phi, \Phi) = (5.85°, 40.91°, 180°) \), \( Cdn = -4.4, A_{pol} = 1.402 \times 10^{-4} \), \( n = 1.4899, D_x = D = 0, dn/n = -4.14 \times 10^{-4} \).
Table 4. Calculated Mueller Matrix Properties of Sample B, Grating #1, and Fitted Incident Stokes Vector
(1, 0, U, 0) for Wavelength 532 nm, n = 1.4942°

<table>
<thead>
<tr>
<th>Experiment</th>
<th>R</th>
<th>ϕ (deg)</th>
<th>M_p(1)</th>
<th>M_p(2)</th>
<th>M_p(3)</th>
<th>M_p(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.8126</td>
<td>45.67</td>
<td>0.8126</td>
<td>0.0190</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Transmission (t)</td>
<td>0.0398</td>
<td>42.10</td>
<td>0.0398</td>
<td>-0.0040</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Diffraction (d)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.7683</td>
<td>39.56</td>
<td>0.7683</td>
<td>-0.1445</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Transmission (t)</td>
<td>0.0376</td>
<td>73.00</td>
<td>0.0376</td>
<td>0.0312</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Diffraction (d)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.7854</td>
<td>49.37</td>
<td>0.7854</td>
<td>0.1193</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Transmission (t)</td>
<td>0.0385</td>
<td>11.40</td>
<td>0.0385</td>
<td>-0.0355</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Diffraction (d)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* (θ, Θ, Φ) = (2.17°, 25.10°, 180°), (θ_p, Φ_p) = (-7.52°, 0°), x_o = 1141.6 nm

\[ n(\Theta_j, \Phi_j) = n_r n_l (n_r^2 \sin^2 \delta_j + n_l^2 \cos^2 \delta_j)^{1/2}, \]

\[ j = ts, tp, ds, dp, \]  \hspace{1cm} (7a)

\[ \cos(\delta_j - \Delta \delta_j) = \sin \Theta_j \cos \theta_K \cos(\Phi_j - \phi_K) + \cos \Theta_j \sin \theta_K. \]  \hspace{1cm} (7b)

\( \delta_j \) and \( \Delta \delta_j \) are the angle between the grating vector \( \mathbf{K} \) and the propagation direction of the diffraction beam \( (\Theta_j, \Phi_j) \); \( dn = n_r - n_l \) is assumed proportional to the grating diffraction-strength parameter \( A_{pol} \),

\[ dn = Cdn(A_{pol}), \]  \hspace{1cm} (8)

where \( Cdn \) is the strength parameter of grating-induced birefringence. For simplicity, we assume that \( \Delta \delta_j \) is the same for \( s \) and \( p \) waves. The parameters \( Cdn, \Delta \delta_0, \Delta \delta_1 \) are determined by fitting the measured \( \Delta \) values. For sample A, these parameters and the grating-induced birefringence \( dn/n \) are listed in Table 2. The calculated parameters \( R, \psi, \Delta, \varrho, \bar{\varrho} \) and \( \eta \) are also listed in Table 2. Agreement with the experiment is good. In Fig. 4 the measured \( \Delta \) shown in Table 2 are plotted as the horizontal lines. The solid and dashed curves are the diffractions and the transmitted beams, respectively. \( A_{pol} = 1.1832 \times 10^{-4} \) is also a best-fit value fitting the \( \Delta \) data of both the diffracted and the transmitted beams.

Since the statistical property of the Mueller matrix\(^5\) was not considered in the dynamical theory,\(^4\) there is no depolarization in the calculated results, \( \varrho = \bar{\varrho} = 0 \) (Table 2). We made similar fitting to the measured data of sample B at 632.8 nm. The results are listed in Table 3. For a wavelength of 532 nm, the incidence angles of the three gratings of sample B are the same. The incident beam at \( \theta = 2.17° \) (Tables 1 and 4) is split into four beams: the transmitted beam and three diffracted beams of angles \( \Theta = 25.10°, 30.93°, \) and \( 36.75° \). Assuming that the three gratings have approximately equal statistical weight, the three diffracted beams’ transmittances versus \( A_{pol} \) are calculated. The results are shown in Fig. 5. Sample thickness 1.71 mm and \( u_o = 0.2821 \mu m \) are used for the calculation.\(^1\)

5. POLARIZATION-SENSING EXPERIMENT

The polarization-filtering effect of the holographic beam splitter (HBS) is shown in the following experiment. As shown in Fig. 6, a rotating half-wave plate, HWP, is placed in the incident 532-nm laser path. Two identical detectors were used to measure the intensities of transmitted (t) and diffracted (d) beams. These measured dif-

![Fig. 5. Calculated transmittance of the transmitted and the three diffracted beams at wavelength 532 nm for sample B. Parameters are \( n = 1.4942, u_o = 0.2821 \mu m \), sample thickness = 1.71 mm. Other parameters are listed in Tables 1 and 4.](image)

![Fig. 6. Polarization-sensing experiment of sample B. HWP, half-wave plate.](image)
fraction signals are due to grating #1, \( (\theta_K, \phi_K) = (5.72^\circ, 0^\circ) \), \( (\theta, \Theta, \Phi) = (2.17^\circ, 25.10^\circ, 180^\circ) \). The Stokes vectors of the incident beam \( (i) \) and those of the transmission \( (t) \) and the diffracted \( (d) \) beams are related by the following relation,

\[
\begin{pmatrix}
I^{(i)} \\
Q^{(i)} \\
U^{(i)} \\
V^{(i)}
\end{pmatrix} = M^{(i)} M_{\text{hwp}}(\delta) \begin{pmatrix}
I_t \\
Q_t \\
U_t \\
V_t
\end{pmatrix}, \quad j = t, d,
\]

where

\[
M_{\text{hwp}}(\delta) = T_{\text{hwp}} \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos 4\delta & -\sin 4\delta & 0 \\
0 & -\sin 4\delta & -\cos 4\delta & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

is the Mueller matrix of the rotating half-wave plate of transmittance \( T_{\text{hwp}} \) and rotating angle \( \delta \) (hwp stands for half-wave plate). The signals measured by the two sensor detectors (with the same detection constant \( s_d \)) are

\[
S^{(j)}(\delta) = s_d I^{(j)}(\delta), \quad j = t, d,
\]

\[
I^{(j)}(\delta) = T_{\text{hwp}} I_{t} M_{t}^{(j)} + (Q_j \cos 4\delta - U_j \sin 4\delta) M_{t12}^{(j)} - (Q_j \sin 4\delta + U_j \cos 4\delta) M_{t13}^{(j)} - V_j M_{t14}^{(j)},
\]

\[
j = t, d. \quad (11b)
\]

\( M_{t1k}^{(j)}(k = 1, 2, 3, 4) \) are the first-row Mueller matrix elements of the two beams \( t \) and \( d \) \([M^{(j)}]\). The diffraction efficiency is

\[
\eta(\delta) = I^{(d)}(I^{(t)} + I^{(d)}) = S^{(d)}(S^{(t)} + S^{(d)}). \quad (12)
\]

For grating #1, the diffraction efficiency \( \eta \) versus \( \delta \) is measured and shown in Fig. 7 (points marked x). The average value \( \langle \eta \rangle = 0.0467 \). With the parameters shown in Table 1, \( \eta(A_{\text{pol}}) \) is calculated and shown in Fig. 8. There are three values of \( A_{\text{pol}} \): (A) \( 4.327 \times 10^{-6} \), (B) \( 3.433 \times 10^{-5} \), and (C) \( 4.167 \times 10^{-5} \), with \( \eta = 0.0467 \). They are marked (x) in Fig. 8. For these three \( A_{\text{pol}} \)’s, the Mueller matrix properties are calculated and shown in Table 4.

We assume that the incident laser is a partially linearly polarized beam with Stokes parameters \((I_i, Q_i, U_i, V_i) = (1, Q, U, 0)\). \( I^{(t)}, I^{(d)} \) and \( \eta \) [Eqs. (11) and (12)] are simulated for fitting the measured \( \eta \) versus \( \delta \) data, with \( A_{\text{pol}} = 4.327 \times 10^{-6} \) and \( (Q, U) = (0, 0.22) \). The fitted \( \eta \) versus \( \delta \) curve is also shown in Fig. 9.
Fig. 7 (solid curve). Assuming $T_{\text{hwp}} = 1$, the calculated transmittances $I(t')$, $I(d)$ versus $\delta$ curves are shown in Fig. 9. Similar fittings were done for the other two parameters $A_{\text{pol}} = 3.439 \times 10^{-5}$ and $4.174 \times 10^{-5}$. The results are $(Q, U) = (0, -0.027)$ and $(0, 0.025)$, respectively. The fitted degrees of linear polarization are then determined to be 0.22, 0.027, and 0.025 for the three cases. The results show that the polarization-filtering power depends on the diffraction strength of the grating. The calculated Mueller matrix properties of the two beams ($t$ and $d$) are listed in Table 4. The first-row Mueller matrix elements $M_{ij}^{(j)} (j = t, d; k = 1, 2, 3, 4)$ for signal calculation [Eq. (11b)] are shown.

6. DISCUSSION AND CONCLUSION

The measured Mueller matrices shown in Tables 2 and 3 demonstrate that there is nonvanishing depolarization ($D > 0$). This depolarization is due to a statistical property that is neglected in the dynamic theory.$^{1,5,6}$ Therefore inclusion of depolarization is needed for a complete theory.

The measured and calculated polarization properties listed in Tables 2 and 3 show the birefringence. On the basis of Eq. (8), $\eta$ of the samples A and B are calculated; they are of the order of $10^{-5}$. The results are listed in Tables 2 and 3. In addition, we measured that the substrate birefringence (reported in section 2) $\eta = 3.42 \times 10^{-5}$. The substrate and grating birefringence of HBCS samples would be an interesting topic for further investigation.

As shown in Fig. 3, $A_{\text{pol}}(\eta)$ is not a unique-valued function; it could not be determined by fitting the measured value of $\eta$ only. As shown in Fig. 4 and Table 2, fitting the polarization parameters $\psi$ and $\Delta$ is also an option. Therefore a scalar-field electromagnetic (EM) theory of strong coupling$^{10-14}$ could not completely determine the diffraction property (including polarization) of the VHG devices. Polarization characterization based on the vector field EM theory and the Mueller matrix$^1$ is necessary. Similarly, the $\eta$ versus $\delta$ fitting curve in Fig. 7 is not unique. In addition to $(A_{\text{pol}}, U) = (4.327 \times 10^{-6}, 0.22)$, we have other solutions, $(A_{\text{pol}}, U) = (3.433 \times 10^{-5}, -0.027)$ or $(4.167 \times 10^{-4}, 0.025)$. The polarization parameters are listed in Table 4. All three cases give the same efficiency, $\eta = 0.0467$, in the absence of the rotating HWP. To get the diffraction-strength parameters $A_{\text{pol}}, Cdn, \Delta \delta_1$, and $\Delta \delta_2$, we fitted the ellipsometric parameters $\psi$ and $\Delta$, which were measured with high accuracy. The results are shown in Tables 2 and 3. Therefore a polarization measurement (or Mueller matrix) is needed for a full optical characterization of the holographic grating devices. The ellipsometric parameter $\Delta$ depends on the anisotropy modeling parameters $Cdn$, $\Delta \delta_1$, and $\Delta \delta_2$, whereas $\eta$ and $\psi$ depend only on the $A_{\text{pol}}$ parameter. Although we have good fitting of $\Delta$, the complicated $A_{\text{pol}}$ behavior shown in Fig. 4 has shown that it is an issue worth further investigation, both theoretically and experimentally. Our model is only the start of this investigation.

As shown in Table 1, the three gratings of sample B were designed to have the same incidence angle 2.17° for wavelength 532 nm. Assuming that the three gratings have approximately equal statistical weights, the transmittances of the transmitted ($t$) and the three diffracted ($d1, d2, d3$) beams and the total transmittance are calculated for $A_{\text{pol}} = 0$ to $5 \times 10^{-5}$. The results, shown in Fig. 5, show that the intensity of each diffracted beam is $\sim 1/3$ of that of total diffraction. The incident beam is almost equally diffracted into three directions ($\Theta = 25.10^\circ, 30.93^\circ, 36.75^\circ$). If operated in reverse, the three beams from different directions would combine to the same diffracted angle 2.17°. This is the principle of a holographic beam combiner.$^9$ Our theory$^1$ could be generally applied for simulating the performance of any HBCS device.

A HBS device can be applied to detect the polarization state of the incident radiation. An experiment and the fitted result were reported in Section 5. A three-hologram HBS device, specifically designed with the same incidence angle, can be applied to measure the Stokes parameters of incident light. The four measured signals of the transmitted ($t$) and the three diffracted ($d1, d2, d3$) beams are

$$S^{(j)}(\delta) = s_d I^{(j)}, \quad j = t, d1, d2, d3, \quad (13a)$$

where

$$I^{(j)} = T_{\text{hwp}}(I_{11} M_{ij}^{(j)} + Q_i M_{ij}^{(j)} + U_i M_{ij}^{(j)} + V_i M_{ij}^{(j)}). \quad (13b)$$

A HBS-based Stokes-meter sensor can be developed on the basis of this principle. An engineering design of an optical system has recently been proposed for the electro-optics–infrared imaging-sensor application.$^15$

In this paper we have concentrated only on the properties of single volume gratings. Properties of $N\times1$ multihologram HBCSs would be worth further investigation. The birefringence and depolarization of the anisotropic substrate and sample with holographic gratings merits further study, as well. The application of the HBS for fabricating polarization-sensor optical devices is a challenging advanced electro-optics–infrared sensor technology.

The major conclusions are as follows:

1. Results of Mueller matrix measurement for holographic volume grating devices are reported we believe for the first time. The agreement with theoretical simulation is a justification of the theory.$^1$

2. The vector EM theory is necessary for completely interpreting the diffraction property of a HBCS device.$^1$

3. The theoretical and experimental foundation of HBS-based Stokes-meter optics for polarization-sensor technology is developed, we believe for the first time.

ACKNOWLEDGMENTS

This research was partially supported by the U.S. Office of Naval Research (ONR) through the NAVAIR/ONR ILIR Program, the NAVAIR Discretionary Fund, and the ONR Electro-Optics/Infrared Sensor Technology Program and by the MDA SBIR-II Program Contract N68936-01-C-0008.

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