Measurement of the amplitude and absolute (i.e., temporal and initial) phase of a monochromatic wave is challenging because in the most general condition the spatial distribution of the field around a point is arbitrary. Therefore, one must know the impedance of the system between the point of interest and the detector, and ensure that there is no interference with the ambient field. It is recently shown in the literature that the absolute phase measurement can be used for accurate qubit rotations and quantum wavelength teleportation. Here we show how such a measurement can indeed be made using resonant atomic probes via detection of incoherent fluorescence induced by a laser beam. This measurement is possible due to self-interference effects between the positive- and negative-frequency components of the field. In effect, the small cluster of atoms here act as a highly localized pickup coil, and the fluorescence channel acts as a transmission line.

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For simplicity, we assume each atom to be an ideal two-level system—i.e., \( \omega t + \phi \). In order to illustrate how this phase can be observed directly, consider a situation where a cluster of noninteracting atoms is at rest at the same location. For simplicity, we assume each atom to be an ideal two-level system where a ground state \( \ket{0} \) is coupled to an excited state \( \ket{1} \) by this field \( B(t) \), with the atom initially in state \( \ket{0} \). The Hamiltonian for this interaction is

\[
\hat{H} = \epsilon (\sigma_z - \sigma_x) / 2 + g(t) \sigma_x,
\]

where \( g(t) = -g_0 \cos(\omega t + \phi) \), \( g_0 \) is the Rabi frequency, \( \sigma_x \) are the Pauli matrices, and the driving frequency \( \omega = \epsilon \) corresponds to resonant excitation. We consider \( g_0 \) to be of the form \( g_0(t) = g_{0M}[1 - \exp(-t/\tau_w)] \) with a switching time \( \tau_w \) relatively slow compared to other time scales in the system—i.e., \( \tau_w \gg \omega^{-1} \) and \( g_{0M} \).

As we have shown before [2,3], without the rotating-wave approximation (RWA) and to the lowest order in \( \eta = (g_0/4\omega) \), the amplitudes of \( \ket{0} \) and \( \ket{1} \) at any time \( t \) are as follows:

\[
C_0(t) = \cos[g_0'(t)t/2] - 2\eta \Sigma \sin[g_0'(t)t/2],
\]

where \( \Sigma = (i/2) \exp[-i(2\omega t + 2\phi)] \) and \( g_0'(t) = (1/\tau_w) \int g_0(t')dt' = g_0[1 - (1/\tau_w)\exp(-t/\tau_w)] \). If we produce this excitation using a \( \pi/2 \) pulse [i.e., \( g_0'(\tau) = \pi/2 \)] and measure the population of state \( \ket{1} \) after the excitation terminates (at \( \tau = \tau \)), we get a signal

\[
C_1(t) = ie^{-i(\omega t + \phi)} \frac{\sin[g_0'(t)t/2]}{2\eta} + \frac{2\eta B^*}{2} \cos[g_0'(t)t/2],
\]
levels are

be written as

the population amplitude of each level

above Hamiltonian. The BSO amplitudes are then calculated

in Fig. 2, uses a thermal, effusive atomic beam. The rf field is

shown in Fig. 1.

While the above analytical model presented here is based

on a two level system, practical examples of which are pre-

sented in Ref. [2], the effect is more generic, and is present
even in three-level or multilevel systems. In particular, we
employed a three-level system to observe this effect, due

primarily to practical considerations. The specific system

employed a three-level system to observe this effect, due

even in three-level or multilevel systems. In particular, we

considered a ladder-type three-level system (0), (1), and (2)).
The transition frequencies for (0)-(1) and (1)-(2) are of the same magnitude \( \epsilon \). We also consider

that a direct transition between (0) and (2) is not allowed.

Now, let the system be pumped by the same field at a fre-

quency \( \omega \). Consider also that the Rabi frequency for the (0)-

(1) transition is \( \omega_0 \) and that for (1)-(2) is also \( \omega_0 \). Then, the

Hamiltonian of the three-level system in a rotating frame can

be written as

\[
\hat{H} = - \omega_0 [1 + \exp(-i2\epsilon t - i2\phi)] (|0\rangle\langle 1| + |1\rangle\langle 2|) + \text{c.c.},
\]

(5)

where \( \omega = \epsilon \). The amplitudes of the three levels are calculated

numerically by solving the Schrödinger equation for

the above Hamiltonian. The BSO amplitudes are then calculated

by subtracting the population amplitude of each level with

the RWA from the population amplitude without the RWA.

The BSO oscillations for all the levels of such a system are shown in Fig. 1.

The experimental configuration, illustrated schematically

in Fig. 2, uses a thermal, effusive atomic beam. The rf field

is applied to the atoms by a coil, and the interaction time \( \tau \) is set by the time of flight of the individual atoms in the rf field

before they are probed by a strongly focused and circularly

polarized laser beam. The rf field couples the sublevels with

\( |\Delta m| = 1 \), as detailed in the inset of Fig. 2. Optical pumping is

employed to reduce the populations of states |1\rangle and |2\rangle com-

pared to that of state |0\rangle prior to the interaction with the

microwave field.

A given atom interacts with the rf field for a duration \( \tau \) prior to excitation by the probe beam that couples state |0\rangle to

an excited sublevel in \( ^8\text{Rb} \). The rf field was tuned to

0.5 MHz, with a power of about 10 W, corresponding to a

Rabi frequency of about 4 MHz for the |0\rangle \rightarrow |1\rangle as well as

the |1\rangle \rightarrow |2\rangle transition. The probe power was 0.5 mW fo-

cused to a spot of about 30 \( \mu \)m diameter, giving a Rabi

frequency of about 60 \( \Gamma \), where \( \Gamma \) (6.06 MHz) is the lifetime

of the optical transition. The average atomic speed is 500 m/s, so that the effective pulse width of the probe, \( \tau_{LP} \), is about 60 ns, which satisfies the constraint that \( \tau_{LP} \ll 1/\omega \).

Note that the resolution of the phase measurement is essen-
tially given by the ratio of \( \min[\tau_{LP}, \Gamma^{-1}] \) and \( 1/\omega \), and can be increased further by making the probe zone shorter. The fluo-

rescence observed under this condition is essentially propor-
tional to the population of level |0\rangle, integrated over a dura-
tion of \( \tau_{LP} \), which corresponds to less than 0.3 Rabi period of

the rf driving field [for \( g_{00}/(2\pi) = 4 \text{ MHz} \)]. Within a Rabi

oscillation cycle, the BSO signal is maximum for \( g_{00}/(\pi)(\pi/2 \equiv (2n+1)\pi/2, \) where \( n = 0, 1, 2, \ldots \), so that there is at least

one maximum of the BSO signal within the region of the probe.
When the rf intensity is increased a component of the BSO at the probe beam is blocked, there is no signal magnetic field applied in the BSO signal amplitude varies as a function of an external period of the Bloch-Siegert oscillation. Thus, there is no washout of the BSO formation is a result of different coupling efficiencies for each of the three ground Zeeman sublevels. We observed that information is not clearly present, since the total population of level \( F = 0 \) is a constant. The observed residual phase information is a result of different coupling efficiencies for each of the three ground Zeeman sublevels. We observed that the BSO signal amplitude varies as a function of an external magnetic field applied in the \( \xi \) direction, with a peak corresponding to a Zeeman splitting matching the applied frequency of 0.5 MHz.

In Fig. 5, we show that the fluorescence signal is phase locked to the second harmonic of the driving field. First, we placed a delay line of 0.4 \( \mu \)s on the cable of the reference field used to trigger the oscilloscope and recorded the fluorescence [Fig. 5(a)]. Then, we put the 0.4-\( \mu \)s delay line on the BSO signal cable and recorded the fluorescence [Fig. 5(b)]. The phase difference between the signals recorded in Figs. 5(a) and 5(b) is approximately 0.8 \( \mu \)s, as expected for a phase locked fluorescence signal. The data presented were for the probe resonant with the transition \( F = 1 \leftrightarrow F' = 1 \), but the same results were observed for \( F = 1 \leftrightarrow F' = 0 \).

To summarize, we report the first direct observation of the absolute phase of the second harmonic of an oscillating elec-
tromagnetic field using self-interference in an atomic resonance. This process is important in the precision of quantum bit rotations at a high speed. The knowledge of the absolute phase of a rf field at a particular point of space may also be useful for single-atom quantum optics experiments. For example, an extension of this concept may possibly be used to teleport the wavelength of an oscillator, given the presence of degenerate distant entanglement, even in the presence of unknown fluctuations in the intervening medium [4–6,12]. Finally, this localized absolute phase detector may prove useful in mapping of radio-frequency fields in microcircuits. Although a particular alkali-metal atom was used in the present experiment, the mechanism is robust and could be observed in virtually any atomic or molecular species.

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