Wavelength locking via teleportation using distant quantum entanglement and Bloch–Siegert oscillation

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Abstract

Recently, we have shown theoretically [M.S. Shahriar, P. Pradhan, J. Morzinski, Phys. Rev. A 69 (2004) 032308] as well as experimentally [G. Cardoso, P. Pradhan, J. Morzinski, M.S. Shahriar, Phys. Rev. A 71 (2005) 063408] how the phase of an electromagnetic field can be determined by measuring the *population* of either of the two states of a two-level atomic system excited by this field, via the so-called Bloch–Siegert oscillation resulting from the interference between the co- and counter-rotating excitations. Here, we show how a degenerate entanglement, created without transmitting any timing signal, can be used to teleport this phase information. This phase-teleportation process may be applied to achieve wavelength locking of remote oscillators.
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The task of synchronizing a pair of oscillators that are separated in space is important for many practical applications, such as the global positioning system (GPS) [3] and the very large base interferometry (VLBI) [4]. Conventionally, this is performed by transmitting timing signals between the oscillators. Consider first the *ideal* situation where the intervening medium is stable and fully characterized. The accuracy of the synchronization process is then limited by the uncertainty in the timing signal. The best result achievable is limited by the signal to noise ratio (SNR). It is generally possible in most cases to eliminate sources of systematic noise, so that the fundamental constraint is the shot noise limit (SNL). In principle, specially prepared quantum states can reduce the effective noise below the SNL. However, since the level of signal in this case is typically much weaker, the actual SNR achievable this way is far below what can be achieved using classical states. Most of the recent proposals [5–9] for achieving

improved oscillator synchronization (OS) using quantum processes suffer from the same constraint, so that in practice they are inferior to classical approaches. Thus, given the current state of technology, quantum mechanical effects is not likely to help in the process of OS under the *ideal* situation.

In a real-life application such as in the field of GPS, the density of the intervening medium fluctuates randomly, leading to a corresponding fluctuation in the time needed for a signal to travel between the oscillators. Under this condition, it is *fundamentally* impossible to synchronize the oscillators to an accuracy higher than the corresponding fluctuation in the travel time. This follows from the principle of special relativity, which is built on the axiom that there exists a maximum speed – namely, the speed of light in vacuum – at which information can propagate. As such, the notion of oscillator synchrony is *defined* with respect to the time it takes for light to traverse the distance between the oscillators. It then follows that if this travel time itself is fluctuating, then the oscillator synchrony is undefined, and cannot be achieved on the timescale of the fluctuation. One can define and establish only an

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average synchrony, valid only for timescales longer than that of the fluctuation. In all situations of practical interest, OS always implies the achievement of this *average synchrony*. This conclusion also holds for the clever technique demonstrated by Ma *et al.* [10].

An alternative way to improve the average synchrony is through frequency locking. Specifically, consider a typical application where each oscillator is locked to a metastable atomic transition. Most of the recent proposals about oscillator synchronization, including the Jozsa protocol [9], make the assumption that each oscillator continues to operate at some ideal transition frequency. In practice, however, this is not the case. The frequency of each oscillator undergoes shifts and drifts due to a host of reasons. These fluctuations lie at the heart of oscillator asynchrony. As such, minimizing the relative drifts in the frequencies is perhaps the most effective way to minimize the error in OS. This approach opens up new possibilities for exploring whether quantum mechanical effects may outperform classical approaches. In this paper, we propose a new technique for locking the wavelengths of two distant oscillators, via phase teleportation.

The process underlying this technique is the so-called Bloch–Siegert Oscillation (BSO), which results from an interference between the co- and counter-rotating parts of a two-level excitation. Recently, we have analyzed the basic features of the BSO theoretically [1], in the context of how it may affect the accuracy of the rotation of a quantum bit. We have also observed the key feature of the BSO using an atomic beam [2], showing specifically that the excited state population of a two-level system driven by a strong microwave field reveals an oscillation that is in phase with the second harmonic of the driving field. In applying the BSO to the task of frequency locking, the phase variation of an oscillator is first mapped by Alice (keeper of the first oscillator) to the wave-functions of an array of atoms, by making use of the fact that the amplitude of the excited state (as well as that of the ground state) depends explicitly on the phase of the driving field. The maximum number of atoms needed to encode the phase variation can be very small, and is given by the Nyquist sampling criterion. Distant entanglement, produced using an asynchronous technique [11], is used to teleport the quantum state of each of these atoms to a matching atom with Bob (keeper of the second oscillator). Bob can thus recreate the exact phase variation of Alice's oscillator locally, and compare with the same for his oscillator. We discuss the potential constraints and advantages of this approach after presenting the scheme in detail.

Consider first a situation where Alice and Bob each has an atom that has two degenerate ground states ($|1\rangle$ and $|2\rangle$), each of which is coupled to a higher energy state ($|3\rangle$), as shown in Fig. 1. We assume the 1–3 and 2–3 transitions are magnetic dipolar, and orthogonal to each other, with a transition frequency ω. For example, in
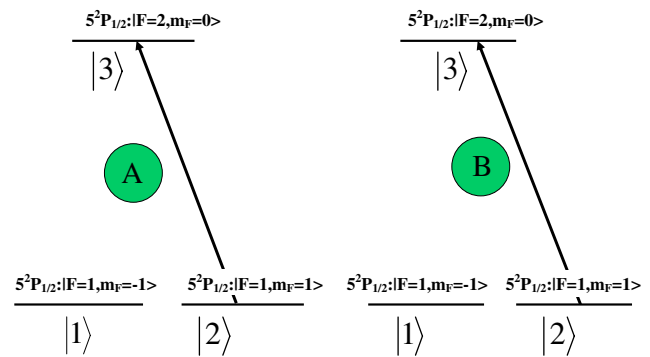

Fig. 1. Schematic illustration of the basic protocol for phase locking two remote oscillators, one with Alice (A), and the other with Bob (B), without transmitting an oscillator signal directly. The model energy levels can be realized, for example, using the metastable hyperfine Zeeman sublevels of ^{87}Rb atoms, as shown in the figure and described in the text.

the case of ^{87}Rb, $|1\rangle$ and $|2\rangle$ correspond to $5^2P_{1/2}$:$|F = 1, m_F = -1\rangle$ and $5^2P_{1/2}$:$|F = 1, m_F = 1\rangle$ magnetic sublevels, respectively, and $|3\rangle$ corresponds to $5^2P_{1/2}$:$|F = 2, m_F = 0\rangle$ magnetic sublevel [2]. Left and right circularly polarized magnetic fields, perpendicular to the quantization axis, are used to excite the 1–3 and 2–3 transitions, respectively. We take ω to be the same as the oscillator frequency ω_c.

We assume that Alice and Bob's fields at ω have the form $B_A = B_{A0}\cos(\omega t + \phi)$ and $B_B = B_{B0}\cos(\omega t + \chi)$, respectively. The origin of the time variable, t, is therefore arbitrary, and does not affect the phase difference, $\Omega \equiv (\phi - \chi)$. The oscillators are assumed to be in phase if $\Omega = 0$, so that if Bob determines that at some instant his magnetic field is maximum and positive in some direction r_B, then Alice will also find her magnetic field to be maximum and positive in some direction r_A at the same instant. As long as Alice and Bob agree on this definition of phase-locking, and use the same definitions all the time, then r_B and r_A do not have to be the same. During the magnetic resonance excitations, the value of any dc magnetic field will be assumed to be vanishing. Symmetry then dictates that any physical observable will be independent of the choice of the quantization axis, as long as it is perpendicular to r_A for Alice, and perpendicular to r_B for Bob. In order to describe our protocol, we now summarize briefly the theory behind the Bloch–Siegert oscillation that occurs when a two-level interaction is considered without the rotating wave approximation (RWA) [12–16], and is presented in greater detail in Ref. [1]. We also describe the condition for the time reversal of an arbitrary evolution under this condition, another necessary element of our protocol.

We consider an ideal two-level system where a ground state $|1\rangle$ is coupled to a higher energy state $|3\rangle$. We assume that the 1–3 transition is magnetic dipolar, with a transition frequency ω, and the magnetic field is of the form $B = B_0\cos(\omega t + \phi)$. In the dipole approximation, the Hamiltonian can be written as

$$\hat{H} = \varepsilon(\sigma_0 - \sigma_z)/2 + g(t)\sigma_x, \tag{1}$$

where $g(t) = -g_0[\exp(i\omega t + i\phi) + \text{c.c.}]/2$, σ_i are the Pauli matrices, and $\varepsilon = \omega$ corresponding to resonant excitation. The state vector is written as

$$|\xi(t)\rangle = \begin{bmatrix} C_1(t) \\ C_3(t) \end{bmatrix}. \tag{2}$$

We perform a rotating wave transformation by operating on $|\xi(t)\rangle$ with the unitary operator $\hat{Q} = (\sigma_0 + \sigma_z)/2 + \exp(+i\omega t + i\phi)(\sigma_0 - \sigma_z)/2$. The Schroedinger equation then takes the form (setting $\hbar = 1$): $\frac{\partial|\tilde{\xi}(t)\rangle}{\partial t} = -i\tilde{H}(t)|\tilde{\xi}(t)\rangle$, where the effective Hamiltonian is given by

$$\tilde{H} = \alpha(t)\sigma_+ + \alpha^*(t)\sigma_-, \tag{3}$$

with $\alpha(t) = -g_0[\exp(-i2\omega t - i2\phi) + 1]/2$, and the rotating frame state vector is

$$|\tilde{\xi}(t)\rangle \equiv \hat{Q}|\xi(t)\rangle = \begin{bmatrix} \tilde{C}_1(t) \\ \tilde{C}_3(t) \end{bmatrix}. \tag{4}$$

The general solution, without RWA, to (4) can be written in the form

$$|\tilde{\xi}(t)\rangle = \sum_{n=-\infty}^{\infty} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \exp(n(-i2\omega t - i2\phi)), \tag{5}$$

with the couplings described by

$$\dot{a}_n = i2n\omega a_n + ig_0(b_n + b_{n-1})/2, \tag{6a}$$

$$\dot{b}_n = i2n\omega b_n + ig_0(a_n + a_{n+1})/2. \tag{6b}$$

We consider $g_0 \equiv g_0(t) = g_{0M}(1 - e^{-t/\tau_{sw}})$ to have a slower time-dependence compared to other characteristic timescales such as $1/\omega$ and $1/g_{0M}$, where g_{0M} is the peak value of g_0 and τ_{sw} is the switching time. Under this condition, one can solve these equations by employing the method of adiabatic elimination, which is valid to first order in $\eta \equiv (g_0/4\omega)$. As derived in Refs. [1,2], the solutions are

$$C_1(t) = \cos(g_0'(t)t/2) - 2\eta\Sigma \cdot \sin(g_0'(t)t/2), \tag{7a}$$

$$C_3(t) = ie^{-i(\omega t+\phi)}[\sin(g_0'(t)t/2) + 2\eta\Sigma^* \cdot \cos(g_0'(t)t/2)], \tag{7b}$$

where $\Sigma \equiv (i/2)\exp[-i(2\omega t + 2\phi)]$ and $g_0'(t) = 1/t \int_0^t g_0(t')\,dt' = g_0[1 - (t/\tau_{sw})^{-1} \exp(-t/\tau_{sw})]$. To lowest order in η this solution is normalized at all times. Note that if Alice were to carry this excitation on an ensemble of atoms through for a $\pi/2$ pulse, and measure the population of the state $|3\rangle_A$ immediately (at $t = \tau$, the moment when the $\pi/2$ excitation ends), the result would be a signal given by $[1 + 2\eta\sin(2\omega\tau + 2\phi)]/2$, which contains information related to the amplitude and phase of her field.

Next, we consider the issue of exact time reversal of such an excitation. The Schroedinger equation (4) has the formal solution

$$|\tilde{\xi}(t_2)\rangle = \exp\left[-i\int_{t_1}^{t_2} \tilde{H}(t')\,dt'\right]|\tilde{\xi}(t_1)\rangle. \tag{8}$$

If the RWA is made, then $\tilde{H}$ is time independent. In that case, if one starts an evolution at t_1, proceed for *any* dura-

tion T, then reverses the sign of $\tilde{H}$ by shifting the phase of the magnetic field by π, and continues with the evolution for another duration T, then the system returns back to the starting state. Here, however, RWA is not made, so that $\tilde{H}$ depends on time. Therefore, the exact reversal can be achieved in this manner only if $T = m\pi/\omega$ for any integer value of m [9,1,2].

Returning to the task at hand, our protocol starts by using a scheme, developed earlier by us [11] to produce a degenerate entanglement of the form $|\psi\rangle = (|1\rangle_A|2\rangle_B - |2\rangle_A|1\rangle_B)/\sqrt{2}$. Recalling briefly, in this technique, a pair of entangled photons, produced by a parametric down converter, for example, are transmitted to Alice and Bob, each receiving one of the photons. The capturing process is checked indirectly, using a quantum non-demolition measurement. If the verification process confirms the capture of the photons, then the quantum states of the atoms remain undisturbed. Otherwise, the process is re-initialized (placing each atom in state $|1\rangle$) and repeated until it succeeds. An optically off-resonant Raman transition (with one leg corresponding to a pump frequency, and the other corresponding to one of the entangled photons) coupling $|1\rangle$ to $|2\rangle$ is used by Alice as well as Bob to capture the photons, resulting in the entangled state shown above. Next, Alice attenuates her field so that the counter-rotating term in the Hamiltonian can be ignored (this assumption is not essential for our conclusion, but merely simplifies the algebra somewhat), and excites a π-pulse coupling $|2\rangle_A$ to $|3\rangle_A$, and then stops the excitation. The degree of attenuation is such that the Rabi frequency is much less than the transition frequency, so that the RWA is valid. Similarly, Bob uses a field, attenuated as above, to excite a π-pulse coupling $|2\rangle_B$ to $|3\rangle_B$, and then stops the excitation. Using digital communications over a classical channel, Alice and Bob wait until they both know that these excitations have been completed. The resulting state is then given by

$$|\psi(t)\rangle = [|1\rangle_A|3\rangle_B \exp(-i\omega t - i\chi) - |3\rangle_A|1\rangle_B \exp(-i\omega t - i\phi)]/\sqrt{2}. \tag{9}$$

The next step is for Alice to make a measurement along the $|1\rangle_A \leftrightarrow |3\rangle_A$ transition. For this process, she chooses a much larger value of g_0, so that the RWA cannot be made. The state she wants to measure is the one that would result if one were to start from state $|1\rangle_A$, and evolve the system for a $\pi/2$ pulse using this stronger g_0

$$|+\rangle_A \equiv \frac{1}{\sqrt{2}}[\{1 - 2\eta\Sigma\}|1\rangle_A + ie^{-i(\omega t+\phi)}\{1 + 2\eta\Sigma^*\}|3\rangle_A], \tag{10}$$

where we have made use of (9). The state orthogonal to $|+\rangle_A$ results from a $3\pi/2$ pulse

$$|-\rangle_A \equiv \frac{1}{\sqrt{2}}[\{1 + 2\eta\Sigma\}|1\rangle_A - ie^{-i(\omega t+\phi)}\{1 - 2\eta\Sigma^*\}|3\rangle_A]. \tag{11}$$

To first order in η, these two states are each normalized, and orthogonal to each other. As such, one can re-express the state of the two atoms in (9) as

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}[|+\rangle_A|-\rangle_B - |-\rangle_A|+\rangle_B] \tag{12}$$

here we have defined

$$|+\rangle_B \equiv \frac{1}{\sqrt{2}}[\{1 - 2\eta\Sigma\}|1\rangle_B + ie^{-i(\omega t+\chi)}\{1 + 2\eta\Sigma^*\}|3\rangle_B], \tag{13a}$$

$$|-\rangle_B \equiv \frac{1}{\sqrt{2}}[\{1 + 2\eta\Sigma\}|1\rangle_B - ie^{-i(\omega t+\chi)}\{1 - 2\eta\Sigma^*\}|3\rangle_B]. \tag{13b}$$

She can measure the state $|+\rangle_A$ by taking the following steps: (i) shift the phase of the B-field by π, (ii) fine tune the value of g_0 so that $g_0'(t) = \omega/2m$, for an integer value of m, (iii) apply the field for a duration of $T = \pi/2g_0'(T)$, and (iv) detect state $|1\rangle_A$. Note that the constraint on g_0 ensures that $T = m\pi/\omega$, which is necessary for time reversal to work in the absence of the RWA. Once Alice performs this measurement, the state for Bob collapses to $|-\rangle_B$, given in (14). Note that if η is neglected, then the measurement produces a $|-\rangle_B$ that contains no information about the phase of Alice's oscillator, which is analogous to the Jozsa protocol [9]. Note also that Bob does not need to know the actual value of g_0 used by Alice in order for this protocol to work.

In the present case, $|-\rangle_B$ does contain information about the amplitude and the phase of Alice's oscillator signal. In order to decipher this, Bob measures his state $|1\rangle_B$. The probability of success is:

$$p_\phi \equiv |_B\langle 1|-\rangle_B|^2 = \frac{1}{2}[1 + 2\eta\sin(2\phi)], \tag{14}$$

where we have kept terms only to the lowest order in η. Of course, the value of $\phi(\text{mod } 2\pi)$, the phase difference, can not be determined from knowing $\sin(2\phi)$ alone. However, this whole process can be repeated after, for example, Alice shifts the phase of her B-field by $\pi/2$, so that Bob can determine the value of $\cos(2\phi)$. It is then possible to determine the value of $\phi(\text{mod } 2\pi)$ unambiguously.

The overall process can be carried out in one of two ways. First, consider the situation where Alice and Bob starts with X pairs of atoms, and entangle each pair in the form of (13). Then, over a digital communication channel, Alice sends Bob a list of the M atoms she found in state $|1\rangle_A$ after performing her measurement process described above. Bob performs his measurement only on this subset of atoms. Suppose he finds L number of atoms in state $|1\rangle_B$. Then

$$\zeta \equiv \left(\frac{L}{M} - \frac{1}{2}\right) \to \eta\sin(2\phi), \quad \text{for large } M. \tag{15}$$

Thus, the value of η determined asymptotically for a large number of entangled pairs will reveal the value of $\sin(2\phi)$. Alternatively, if only a single pair of atoms is available, then the same result can be obtained by repeating the whole process X times, assuming that ϕ remains unchanged during the time needed for the process.

Note that what is determined by Bob is ϕ, not Ω. Thus, it is not possible to measure the absolute phase difference in this manner. However, one could use this approach of phase teleportation in order to achieve frequency locking of two remote oscillators. This is illustrated in Fig. 2. Briefly, assume that Bob has an array of N atoms. Assume further that Alice also has an identical array of atoms. For our protocol, the physical separations between the neighboring atoms do not have to match. In principle, one can create such an identical pair of arrays by embedding N rows of atoms (or quantum dots) in a substrate patterned lithographically, with two atoms in each row, and then splitting it in two halves. To start with, the corresponding atoms in each array are entangled with each other using the asynchronous approach of [11]. Here, we assume that the two oscillators may differ in frequency. The frequency-locking algorithm then proceeds as follows. Alice and Bob both apply their fields parallel to their arrays of atoms, so that the phase variation is 2π over their respective wavelengths. After Alice makes her measurements of the state $|+\rangle_A$, using the same set of steps as described above, she informs Bob, over a classical communication channel, the indices of her atoms that were found in this state. Bob now measures the state $|-\rangle_B$ for this subgroup of atoms only, using an analogous set of time-reversed excitation steps which ends in observing his atom in state $|3\rangle_B$. For a given atom in this subgroup, the phase of his field at that location at the time Bob starts the measurement affects the probability of success in finding the atom in state $|3\rangle_B$ at the end of the measurement process. This phase is varied as Bob repeats the measurement for different measurement-starting-times (modulo $2\pi/\omega_B$, where ω_B is the frequency of Bob's oscillator). It is easy to show that there exists a choice of this phase for which the probability of success is 100%. However, the success probability for atoms (in the post-selection subgroup) would vary with location if the frequencies of Bob's and Alice's oscillators are not
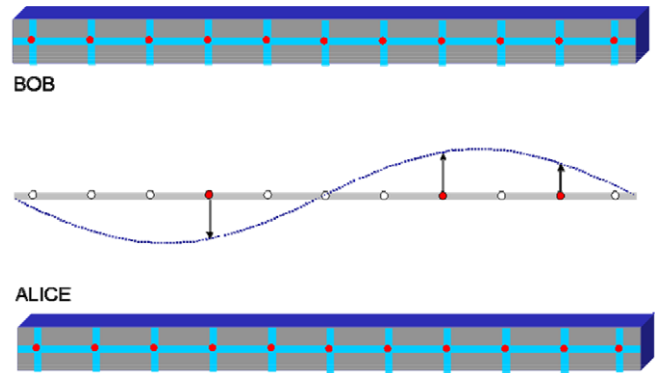

Fig. 2. Schematic illustration of the process to be employed for remote frequency locking. The top (bottom) array shows the atoms co-located with and excited by Bob (Alice). The degree of correlation observed after executing the frequency-locking protocol displays a spatial variation only if the frequencies of Bob's and Alice's oscillators are different, as shown in the middle. Elimination of such a variation leads to frequency locking.

the same. This effect can be used by Bob to adjust his oscillator frequency, thereby achieving frequency locking. The Nyquist sampling criterion dictates that the number of atoms in this subgroup can be as low as only two, so that N can be quite small, thus making this protocol potentially practicable.

The optimal SNR that may be achievable in the protocol outlined here will be determined fundamentally by the SNL. For a measurement interval of τ, the SNR in this case would be $\eta \sqrt{(M\tau/\tau_o)}$, where $\tau_o (\ll \tau)$ is the time needed to carry out a single sequence in the protocol. The values of all these parameters (η, M and τ_o) would depend on the actual technology to be employed in realizing the protocol. As a concrete example, let us assume a value of η to be 0.025, corresponding to a case where $(g_0/\omega) = 0.1$. The values of M and τ_o are not independent of each other; the bigger the M, the bigger the amount of time that would be necessary to carry out a single sequence of the protocol. Note that τ_o determines the speed with which one wants to update the frequencies of the oscillator (*i.e.*, the inverse of the bandwidth of the frequency locking servo). Let us assume a servo bandwidth of 1 kHz, corresponding to a value of $\tau_o = (1 \text{ ms}/2\pi)$. For a lithographically patterned substrate, it is easy to envision a value of M as large as 10^4. For an averaging time of 1 s, the SNR is then close to 200. Of course, such an SNR is expected only under ideal conditions. In practice, the SNR can be degraded by imperfect fidelity in the generation of the entanglement, for example.

Given that, to the best of our knowledge, this is the only technique for performing frequency locking in a manner that is independent of the fluctuations in the intervening optical path length, it is not possible to compare this value to any other technique directly. An alternative way of achieving the wavelength locking would be to transmit a radio frequency signal, corresponding to the local clock frequency, from one location to another, for example. In this case, one must take into account effects such as fluctuating dispersion in the intervening atmosphere, which will add an essentially random source of noise, thus degrading the SNR. Another effect to be taken into account is the Doppler shift attributed to the relative motion of the clocks. This in turn requires precise measurement of relative velocities, which is also affected by random fluctuations of the intervening path-lengths. In contrast, for the technique proposed here, no such radio signal transmission

is necessary, thereby circumventing the problem of random density fluctuations in the atmosphere.

To summarize, previously we have shown how the phase of an electromagnetic field can be determined by measuring the population of either of the two states of a two-level atomic system excited by this field, via the so-called Bloch–Siegert oscillation. Here, we show how a degenerate entanglement, created without transmitting any timing signal, can be used to teleport this phase information. This in turn makes it possible to achieve wavelength locking of remote oscillators.

Acknowledgements

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