

corresponds to $f(z_L) \left[1/\sqrt{T} + \sqrt{R}/\sqrt{T} \right]$ and the minimum ($\equiv E_{\min}$) to $f(z_L) \left[1/\sqrt{T} - \sqrt{R}/\sqrt{T} \right]$. In particular, the lower inset in Fig. 4(b) indicates that for $f(z_L) \approx 1$, $|S_2(z,t)|^2$ oscillates between $|E_{\min}|^2 = 0.089$ and $|E_{\max}|^2 = 11.24$.

Another important aspect of the result shown in Fig. 4 is the absence of any reflection by the cavity. Of course, this follows from the fact that each constituent wave is resonant for the ideal WLC condition of $n_g = 0$, as discussed above. We can also see this explicitly by considering the expressions for E_{r1} given by Eq. (2) and E_{r2} given by Eq. (7.a), keeping in mind that the net reflected beam is given by the sum of these two. By setting $2k_d L = (2m+1)\pi$ on resonance in Eq. (7.c), we get $E_{r2} = \exp(-j\pi/2)\sqrt{R}\exp(j\phi_{r2})$ where $\phi_{r2} = k_1(P-z_1) + k_1(P-z) - \omega(t-t_1) + \pi$. Comparing this to Eq. (2), we see that $E_{r2} = -E_{r1}$, so that the net reflected field is zero. Since this conclusion is true for each constituent wave, there is no reflection by the ideal WLC.

Finally, Fig. 4(d) illustrates that the pulse emerging from the ideal WLC (black trace) is advanced compared to a reference pulse (blue trace) propagating through a non-dispersive medium of index n_0 . The distance of advancement $(n_0 - n_g)L = n_0 L$, as expected, meaning that the pulse suffers no time delay at all in crossing the WLC. In Fig. 4(e), we show the corresponding result in time domain, again calculated by using the TTF method. The advance in time compared to the reference pulse is in agreement with the result of Fig. 4(d). Of course, we would see the same amount of advancement, in time or space, if we did not use a cavity, but still used a dispersive medium with $n_g = 0$.

We note that this type of advancement using a free space medium cannot be used to violate causality for any real system [18, 20]; the same conclusion holds for an ideal WLC. The apparently unphysical nature of this advancement is attributed to the unavoidable limitation of our model, since the Gaussian input pulse has an infinite temporal extent. If it were possible to model the propagation of a pulse with a true front, we expect to find that the front of the pulse would never propagate faster than the vacuum speed of light. However, such a pulse has an infinite bandwidth, and cannot be studied using the spectral decomposition method employed here. This interpretation is essentially the same as what is offered in explaining the superluminal propagation of a Gaussian pulse envelope through a free-space fast-light medium.

To summarize, in order to understand the behavior of a pulse inside a cavity, we have developed an approach that starts by finding a self-consistent solution for a monochromatic field of infinite spatial and temporal extents, and determine its amplitudes before, inside, and after the cavity. We then construct a Gaussian input pulse by adding a set of these waves, properly phased and weighted, to represent a moving pulse before the cavity. Adding these waves at various time intervals then yields the complete spatial profile everywhere, including before, inside and after the cavity. In particular, it reveals waves in both forward and backward directions, including multiple bounces occurring inside the cavity. This approach is generic, and can be applied to any situation, including an empty cavity. We first confirm the prediction of this model by analyzing the behavior of a pulse passing through an empty cavity, and comparing the prediction of the output with the one produced by the TTF method. We then apply the technique to a cavity containing a fast-light medium. The output pulse produced this way again agrees with the prediction of the TTF method. The resulting model allows us to visualize the behavior of the pulse as it propagates superluminally inside the cavity, and interferes with itself through multiple bounces. For the limiting case of a vanishing group index over the entire bandwidth of the pulse, an interference pattern is formed immediately after the pulse enters the cavity, with an output pulse emerging with no

time delay or distortion. The results obtained here illustrates the physical mechanism behind pulse propagation through a white light cavity, a process we have proposed earlier for realizing a high bandwidth, long delay data buffering system

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