Optics Communications 281 (2008) 4931–4935

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Demonstration of displacement–measurement–sensitivity proportional to inverse group index of intra-cavity medium in a ring resonator

G.S. Pati *, M. Salit, K. Salit, M.S. Shahriar

EECS Department, Northwestern University, Evanston, IL 60208, United States

ARTICLE INFO

Article history:
Received 6 September 2007
Received in revised form 23 June 2008
Accepted 24 June 2008

OCIS:
020.1670
020.3690
300.6450

ABSTRACT

We show that an intra-cavity medium with dispersion modifies the sensitivity of the cavity resonance frequency to a change in its length by a factor inversely proportional to the group index, n_g, in the medium. For a positive group index characteristic of the slow-light media, with a very large value of n_g, this effect can help in constructing highly frequency-stable cavities for various potential applications without taking additional measures for mechanical stability. For a negative group index characteristic of the fast-light media, with n_g close to a null value, this implies enhancement in sensitivity to change in cavity length. This enhancement in turn can be employed to increase the sensitivity of a ring laser gyroscope.

Optical cavities are used in applications such as optical gyroscopes, laser frequency stabilization, cavity ring-down spectroscopy, and quantum information processing [1–4]. The response of a cavity can be significantly modified by using a dispersive intra-cavity medium. In a recent study [5] we discussed a cavity-based optical gyroscope containing a negative dispersive medium, where the shift in the cavity resonance frequency due to a rotation varies inversely as the group index, n_g. Our analysis in Ref. [5] is based on a relativistic formulation of the Sagnac effect, and does not contradict any existing theoretical and experimental result. For example, if a gyroscope is configured as a Sagnac interferometer, it has been shown theoretically [6–9] and verified experimentally [7,9] that the rotation sensitivity does not depend on the index, nor the dispersion, of the medium. Our analysis in Ref. [5] also confirms this conclusion. This effect can be understood in many different ways, including the invocation of the concept of Fresnel drag [5,6,8,10]. An even more subtle effect occurs when the gyroscope rotates while the medium remains stationary. Under this condition, the Fresnel drag is modified, and is called the Laub drag. Under this scenario, the rotation sensitivity does indeed depend on the dispersion, and is enhanced for positive dispersion ($n_g > 1$), as shown in Ref. [11]. This conclusion is also confirmed by our analysis in Ref. [5]. Of course, this scenario is not very useful for rotation sensing, and is merely of intellectual curiosity.

On the other hand, the situation is quite different when a gyroscope is configured as a resonator, either passive [12] or active [13]. In this case, the rotation sensitivity does indeed depend on the index of the medium, as discussed in detail in Ref. [7]. The reason for this distinction is not difficult to see. In the case of an interferometric configuration, the optical field moving in one direction interferes with the optical field moving in the opposite direction. The form of the Fresnel drag (which results directly from the relativistic addition of velocities) is such that any dependence on the index of the medium cancels out. On the other hand, for a resonator version of the Sagnac gyroscope, optical field moving in one direction interferes with multiple iterations of itself, and this cancellation does not occur. Hence, the dependence on the index survives. In our analysis in Ref. [5], we confirm this conclusion, and take it one step further by studying how the sensitivity depends not only on the mean value of the index, but its variation with frequency (i.e., dispersion). Specifically, as stated above, we find that the sensitivity scales as the inverse of the group index.

In particular, we showed that the sensitivity of the gyroscope is enhanced when the group index approaches a null value, a condition known as the critically anomalous dispersion (CAD). The apparent divergence of the sensitivity as the group index reaches the null value is avoided by the fact that the dispersion is linear over a finite range. In Ref. [5], we have also shown that the enhancement in rotational sensitivity is equivalent to a corresponding increase in displacement–measurement–sensitivity for a cavity containing a negative dispersive medium. Furthermore, while the enhancement occurs for negative dispersion, the process is a manifestation of the general property that the sensitivity (for both rotation and displacement measurement) varies as the inverse of the group index of the intra-cavity medium. Therefore, the general principle behind this enhancement can be tested with any dispersion, positive or negative.

* Corresponding author. Address: Delaware State University, Department of Physics and Pre-engineering, Dover DE-19901, United States. Tel.: +1 302 857 6714; fax: +1 302 857 7482.
E-mail address: gspati@desu.edu (G.S. Pati).

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doi:10.1016/j.optcom.2008.06.075

We have recently demonstrated a cavity with an intra-cavity medium near the CAD condition, in the context of studying the so-called white light interferometer [14]. In principle, this system could also be used to demonstrate directly the enhancement in displacement–measurement–sensitivity. However, as shown in detail in Ref. [5], the enhancement near the CAD condition becomes very large, so that the effect would be evident only for a very small change in the cavity length, since the negative dispersion is present only over a limited range. This is not a concern regarding the utility of this process, since the enhancement in sensitivity is only useful when measuring very small rates of rotation. Nonetheless, this fact poses significant experimental constraints in demonstrating the enhancement directly. As discussed later on in this paper, in order to see the enhancement effect with the type of apparatus reported in Ref. [14], it would be necessary to reduce the empty cavity linewidth by more than three orders of magnitude (from 3 MHz down to about 1 kHz). Achieving such a linewidth requires the use of very high reflectivity mirrors, a monolithic base made of Zerodur material, for example, and a very high degree of vibration isolation. Furthermore, the Ti:Sapphire laser used for the experiment has to be stabilized to a linewidth of less than 1 kHz. Alternatively (as discussed in Ref. [5] as well as later on in this paper), one could use an active ring laser cavity augmented with the negative dispersion material to demonstrate this effect. In our laboratory, we are currently pursuing both of these very arduous approaches.

While this effort is underway, here we report an experiment that confirms the essential aspect of this enhanced sensitivity, by demonstrating that the displacement–measurement–sensitivity varies inversely as the group-index of the intra-cavity medium. The experiment is performed using a steep dispersion produced via electromagnetically induced transparency (EIT) [15]. The effect of dispersion on the cavity response has been studied earlier. In particular, it has been shown that the cavity linewidth gets narrowed under positive dispersion [16–19]. Furthermore, Ref. [19] has reported the effect of the medium dispersion on the cavity sensitivity. However, it does not show that the displacement–measurement–sensitivity varies as the inverse of the group index, thereby failing to realize that the sensitivity can be increased considerably by using a negative dispersion medium where the group index can have a null value over some spectral range.

As mentioned above, the key purpose of this experiment is to establish indirectly the validity of the enhancement in sensitivity under the CAD condition. However, the reduced-sensitivity demonstrated here may find applications of its own. For example, the insensitivity achieved by a large group index may be used in forming highly stable cavities. Such cavities may find applications in laser frequency stabilization, producing squeezed light for gravitational wave interferometers [20], and in other applications that rely on narrow linewidth, low loss optical cavities immune to external perturbations.

In our experiment, a 10 cm long rubidium vapor cell is used inside a 100 cm long, four-mirror ring cavity as shown in Fig. 1a. The windows of the cell are anti-reflection coated. The empty cavity finesse and linewidth are close to 100 and 3 MHz, respectively. One mirror is attached to a piezoelectric transducer (PZT) used in adjusting the cavity length. The Λ-system is realized in the D_2 lines of ^{85}Rb vapor as shown in Fig. 1b. The probe and the pump are obtained from a Ti:Sapphire laser. The probe is aligned to resonate in the cavity and is combined with a co-propagating, orthogonally-polarized pump using an intra-cavity polarizing beam splitter (PBS). A second intra-cavity PBS separates the pump from the probe before detection. The probe frequency is scanned using an acousto-optic modulator (AOM) in a double-pass configuration. A flipper mirror was used in the cavity beam path to interrupt the cavity resonance, in order to monitor, when necessary, the probe field under the EIT condition. First, the probe frequency ω_0 is set to satisfy the two-photon resonance condition. The cavity is then made resonant at this frequency by adjusting its length L. The cavity length is now actively held fixed at this value by using the cavity output produced by a resonating lock beam, with its frequency set at multiples of the free spectral range (FSR) away from the probe frequency.

The linewidth of the cavity containing the vapor cell, in the absence of EIT, is measured to be about 8 MHz. This is nearly three times broader than the empty cavity linewidth. This broadening is attributable primarily to the absorption by the vapor. The effect

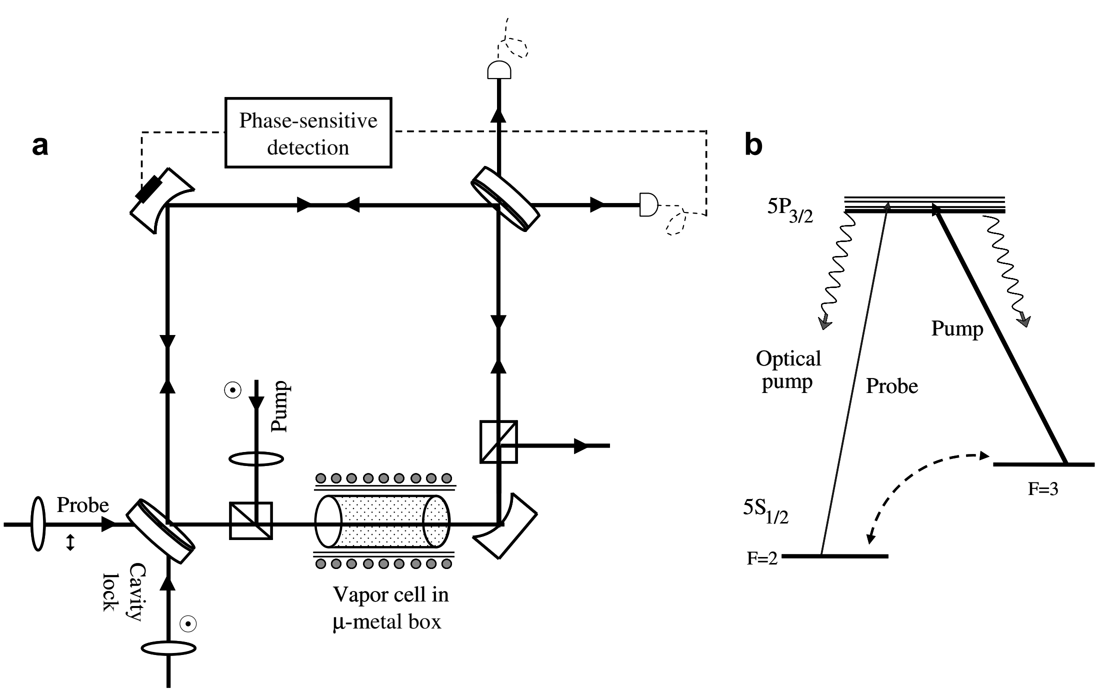

Fig. 1. (a) Schematic of experimental set-up, (b) two-photon transitions used in ^{85}Rb D_2 line for EIT.

of dispersion is observed by switching the pump on. The EIT-affected cavity linewidth (∼1.5 MHz) is narrower by a factor of nearly five than the cavity linewidth without dispersion (Fig. 2a). This is also narrower than the empty cavity linewidth (3 MHz), as expected [5]. Similar narrowing has also been observed in earlier experiments [15,17,19]. For a medium of length ℓ, the narrowing can be predicted from the dispersion-induced change of phase $\delta\varphi = (\partial\varphi/\partial\omega)\cdot\delta\omega$ for a deviation $\delta\omega$ away from the resonant frequency ω_0, with the phase expressed as $\varphi = \omega\cdot(L-\ell)/c + \omega\cdot n(\omega)\cdot\delta\ell/c$. The modified cavity linewidth $\delta\omega'_{1/2}$ can be written as [18]:

$$\frac{\delta\omega'_{1/2}}{\delta\omega_{1/2}} = \left|\left(\sin^{-1}\left[\frac{1-R\rho}{2\sqrt{R\rho}}\right]\Big/\sin^{-1}\left[\frac{1-R}{2\sqrt{R}}\right]\right)\Big/\left[1+(n_g-1)\cdot\frac{\ell}{L}\right]\right|,$$
$$n_g \equiv 1+\omega\cdot\frac{\partial}{n}\partial\omega\bigg|_{\omega=\omega_0}, \quad \rho \equiv e^{-\frac{\alpha\ell}{2}} \tag{1}$$

where $\delta\omega_{1/2}$ is the empty cavity linewidth, n_g is the group index, R is the reflectivity of each of the beam-splitters, and α is the loss coefficient. If dispersion is negligible (i.e., $n_g = 1$), then the linewidth is broadened due to the loss induced attenuation. To see the effect of dispersion, consider first the simplest case of $L = \ell$, for which the narrowing is inversely proportional to the group index n_g. When $\ell < L$, the behavior of the broadening is qualitatively the same, except that the value of n_g needed to achieve the same degree of narrowing is larger by a factor close to (L/ℓ). Note that narrowing occurs for any positive dispersion: $n_g > 1$. It can also occur for negative dispersion under certain conditions. For $L = \ell$, the necessary condition is that the negative dispersion has to be steep enough so that $n_g < (-1)$. For $\ell < L$, the negative dispersion has to be steeper yet, so that $n_g < (1-2L/\ell)$. Fig. 2b shows a simulation illustrating the narrowing, obtained by using an analytic expression [5,21] for $n(\omega)(=\sqrt{1+\mathrm{Re}[\chi]})$ and $\alpha(=[\mu_0\omega^2/k^2]\mathrm{Im}[\chi])$, where χ is the susceptibility. A close match to the observed broadening is obtained for the estimated values of the experimental parameters.

Next, we describe the effect of the cavity length variation on its resonance frequency. Fig. 2a shows the shift in the center of cavity resonance both in the presence and in the absence of the medium dispersion, as L is changed. L is reduced (increased) by simply red-(blue-) detuning the lock frequency ω_c away from its original value. In the absence of dispersion, the center of cavity resonance gets shifted by $\Delta\omega = \omega_c - \omega_0$. However, in the presence of dispersion, the center of cavity resonance is shifted by a *smaller* amount $\Delta\omega'$ from ω_0. The magnitude of this shift is a measure of the cavity sensitivity to its length change. This is quantified by calculating the change in the resonance frequency for a given ΔL, using the resonance condition $\omega = 2\pi Nc/(n(\omega)L)$, (where N is a large integer $O(L/\lambda)$), and is given by

$$\Delta\omega'_o = \Delta\omega_o/[1+(n_g-1)\cdot(\ell/L)] \tag{2}$$

where $\Delta\omega_0$ and $\Delta\omega'_0$ are the shifts in the resonance frequency for the empty cavity and the loaded cavity, respectively. The frequency shift, similar to the narrowing, is inversely proportional to n_g, and this effect is also scaled nearly by (L/ℓ). This effect is related to frequency-pulling described theoretically in earlier Refs. [16,18].

Fig. 3 shows a sequence of such results when the resonance frequency of the cavity is gradually changed from ω_0. Two effects are observed. First, the peak amplitude of the transmitted signal for the loaded-cavity resonance is reduced with increasing $\Delta\omega_0$ (or ΔL). This is because at the new resonance frequency, the probe now does not satisfy the two-photon resonance condition for EIT. In particular, if the shift $\Delta\omega'_0$ (corresponding to a given $\Delta\omega_0$) exceeds the EIT linewidth, the signal level corresponding to maximum probe transmission for the loaded-cavity resonance is reduced. Second, for increasing $\Delta\omega_0$, the reduced shift $\Delta\omega'_0$ is no longer determined strictly by the expression in Eq. (2). This is because Eq. (2) assumes the variation of the index to be linear. In practice, the linearity assumption fails to hold once $\Delta\omega'_0$ becomes comparable to the EIT linewidth (∼1 MHz). A more general expression for the reduced shift $\Delta\omega'_0$ can be obtained by going beyond this assumption. It can be found by solving the following self-consistent equations for $\Delta\omega'_0$:

$$\Delta\omega'_0 = \frac{\Delta\omega_0}{1+[n_{g,\mathrm{eff}}(\Delta\omega'_0)-1].\frac{\ell}{L}};$$
$$n_{g,\mathrm{eff}}(\Delta\omega'_0) \equiv 1+\omega'_0\left[\frac{n(\omega'_0)-n(\omega_0)}{\Delta\omega'_0}\right]$$
$$= 1+(\omega_0+\Delta\omega'_0)\left[\frac{n(\omega_0+\Delta\omega'_0)-1}{\Delta\omega'_0}\right] \tag{3}$$

To understand this expression, consider first the simple case where $L = \ell$. The linewidth reduction factor is then given by the inverse of the effective value of the group index, which is determined by the difference of the index value at the frequency ω'_0 corresponding to the shifted loaded-cavity resonance, and the index value (≅1) at the central frequency ω_0. For $\ell < L$, a larger value of n_g, eff is

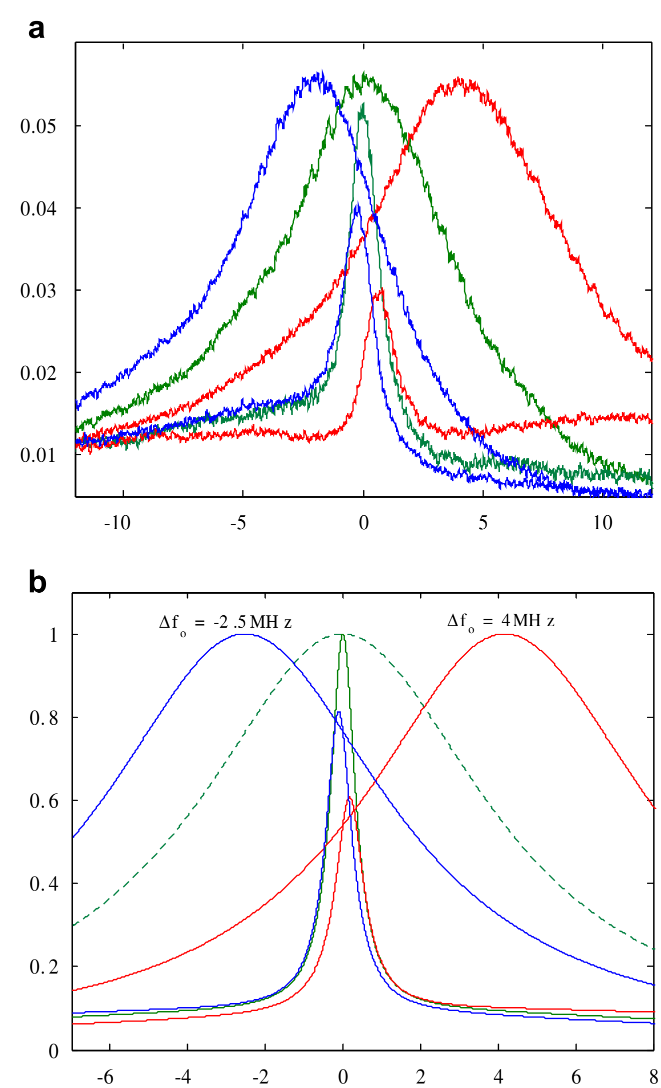

Fig. 2. (a) Experimental results showing linewidth narrowing and reduced shift in cavity resonance for $\Delta f_0 = -2.5$, 0 and 4 MHz, respectively (b) theoretical model for cavity response (cavity linewidth = 8 MHz, EIT Linewidth = 1 MHz, $L/\ell = 10$ and $n_g \approx 50$). In the figures, the horizontal axis represents the frequency in MHz and the vertical axis represents the cavity transmission in arbitrary units.

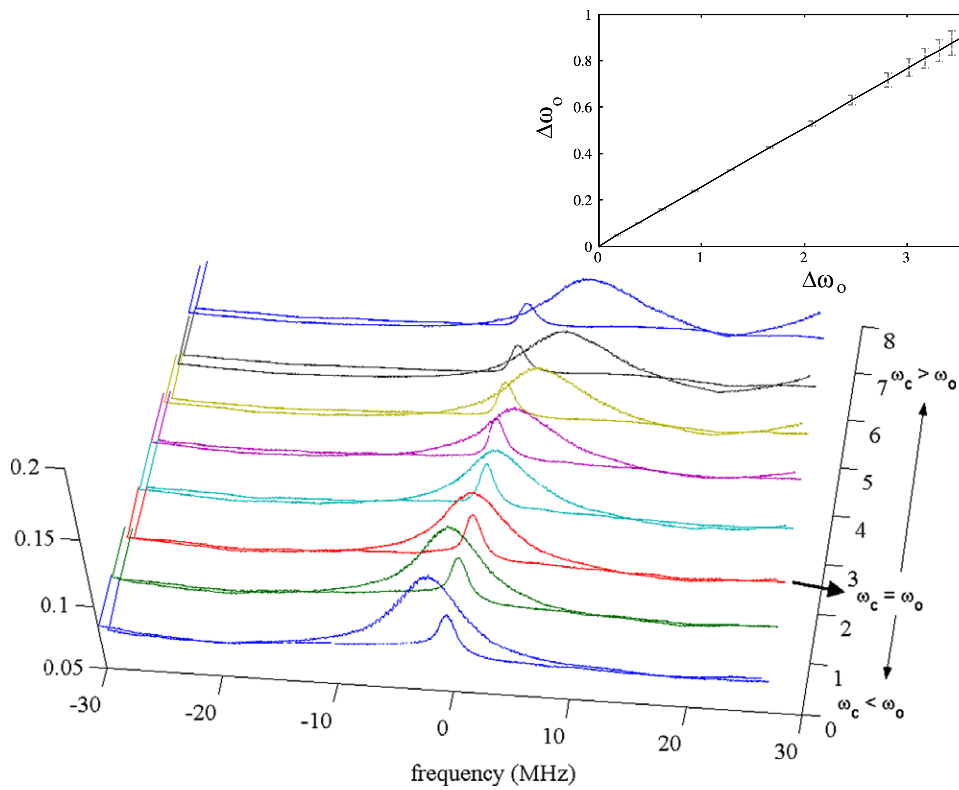

Fig. 3. Cavity resonances with (narrow) and without (broad) the effect of the intra-cavity medium dispersion. The inset in the figure shows measured frequency shifts corresponding to the peaks of resonances, illustrating reduced sensitivity to length change. The straight line superimposed on the data represents the fit to Eq. (2).

needed to get the same effect, just as explained in the case of Eq. (2). An exact solution of this equation can be found graphically by finding the point of intersection between the following pair of equations plotted as a function of $x(\equiv \Delta\omega_0')$

$$y = \Delta\omega_o/[1 + (\omega_o + x)[(n(\omega_o + x) - 1)/x] \cdot (\ell/L)]; \quad y = x \tag{4}$$

The linewidth at the shifted resonance is still given by Eq. (1), except that n_g is replaced by the local group index at ω_0', defined as $n_{g,local} = 1 + \omega_o'(\partial n/\partial\omega)|_{\omega_o'}$ [5]. The inset in Fig. 3 shows experimentally measured values of the reduced shift $\Delta\omega_0'$ as a function of $\Delta\omega_0$. Here, the range of $\Delta\omega_0$ is chosen such that $\Delta\omega_0'$ is restricted to within the EIT linewidth. The slope of the linear fitting shows sensitivity reduction in agreement with the theoretically expected reduction factor, S, defined as $S \equiv 1+(n_g - 1)\cdot(\ell/L) \approx 5$. Experimentally, it should be possible to achieve a much larger value of S via optimization of the experimental parameters such as (ℓ/L) and n_g.

A cavity of this type has been proposed for frequency dependent squeeze-amplitude attenuation and squeeze-angle rotation for gravitational wave detection (GWD) [20]. In Ref. [20], it has been pointed out that such a cavity has significant advantages over other techniques due to the narrowness and tunability of the linewidth, coupled with low optical losses. The results here point out an additional advantage of this filter in that it is highly insensitive to fluctuations in the position of the cavity mirrors.

While the results presented here is for positive dispersion only, the theoretical model is valid for negative dispersion as well. We already described above the conditions under which negative dispersion also causes linewidth narrowing and reduced sensitivity. However, there is a range of negative dispersion for which the effect is reversed. For simplicity, let us restrict the discussion to the case where $\ell = L$. In that case, if the dispersion is such that $|n_g| < 1$, the linewidth is broadened, and the sensitivity is enhanced. The effect is most pronounced at the CAD condition where $n_g = 0$. As can be seen from Eq. (1), the width of the cavity resonance becomes infinite in this case. Similarly, as can be seen from Eq. (2), the sensitivity enhancement also becomes infinite. The divergence is a result of the assumption that the index variation is linear around ω_0 for all frequencies. Once the true behavior of the index variation is taken into account, it can be shown that the cavity linewidth is large but finite [5]. Similarly the sensitivity enhancement is also large but finite [5]. The enhanced bandwidth under this condition has been proposed for constructing the so-called white-light-cavity (WLC) [22,23], which can be useful for enhancing the sensitivity-bandwidth product for GWD, for example. The WLC effect has been demonstrated using a whispering gallery mode cavity [24]. However, this device is unsuitable for application to GWD [14]. We have demonstrated [14] a version of the WLC that is suited for GWD, using a variation of the experiment we have described above. Specifically, we used a dual-peaked Raman [25] gain to realize the negative dispersion necessary for this demonstration.

The enhanced sensitivity under the CAD condition may be used to increase the signal for a given rotation rate (i.e., enhanced sensitivity) in a ring resonator gyroscope. This is because the effect of rotation can be shown to be equivalent to a change in the cavity length, as shown in Ref. [5]. However, for a passive resonator, this does not lead to an actual enhancement in the capability of a rotation sensor, which is characterized by the minimum measurable rotation rate. This results from the fact that while the signal level is enhanced, the linewidth broadening compensates for it, so that there is no net improvement in the sensitivity. However, as we have discussed in detail in Ref. [5], this is not the case if an active resonator (i.e., a ring laser gyroscope: RLG) is used. To see why, note that for an RLG, the linewidth depends only on the cavity decay time [26], which is unaffected by the CAD condition [22,23,26]. Thus, in order to demonstrate fully that the CAD condition can be used for ultra-precise rotation sensing, it is necessary to realize an

RLG with a built-in CAD medium. This is a difficult challenge, and efforts are underway in our laboratory to realize such a device, as mentioned earlier.

While a passive cavity cannot be used to improve the performance of a rotation sensor, it should nonetheless be possible to use it to establish the idea that the CAD condition leads to an enhancement in sensitivity. Thus, one might expect that the experimental apparatus used by us to observe the WLC effect under the CAD condition in Ref. [14] could easily be used to demonstrate this enhancement. However, there are some significant difficulties in doing so, for the following reasons. The enhancement factor is non-linear: it decreases with increasing values of the empty cavity frequency shift, $\Delta\omega_0$ (corresponding to ΔL, or equivalently, a rotation rate) [5]. Furthermore, in order for the enhancement to be evident, the value of the loaded-cavity frequency shift, $\Delta\omega_o'$ (i.e., the enhanced shift) must be less than the dispersion bandwidth. Thus, for the limited dispersion bandwidth realized in Ref. [14], one must use a very small value of ΔL in order to observe an enhancement. This in turn requires a resonator with a linewidth as narrow as 1 KHz, which is more than three orders of magnitude smaller than the linewidth (~3 MHz) of the cavity used in Ref. [6]. In order to achieve such a linewidth, mirrors with much higher reflectivities have to be used, the base of the resonator has to be constructed with temperature insensitive material such as Zerodur, and high degree of vibration isolation has to be employed. Furthermore, the Ti:Sapphire laser used for the experiment has to be stabilized further using the so-called Pound–Drever–Hall technique [27], to produce a linewidth of less than 1 KHz. The required modifications are non-trivial, and efforts are underway in our laboratory to implement these changes to the apparatus.

Acknowledgements

This work was supported in part by the Hewlett–Packard Co. through DARPA and the Air Force Office of Scientific Research under AFOSR Contract No. FA9550-05-C-0017, and by AFOSR Grant No. FA9550-04-1-0189.

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