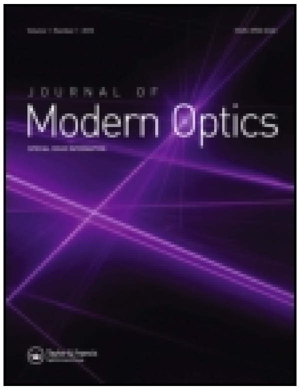

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Recently, we have been investigating the development of a superluminal ring laser, where finely tuned anomalous dispersion leads to an enhancement in the sensitivity of the laser frequency to a change in the cavity length, by as much as six orders of magnitude, for applications such as hypersensitive rotation sensing and accelerometry, as well as for gravitational wave detection. For such a laser, as well as other lasers that are used for precision metrology, the effective dispersion – manifested in the manner in which the lasing frequency varies as a function of a change in the cavity length due to the index induced by the medium under saturated gain corresponding to steady-state lasing – is of utmost significance. In determining the effective dispersion, the role of inhomogeneous broadening (IB) must be taken into consideration carefully. In this work, we consider an inhomogeneously broadened gain medium in a single mode optical cavity, and study the effective dispersion experienced by the lasing field. It is well known that the steady state index for such a laser cannot be expressed analytically. Previous studies have employed approximate models to interpret the effective dispersion, in two limits: IB is much larger than homogeneous broadening (HB), and IB is insignificant compared to HB. Here, we use an iterative but quickly converging numerical code to determine the exact behavior of the effective dispersion under all conditions, and show that the results agree with the expected behavior in these two limits. This technique paves the way for taking into account the effective dispersion in any inhomogeneously broadened laser, including the superluminal laser, in determining accurately its sensitivity to change in cavity length, as well as it quantum noise limited linewidth.

Keywords: laser; gyroscope; laser sensor; inhomogeneous broadening; dispersion

1. Background and introduction

Currently, lasers are used for a wide range of applications. In most cases, minor variations in the frequency of the laser as function of a change in the cavity length, L, are not important [1–4]. However, for some applications, such as in rotation sensing with a ring laser gyroscope – for which a rotation induces an effective change in L – it is important to understand, as precisely as possible, the variation in the lasing frequency as a function of L. The frequency change depends on the value of the effective index of the saturated gain medium under steady-state lasing condition: the effective dispersion. Recently, we have shown that if the gain medium produces an anomalous effective dispersion, the sensitivity of the laser frequency as a function of L can increase significantly, by as much as six orders of magnitude when the dispersion is tuned to a critical value corresponding to a superluminal group velocity, for application to rotation sensing, accelerometry and gravitational wave detection [5–9]. Such a dramatic effect has rekindled the interest in understanding the precise nature of effective dispersion in a laser.

For a single mode laser with primarily homogeneous broadening (HB), it is relatively simple to develop an exact expression for the effective dispersion, by solving the laser equation while taking into account the saturation of the real and imaginary parts of the electric susceptibility. Similarly, if the inhomogeneous broadening (IB) is much larger than the HB, it is possible to develop an approximate but fairly accurate expression for the effective dispersion. However, in the intermediate regime where IB is comparable to HB, it is not possible to express the effective dispersion analytically. A typical example of a laser where both HB and IB have to be taken into account is a gas laser [4,5]. In such a laser, the natural decay rate of the upper level yields the HB, while the motion of the atoms leads to the Doppler effect induced IB [4,10,11]. In some studies, the Doppler induced IB has been considered under certain approximations [10,12–15].

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In this paper, we use an iterative but rapidly converging numerical code to study the effective dispersion for the general case where no assumption is made regarding the relative magnitudes of IB and HB. We find that the results agree well with the limiting cases where either HB or IB dominates. The general result we produce would be useful for precise modeling of systems such as a helium–neon ring laser gyroscope. This work also paves the way for determining, in the near future, the optimal operating condition for a superluminal ring laser using a gaseous medium [7], and establishing precisely the degree of sensitivity enhancement achievable in such a system.

The remainder of the paper is organized as follows. In Section 2, we study the case of a single mode laser, in the regime of a negligible inhomogeneous broadening, and solve analytically the set of equations that represents the laser system. In Section 3, we consider the inhomogeneously broadened case and show that the given theoretical model is unsolvable analytically, to the best of our knowledge. A numerical solution is discussed and plots of the gain and dispersion of different broadening regimes are presented in the same section. The main findings of this work are summarized in Section 4.

2. Overview of a homogeneously broadened single mode laser

The absorption and dispersion phenomena experienced by light propagating through a medium are governed by the imaginary (χ'') and real (χ') parts of the susceptibility, $\chi(E, v)$, of the medium, respectively. The susceptibility, $\chi(E, v)$, is a function of the electric field envelope, E, and frequency, v, of the propagating light. We consider first a basic, inverted two-level atomic system of the following imaginary and real parts of χ:

$$\chi''(E, v) = -\frac{G\Gamma^2}{2\Omega^2 + \Gamma^2 + 4(v - v_0)^2}, \tag{1}$$

$$\chi'(E, v) = \frac{2G(v - v_0)\Gamma}{2\Omega^2 + \Gamma^2 + 4(v - v_0)^2}, \tag{2}$$

where G is the gain coefficient, Ω is the field Rabi frequency, v_0 is the atomic transition frequency, and Γ is the population decay rate from the excited to the ground level. Here, we consider a limiting case where the medium experiences homogeneous broadening only.

If the gain and cavity parameters are chosen to ensure lasing in a single longitudinal mode (e.g. a ring laser operating unidirectionally with the aid of an optical diode), the real and imaginary parts of the susceptibility of the medium are related to the field envelope, E, and phase, φ, by the laser equations [16]

$$\dot{E} = -\frac{1}{2}\frac{v}{Q}E - \frac{1}{2}vE\chi''(E, v), \tag{3}$$

$$v + \dot{\varphi} = v_c - \frac{\chi'(E, v)}{2}v, \tag{4}$$

where v_c is the resonance frequency for the empty cavity for the corresponding mode. Q is the empty cavity quality factor which is defined as $Q = v/\gamma$ where γ is the amplitude attenuation coefficient.

In steady state, $\dot{E} = 0$, and in the case of a non-zero field, $E \neq 0$, Equation (3) leads to

$$\chi''(E, v) = -\frac{1}{Q}, \tag{5}$$

which is a condition that follows from the fact that the gain $\left(-v\chi''_{IB}/2\right)$ is equal to the cavity loss $(-v/2Q)$ in steady state. This condition can be used to find the field amplitude E, and by extension Ω, as a function of the frequency v. Equations (1) and (5) lead to:

$$\Omega^2 = \frac{G\Gamma^2 Q - \Gamma^2 - 4(v - v_0)^2}{2}. \tag{6}$$

Since Ω^2 is a positive number, Equation (6) yields the following lasing frequency range:

$$-\sqrt{\Gamma^2\frac{GQ - 1}{4}} \leq v - v_0 \leq \sqrt{\Gamma^2\frac{GQ - 1}{4}}. \tag{7}$$

In order to study the dispersion, χ', we take the ratio of Equations (1) and (2) to obtain

$$\frac{\chi'}{\chi''} = \frac{-2(v - v_0)}{\Gamma}. \tag{8}$$

In the lasing region (frequency range given in Equation (7)), substituting Equation (5) into the ratio of χ' and χ'' (Equation (8)) leads to:

$$\chi' = \frac{2(v - v_0)}{Q\Gamma}. \tag{9}$$

We plot in Figure 1 the gain and dispersion as given by Equations (1) and (2) outside of the lasing range and by Equations (5) and (9) within the lasing range (Equation (7)). The lasing range is marked with the two solid vertical straight lines. To the left and right of the vertical lines the gain and dispersion are those of an unsaturated two-level system (Equations (1) and (2)). In between the vertical lines, which is the lasing range, the gain is constant and equal to loss (Equation (5)), while the dispersion has a constant negative slope (Equation (9)).

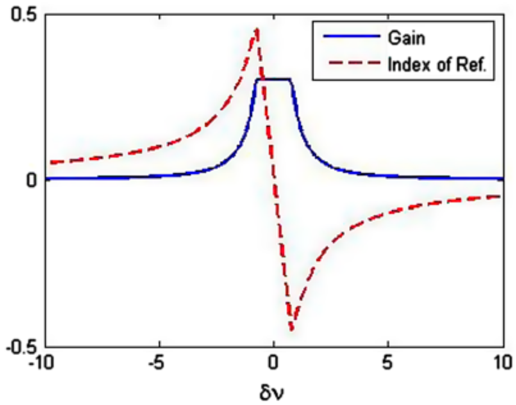

Figure 1. Homogeneous gain (solid line) and dispersion (dashed line) in a laser under steady state operation. (The color version of this figure is included in the online version of the journal.)

3. Model of an inhomogeneously broadened single mode laser

Next, we consider the situation where the medium is inhomogeneously broadened. A specific example would be a vapor gain medium at room temperature. In such a medium, the moving atoms see the frequency of light Doppler shifted from its original value. This shift is accounted for by a frequency distribution, $M(\nu)$, which corresponds to a Gaussian velocity distribution, given by

$$M(\nu) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(\nu-\nu_0)^2/\sigma^2}. \tag{10}$$

Here σ is the Doppler broadening linewidth,

$$\sigma = \frac{\nu_0}{c}\sqrt{\frac{2K_B T}{M}}, \tag{11}$$

where K_B is the Boltzmann constant, T is the temperature of the medium, M is the mass of the atom, c is the speed of light in vacuum, and ν_0 is the central frequency of the distribution. An example of such a profile, with a linewidth of 300 MHz is plotted in Figure 2 as a function of $\delta\nu \equiv \nu - \nu_0$.

The Doppler broadening is accounted for by integrating the real and imaginary parts of the susceptibility over all possible frequencies weighted by the Gaussian distribution, $M(\nu)$, in the form

$$\chi''_{IB} = \int_{-\infty}^{+\infty} M(\nu')\chi''(\nu - \nu')d\nu' \tag{12}$$

$$\chi'_{IB} = \int_{-\infty}^{+\infty} M(\nu')\chi'(\nu - \nu')d\nu' \tag{13}$$

where 'IB' stands for inhomogeneously broadened. The equations for the single mode laser can now be written as

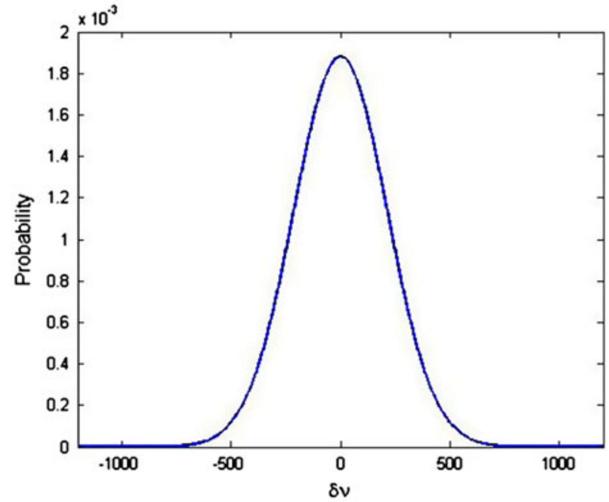

Figure 2. Gaussian frequency distribution (Equation (10)). σ = 300 MHz. (The color version of this figure is included in the online version of the journal.)

$$\dot{E} = -\frac{1}{2}\frac{\nu}{Q}E - \frac{1}{2}\nu E\chi''_{IB}(E,\nu), \tag{14}$$

$$\nu + \dot{\varphi} = \nu_c - \frac{\chi'_{IB}(E,\nu)}{2}\nu. \tag{15}$$

In steady state ($\dot{E} = 0$), we set

$$\chi''_{IB}(E,\nu) = -\frac{1}{Q}. \tag{16}$$

Given the convoluted form of Equations (12) and (13), these laser equations cannot be solved explicitly. In what follows, we therefore find the steady state solutions of Equations (14) and (15) numerically and determine the effective form of the dispersion experienced by the laser.

In our numerical code, we first scan over a desired range of frequencies and calculate the free space (no cavity) homogeneously broadened gain, χ'' (Equation (1)) and dispersion, χ' (Equation (2)), followed by the inhomogeneously broadened gain, χ'_{IB} (Equation (12)), and dispersion, χ''_{IB} (Equation (13)), as shown in Figure 3.

These pre-computed homogeneously and inhomogeneously broadened profiles are then used to study the single mode lasing operation exhibited by the medium when placed in a cavity, such as the case of a ring laser. Note that while the HB case can be solved analytically, we include that in our numerical study in order to provide convenient comparison with the IB case.

Outside the lasing frequency range (Equation (7)) where the gain $(-\nu\chi''_{IB}/2)$ is less than the cavity loss $(-\nu/2Q)$, the lasing Rabi frequency (Ω) has a value of zero and the effective gain is given simply by Equation (1) for the HB case and Equation (12) for the IB case. In the

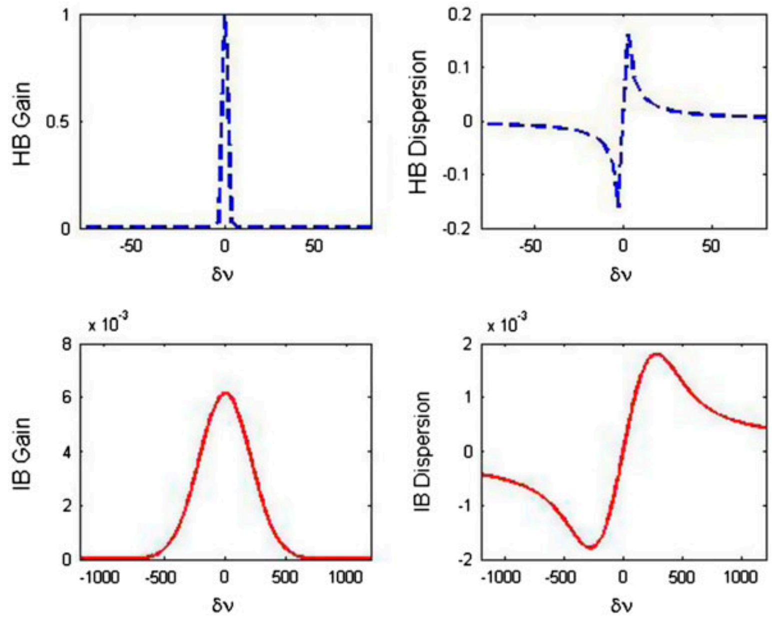

Figure 3. Effective gain and dispersion in the homogeneous (dashed lines) and inhomogeneous (solid lines) regimes, in free space. σ = 300 MHz, Γ = 1 MHz, G = 1, Q = 0.0252. (The color version of this figure is included in the online version of the journal.)

lasing range $(-\chi''_{IB} > Q)$ we search for the value of the Rabi frequency that satisfies the corresponding laser equation (Equation (3)), using an iterative code that converges rapidly. Note that we re-calculate χ''_{IB} every time the Rabi frequency is updated. Using this process, we obtain the Rabi frequency profile due to the lasing field for the HB (dashed line in Figure 4) and the IHB (solid line in Figure 4) cases, which is zero outside of the lasing frequency range. As expected, the inhomogeneous broadening widens the Rabi frequency distribution and reduces its peak value.

These Rabi frequency profiles are then used to calculate the effective indices of refractions: χ' and χ'_{IB}. Figure 5 shows the gain and dispersion profiles in the HB (dashed line) and IB (solid line) cases. The homogeneous and inhomogeneous gain profiles are constant and equal to loss over their corresponding lasing frequency ranges. One expected difference between the two gain profiles is that the inhomogeneous line is broader due to the large inhomogeneous linewidth (σ = 300 MHz) compared to the homogeneous one (Γ = 1 MHz). Unlike the homogeneous dispersion (dashed line in Figure 5) which has a discontinuous dispersion (slope) at the lasing boundaries, the inhomogeneous line has a continuous dispersion and is qualitatively similar to its free space counterpart (solid line in Figure 3).

All of the IB case studies that we have presented so far (Figures 2–4) used a Doppler linewidth that is significantly larger than the homogeneous one, σ >> Γ. Figure 6 is a study of the opposite case, where the Doppler linewidth is significantly smaller than the homogeneous one (σ << Γ). In this limit, as expected, both gain and dispersion show a nearly perfect match between the HB and IB cases. As expected, we can conclude from this part of the study that the inhomogeneous broadening and its mathematical convolution can be ignored in the case of Doppler linewidth that is significantly smaller than its homogeneous counterpart.

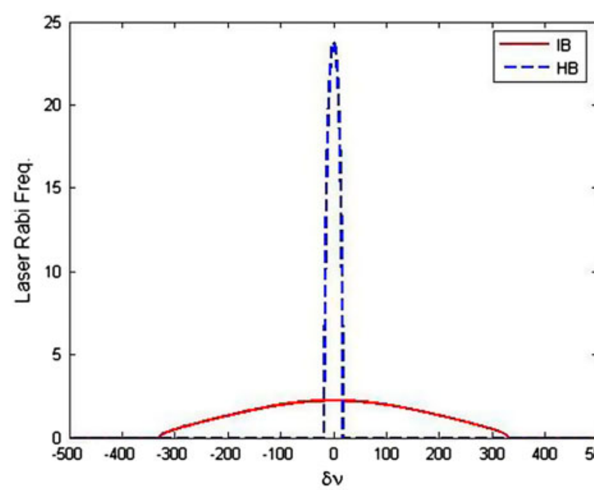

Figure 4. Homogeneously (dashed line) and inhomogeneously (solid line) broadened laser Rabi frequencies, Ω. σ = 300 MHz, Γ = 1 MHz, G = 1, Q = 0.0252. (The color version of this figure is included in the online version of the journal.)

These two limiting cases (i.e. an IB that is significantly larger or smaller than the HB), can be modeled

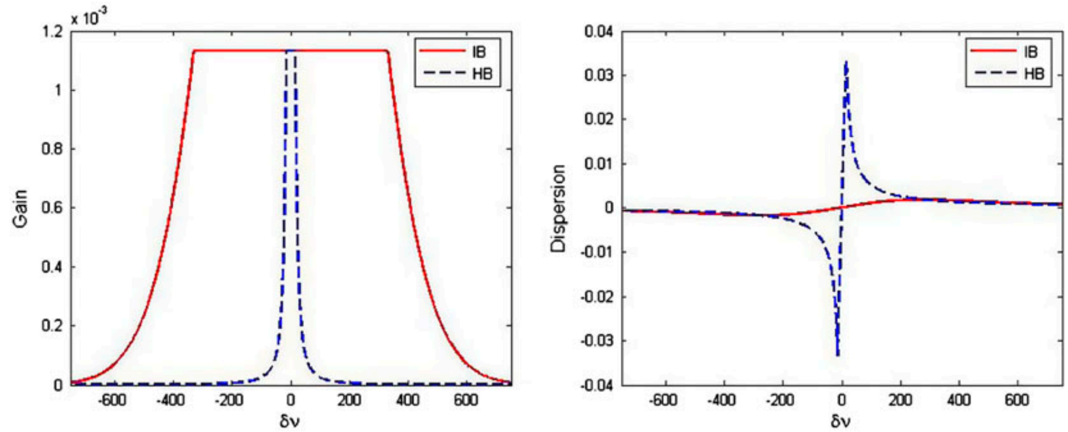

Figure 5. Effective gain and dispersion in the homogeneous (dashed lines) and inhomogeneous (solid lines) broadening cases, in a single mode laser cavity. σ = 300 MHz, Γ = 1 MHz, G = 1, Q = 0.0252. (The color version of this figure is included in the online version of the journal.)

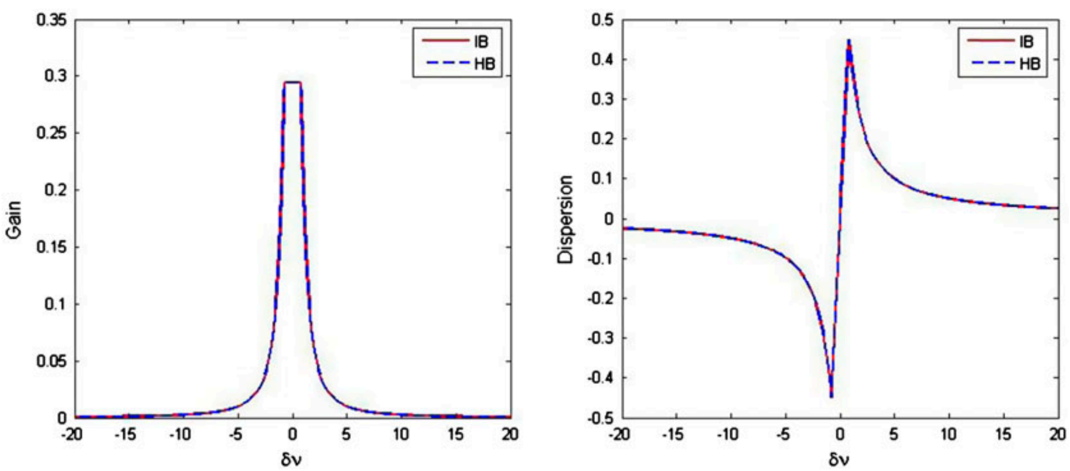

Figure 6. Effective gain and dispersion in the homogeneous (dashed lines) and inhomogeneous (solid lines) broadening cases, in a single mode laser cavity. σ = 0.1 MHz, Γ = 1 MHz, G = 1, Q = 0.0252. (The color version of this figure is included in the online version of the journal.)

under different approximations, as done before in the literature [15]. The situation becomes more challenging when the two linewidths are comparable. Figure 7 shows a plot of the gain and dispersion profiles in the case of an inhomogeneous linewidth that is twice the homogeneous one ($\sigma = 2\Gamma$). While the gain profiles continue to follow the same behavior (i.e. constant and equal to loss over the lasing frequency range), the inhomogeneous dispersion has a profile that is not obvious from any of the simple physical models [15]. The effective dispersion of the type shown in Figure 7 is relevant to understanding how the sensitivity of devices such as a ring laser gyroscope depends on the degree of inhomogeneous broadening.

For the superluminal laser [6], which is a candidate for realizing an ultrasensitive ring laser gyroscope, it is necessary to carry out a similar analysis, taking into account the effect of any IB. The degree of IB in such a laser would depend, of course, on the details of the physical media used in implementing the requisite gain profile with a dip in the center. Using the numerical approach presented here, we have been carrying out such an analysis for several different implementations of the superluminal laser. The result of this analysis will be presented in the near future.

Finally, we point out that the effective dispersion is also of significance in determining the quantum noise limited (QNL) linewidth in any laser, which in turns determines the minimum measurable rotation rate by a ring laser gyroscope [17–20]. This linewidth is influenced by two effects that are related to the effective dispersion. First, the addition of a spontaneously emitted photon (which is the source of the finite QNL linewidth in a laser) modifies the amplitude of the lasing field,

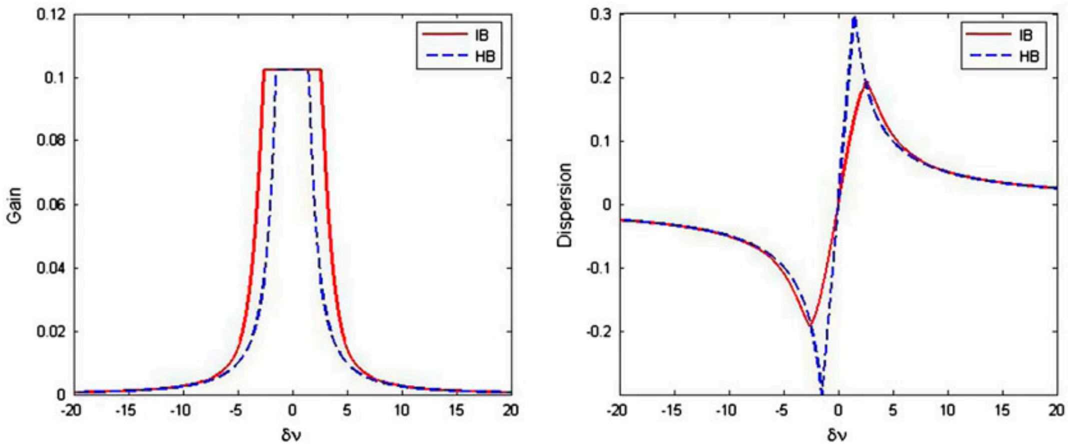

Figure 7. Effective gain and dispersion in the homogeneous (dashed lines) and inhomogeneous (solid lines) broadening cases, in a single mode laser cavity. σ = 2 MHz, Γ = 1 MHz, G = 1, Q = 0.0252. (The color version of this figure is included in the online version of the journal.)

which then undergoes relaxation oscillation to restore the steady state value [20]. This restoration process depends on the saturated gain and saturated (i.e. effective) dispersion of the lasing medium. Second, the effective lifetime of the spontaneously emitted photon is determined by a combination of the cavity parameters and the propagation speed of the photon [5,17], the latter depending on the saturated (effective) dispersion. Thus, the technique we have shown here for finding the effective dispersion for an arbitrary degree of IB is likely to play an important role in determining accurately the QNL linewidth. A detailed analysis of the degree to which the QNL linewidth is affected by the effective dispersion is also in progress, for lasers in general and the superluminal laser in particular, and will be presented in the near future.

4. Conclusion

We have considered an inhomogeneously broadened gain medium in a single mode optical cavity, and studied the effective dispersion experienced by the lasing field. Previous studies have employed approximate models to interpret the effective dispersion, in two limits: IB is much larger than homogeneous broadening (HB), and IB is insignificant compared to HB. Here, we use an iterative but quickly converging numerical code to determine the exact behavior of the effective dispersion under all conditions, and show that the results agree with the expected behavior in these two limits. This technique paves the way for taking into account the effective dispersion in an inhomogeneously broadened laser in determining accurately its sensitivity to change in cavity length, as well as the QNL linewidth. Application of this technique to determining the effective dispersion as well as the QNL linewidth of a superluminal ring laser is in progress, and will be reported in the near future.

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