STUDY OF WHITE LIGHT CAVITY EFFECT VIA STIMULATED BRILLOUIN
SCATTERING INDUCED FAST LIGHT
IN A FIBER RING RESONATOR

A Dissertation

by

HONAM YUM

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2009

Major Subject: Electrical Engineering
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Approved by:
Chair of Committee, Philip Hemmer and Selim Shahriar
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ABSTRACT

Study of White Light Cavity Effect via Stimulated Brillouin Scattering Induced Fast Light in a Fiber Ring Resonator. (August 2009)

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Techniques to control dispersion in a medium have attracted much attention due to potential applications to devices such as ring laser gyroscopes, interferometric gravitational wave detectors, data buffers, phased array radars and quantum information processors. Of particular interest is an optical resonator containing a medium with an anomalous dispersion corresponding to fast-light, which behaves as a White Light Cavity (WLC). A WLC can be tailored to improve the sensitivity of sensing devices as well as to realize an optical data buffering system that overcomes the delay-bandwidth product of a conventional cavity.

This thesis describes techniques to tailor the dispersion for fast-light in intracavity media. We present first a demonstration of fast-light a photorefractive crystal. When placed inside a cavity, such a medium could be used to enhance the bandwidth of a gravitational wave detector. We then describe how a superluminal laser can be realized by adding anomalously dispersive medium inside a ring laser. We identify theoretically conditions under which the sensitivity of the resonance frequency to a
change in the cavity length is enhanced by as much as seven orders of magnitude. This paves the way for realizing a fast-light enhanced ring laser gyroscope, for example. This is followed by the development of a novel data buffering system which employs two WLC systems in series. In this system, a data pulse can be delayed for essentially an arbitrary amount of time, without significant distortion. The delay time is independent of the data bandwidth, and is limited only by the attenuation experienced by the data pulse as it bounces between two high-reflectivity mirrors. Such a device would represent a significant breakthrough in overcoming the delay-time bandwidth product limitation inherent in conventional data buffers.

We then describe our experimental effort to create a fiber-based WLC by using stimulated Brillouin scattering (SBS). Experimental results, in agreement with our theoretical model also presented here, show that the WLC effect is small under the conditions supported by current fiber optic technology. We conclude that future efforts to induce a large WLC effect would require fibers with high Brillouin coefficient and low transmission loss, as well as optical elements with very low insertion loss and high power damage thresholds.
DEDICATION

To my parents and family
ACKNOWLEDGEMENTS

First of all, I want to say that this dissertation is produce of cooperation. I could not finish this dissertation without my family, mentors and friends. I would like to express special thanks my advisor Dr. Philip Hemmer for providing me exposure and opportunity to work in various technical areas. My co-advisor, Dr. Selim Shahriar in Northwestern University, showed me what scientists pursue. I learned from how to explain and how to tackle some of the hardest problems in engineering, science and even affairs that humans stumble on. I also give many thanks to Dr. Mary Salit and Dr. Ken Salit. They were Dr. Shahriar’s students. Dr. Ken Salit helped me settling down here in Northwestern. It was my pleasure to work with Dr. Mary Salit on the super-sensitive gyroscope project. I also learned a lot from colleagues Dr. Zhijie Deng, Dr. Mughees Khan, Dr. Chang-Seok Shin, Huiliang Zhang, Chang-Dong Kim (Both are soon to be Dr.). I want to thank my previous committee members, Dr. Ohannes Eknoyan, Dr. Suhail Zubairy, and current committee members, Dr. Laszlo Kish, Dr. Robert Nevels, and Dr. George Welch for their valuable instructions.

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I. INTRODUCTION

Fast-light and slow-light have been of significant interest to scientific community for the last decade. A number of experimental demonstrations have been reported based on electromagnetically induced transparency (EIT) in cold atomic vapor[1,2] or solid[3], stimulated Raman scattering in hot atomic vapor[4], two wave mixing in photorefractive crystal[5] and stimulated Brillouin scattering(SBS) in fibers[6, 7]. Significant research activities in these fields are still ongoing due to numerous potential applications such as optical buffers, optical variable delay lines, optical memories, quantum information processing, gravitational wave detection, and super-sensitive rotation sensing.

The study presented in this thesis is about finding the method to create slow- and fast-light effect inside optical cavities for the application to gravitational wave detection, data buffering system, and rotation sensing. In order to see how these applications emerge and the related issued, it is important first to review the basic concepts of dispersion and fast- and slow-light, the White light cavity (WLC) effect.

If light propagates in a material with refractive index n, its wavelength is given by $\lambda/n$, where $\lambda$ is the wavelength in vacuum. In general, the refractive index varies with the frequency of light, a process known as dispersion. In some materials, it can be large as well as artificially controlled by applying external optical beams. Under certain conditions, dispersion can have significant effects on optical pulses made up of many frequency components. Each individual frequency component feels a different refractive index and thus experiences a different phase shift. The peak of the pulse coincides with

This thesis follows the style and format of IEEE Transactions on Antennas and Propagation.
the point where all constituent waves meet in phase. The point where the in-phase condition occurs can move faster (in a fast-light medium) or slower (in a slow-light medium) than it would in free space. As such, the optical pulse is seen to propagate in such a medium faster or slower than the speed of light in free space, corresponding to so-called superluminal (fast-light) or subluminal (slow-light) propagation, respectively. In the case of fast-light argument, one might worry about violating Special Relativity. However, since information is encoded in the leading edge of a pulse, corresponding to a bandwidth that always exceeds the bandwidth of any dispersion, information cannot be sent faster than the free space speed of light, so that there is no violation of Special Relativity.

The presence of a fast-light medium can modify the behavior of an optical cavity very significantly. Consider an optical cavity consisting of two high reflectivity partial mirrors. If the length corresponds to an integer times the wavelength of light, the cavity is said to be on resonance, and any light incident on it will transmit fully. Now, consider the same cavity filled with a fast-light medium. The index variation can be tailored in a way so that a large range of frequencies would have the same wavelength. If this wavelength fulfills the resonance condition, the cavity will resonate over the whole frequency range. At resonance, the light intensity builds up in the cavity. In general, the build-up factor is inversely proportional to the cavity linewidth, both being due to the finite losses and mirror transmittivities. In the presence of the fast-light medium, the cavity linewidth becomes broader, while the buildup factor remains unchanged. In other words, the cavity lifetime remains the same as before, since the losses and the mirror
transmittivities remain unchanged. This phenomenon is called the ‘White Light Cavity’ (WLC) effect. The system described here is called a passive WLC. In contrast, an active WLC is a cavity with similar properties, but contains an active gain material as well.

The thesis is organized as follows. In chapter II, we demonstrate superluminal and subluminal pulse propagation using a photorefractive crystal. We use bi-frequency pumps to produce the anomalous dispersion. Fast light in such a medium can be used to enhance the bandwidth of a gravitational wave interferometer without reducing its sensitivity.

In chapter III, we theoretically investigate fast-light in a laser cavity, i.e. an active WLC. We use a narrowband depletion embedded in broadband gain to produce the necessary dispersion for the fast light. The broadband gain is provided by the active gain medium. For the application to gyroscope, the resonance property of the cavity is discussed. The active WLC exhibits high sensitivity to cavity length change such that it provides the prospect to realize a super-sensitive optical gyroscope.

In chapter IV and V, we propose a data buffering system using a pair of passive WLCs. The probe pulse corresponds to a data pulse. In the basic model, a Fabry-Perot(FP) cavity is used to realize each WLC. Numerical simulation indicates that the data pulse is delayed by thousands times the pulse duration, for a given choice of parameters. Other choices would lead to even longer storage time. The proposed buffering system exceeds the delay time-bandwidth product constraint encountered in a conventional data buffer. Next, we describe the design of a buffering system based on fiber ring resonators. The SBS process can be used to control the dispersion [6,7,8,9] in a
fiber resonator. As such, the fiber itself can become the fast-light medium. For the data buffer, two fiber resonators are connected in series. The pump beams are inserted into the resonators to turn them into passive WLCs. As in the FP based system, the delay time can be controlled independently of the bandwidth of the data pulses. Furthermore, we describe how the bandwidth of the system can be extended to be as large as several times the Brillouin frequency shift of nearly 10 GHz in a typical fiber.

In chapter VI and VII, we investigate the parameter constraints in realizing a passive WLC in fiber resonators. First, we measure the Brillouin characteristics of a conventional silica fiber. We describe the construction of a fiber resonator and the experimental demonstration of a passive WLC. In the experiment, the amount of linewidth broadening (i.e., WLC effect) observed is rather small, in agreement with the theoretical model. Numerical analysis indicates that, using the current approach, a fiber with a high Brillouin coefficient and a low transmission loss, as well as fiber components with high damage thresholds are required to induce large WLC effect in fiber resonators. However, current fiber fabrication techniques do not yet yield such a fiber. For future works, we propose an alternative experimental setup to realize an efficient WLC under conditions which current fiber technology can support.
II. FAST LIGHT DEMONSTRATION IN A PHOTOREFRACTIVE CRYSTAL

A. Pulse propagation and Group velocity

Scientists have discovered that the pulse can propagate extremely slower or faster than the speed of light in vacuum\([1, 2, 4, 10]\). To explain this observation, it is crucial to understand a nonperiodic wave (pulse), and the group velocity of the pulse.

Fig. 2.1 displays the addition of two sine waves with different frequencies produce the beating output wave. The amplitude of the resultant wave becomes zero when two waves are completely out-of-phase. It gives maximum value as the two waves return in-phase. This phenomenon occurs periodically, so-called beating.

Let us extend this concept to a pulse made up of multiple frequencies. Fig. 2.2(a)–(e) illustrate how the localized pulse is generated\([11]\). As adding more frequency components, the separation between neighboring beats increases. Note that a carrier
frequency in the beating is equal to the average frequency of the harmonic components. If one adds more harmonics symmetrically around $\omega_0$, the beats (envelopes for the carrier wave) separate more without changing the carrier frequency. Fig. 2.2(e) illustrates the amplitude of the perfectly localized pulse. If numerous different frequency harmonics are superimposed with specifically shaped amplitudes (Gaussian), the sinusoids are in-phase in the localized region and out-of phase everywhere except over the region. As such, a single solitary pulse remains i.e. the pulse is localized. The amplitudes of the harmonics are enveloped with the Gaussian spectrum centered at $\omega_0$ thereby the addition of the harmonics generating the Gaussian enveloped pulse in time domain.

![Fig. 2.2](image)

Fig. 2.2 1st column shows the frequency spectrums. The beats in time domain is displayed in the 2nd column.

Here, let us next imagine that the pulse propagates inside medium[12]. The harmonic components are added up in phase at the peak of the pulse. Define the phases of the waves $\phi = n(\omega)\omega z/c - \omega t$, where $\omega$ is the angular frequency of each wave, $n(\omega)$ is
refractive index of the medium, $c$ is speed of light in vacuum. Assuming that the pulse propagates without distortion, all harmonics are in-phase at the peak of the pulse. Note the propagation distance of $z$ corresponds to the position where the pulse peak is present. As such, the harmonics are in-phase for all values of the propagation thereby $\phi$ being constant with respect to $\omega$. It can be mathematically expressed as

$$\frac{dn}{d\omega} \frac{\omega z}{c} + \frac{n(\omega)z}{c} - t = 0 \quad \text{i.e.} \quad \frac{d\phi}{d\omega} = 0 \quad (2.1)$$

Using this equation, we express $z$ in terms of time $t$. Defining the group velocity $v_g$, it leads to $z = v_g t$. Note that $v_g$ tells where the peak of the pulse is located after time $t$. From Eq.(2.1), the group velocity is given by

$$v_g = \frac{c}{n_0 + \omega \frac{dn}{d\omega}} \quad (2.2)$$

where $n_0$ is the mean index of the material. We define the denominator $n_g = n_0 + \omega (dn/d\omega)$ as a group index. In a non-dispersive material system i.e. $dn/d\omega = 0$, the group velocity corresponds to the phase velocity of a monochromatic wave. We are interested in the pulse propagation inside dispersive medium. The pulse propagates in normal dispersion medium $dn/d\omega > 0$ slower than $c$, i.e. $v_g < c$ or in negative dispersion medium $dn/d\omega > 0$ greater than $c$, i.e. $v_g > c$. In next sections, we demonstrate an experiment for pulse propagation in the dispersive medium.
B. Motivation for fast light in photorefractive crystal

In a so called fast-light medium, the dispersion is anomalous over a limited bandwidth. In such a medium, the group velocity for a pulse made up of frequency components within this bandwidth can be greater than the free space velocity of light. Scientists have been exploring a range of applications for such a medium[13,14,15,16,17,18,19]. These include enhancement of the sensitivity-bandwidth product of a LIGO-type gravitational wave detector, optical data buffering with a delay far exceeding the limit imposed by the delay-bandwidth product of a conventional cavity, a zero-area Sagnac ring laser gravitational wave detector with augmented strain sensitivity, and a super-sensitive ring laser gyroscope.

These applications are all based on so-called the White Light Cavity (WLC). A WLC is a cavity which resonates over a broader range of frequencies than ordinary empty cavities of equal length and finesse without a reduction in the cavity lifetime. As such, it can circumvent the tradeoff between the resonance bandwidth and the field build-up factor that ordinary cavities entail. A WLC also has the property that if the cavity length is moved away from the condition for empty cavity resonance, the frequency offset needed to restore the resonance is much larger than that for a conventional cavity, thus making it a more sensitive displacement and rotation sensor than an empty cavity can be.

A variety of approaches have been proposed and studied for realizing a white light cavity experimentally[20,21,22]. For example we have previously demonstrated one approach that uses a dispersive vapor medium within the cavity[15]. Specifically,
the anomalous dispersion was produced by a rubidium vapor with bi-frequency pumped Raman gain. A WLC operating at the wavelength of the Rb-transition, however, is not suitable for many of the applications listed above. For example, in order to apply the WLC concept for enhancing the bandwidth-sensitivity product of a LIGO-like gravitational wave detector, it is necessary to realize a WLC that operates at 1064 nm.

In this chapter, we show that the two wave mixing between pump and probe pulses in a photorefractive crystal creates a double gain profile similar to that of the bi-frequency pumped Raman gain in rubidium we have used previously, with the corresponding anomalous dispersion. We demonstrate superluminal propagation of pulses in such a medium. The anomalous dispersion produced in this way can also be employed to realize a WLC. The experiment reported here used a green laser at 532 nm, with a photorefractive crystal that has a relative slow (~seconds) response time. However, the technique demonstrated here is generic enough so that it can be employed with other photorefractive crystals as well. For example, a crystal of SPS[5],[23] is sensitive at 1064 nm, and has a much faster (~msec) response time. An extension of this technique to the SPS crystal at 1064nm could thus be used to make a WLC for enhancing the sensitivity-bandwidth product of a LIGO-type gravitational wave detector as mentioned above.

C. Gain doublet in photorefractive crystal

The gain doublet and corresponding anomalous dispersion are the product of non-degenerate two-wave mixing and angular multiplexing in a photorefractive crystal.
Here, we summarize briefly the physical model used to study this process. A space charge field is generated by the interference of two strong pump beams with a weak probe, so that the refraction index is modulated by the electro-optic effect. The probe is coupled with the pump beams due to these refractive index gratings[24]. Assuming that the intensity of each pump is much higher than that of the probe, in undepleted pumps approximation the phase and intensity coupling coefficients can be written as functions of the angular frequencies of the pumps and the probe[25]:

\[
\Gamma_{in} = \sum_{j=1,2} \frac{\Gamma_{0j}}{2} \left[ \frac{d}{1 + (\omega_s - \omega_{pj})^2 \tau_j^2} \right] \quad (2.3a)
\]

\[
\Gamma_{ph} = \sum_{j=1,2} \frac{\Gamma_{0j}}{2} \left[ \frac{d(\omega_s - \omega_{pj})\tau_j}{1 + (\omega_s - \omega_{pj})^2 \tau_j^2} \right] \quad (2.3b)
\]

where \(\Gamma_{in}\) and \(\Gamma_{ph}\) are the intensity and phase coupling coefficients respectively. \(\Gamma_{0j}\) depends on the incident angle between the jth pump and the probe[26]. \(\tau_j\) is the rise time of the space charge field induced by the pumps, and \(d\) is the effective interaction length. \(\omega_s\) and \(\omega_{pj}\) are the angular frequencies of the probe and the jth pump respectively. The intensity gain and the phase shift of the probe are determined by these coefficients. For our experiment, the 1st and 2nd pump beams are up-shifted and down-shifted respectively by \(\Delta\omega\) from the source frequency of \(\omega_0\), so that \(\omega_{p1} = \omega_0 + \Delta\omega\), \(\omega_{p2} = \omega_0 - \Delta\omega\). This creates two gain lines separated by \(2\Delta\omega\), and a region of anomalous dispersion occurs between them. The probe is pulsed, with a Gaussian spectrum centered around \(\omega_0\). The group velocity of the Gaussian pulse can be expressed as \(v_g = c/(n_0 + c \times \partial \Gamma_{ph}/\partial \omega)\) [12]. If the
carrier frequency of the probe pulse is placed in the middle of the anomalous dispersion region and the pulse bandwidth is smaller than the negative dispersion bandwidth, the group velocity in the photorefractive medium can become larger than \( c \), or may even become negative. It is instructive to view these parameters graphically. We consider the probe pulse to be of the form \( \exp(-t^2/t_0^2) \). We choose the pump intensities to be equal so that \( \Gamma_0 = \Gamma_0 = \Gamma \) and \( \tau_1 = \tau_2 = \tau_M \). For illustration, we consider \( \Gamma_0 d = 6 \), \( \tau_M = 1.1 \text{sec} \), and \( t_0 = 0.6 \text{sec} \). Fig. 2.3(a) shows the normalized intensity and phase coupling coefficients \( (\Gamma, \Gamma_{\text{ph}}) \), and the Fourier Transform \( (S) \) of the probe as functions of \( \omega_s - \omega_0 \), for \( \Delta\omega = 0 \). Here the two gains overlap exactly and behave like a single gain line. The dispersion around the probe carrier frequency in this case is normal, and the \( v_g \) of the probe becomes smaller than \( c \). The width of the gain here is close to \( \tau_M^{-1} \). In Figs 3(b-d), we plot the same parameters for increasing separation between the pump frequencies. For \( \Delta\omega = 1 \text{Hz} \) (Fig. 2.3(c)), the separation is comparable to the gain width, so that two gain peaks are clearly distinguishable. This leads to negative dispersion between the peaks, as shown. In Fig 2.3(d), the separation becomes sufficiently wide so that that negative dispersion region covers the whole bandwidth of the Gaussian spectrum of the pulse.
Fig. 2.3. Numerical, normalized plots of $S(\omega)$, the Fourier transform of the input pulse (solid line), $\Gamma_{in}$ (dashed line), and $\Gamma_{ph}$ (dotted line). The input pulse is assumed to be of the form $\exp(-t^2/t_0^2)$. For these plots, we have used for $t_0=0.6\text{sec}$, $\Gamma_0d=6$ and $\tau_i=\tau_s=\tau_M=1.1\text{sec}$. The four sets are for four different gain separations, $2\Delta\omega$ (a) 0Hz, (b) 1Hz, (c) 2Hz, (d) 4Hz

D. Pulse propagation in gain doublet system

For the case of the Gaussian input pulse of the form $\exp(-t^2/t_0^2)$, the output pulse coupled in the frequency domain can be expressed as [5],[23]

$$S(d,\omega_s)=S(0,\omega_s)\exp(\Gamma_{in1}+\Gamma_{in2})\exp[i(\Gamma_{ph1}+\Gamma_{ph2})]$$  (2.4)
Here $d$ is the propagation distance of the probe inside a photorefractive crystal. $S(0, \omega)$ is the Fourier transform of the input pulse at the entrance of the material. Hence, one can obtain the total output signal intensity in the time domain by squaring the inverse Fourier transform of $S(d, \omega)$. Figs. 2.4(a)–(d) show the numerical simulation of the normalized output signal intensity for different gain line separations corresponding to each case in Figs. 2.3(a)–(d). In Figs. 2.4(a) the output is clearly delayed compared to the pulse propagating in free space, as expected. Fig. 2.4(b) shows the resultant output from propagation under the two gain peaks configuration associated with Fig. 2.3(b). To understand the behavior of the probe in this case, note brief that the gain in Fig. 2.3(b) is peaked at $\omega_0 \pm \Delta \omega$ with a deep valley in the middle. As such, the components of the probe spectrum at $\omega_0 \pm \Delta \omega$ get amplified disproportionately, leading to a two-peaked spectrum. This leads to the beat note at 1Hz in Fig. 2.2(b). Furthermore, each of these peaks experiences normal dispersion, which leads to slowing of the probe, also evident in Fig. 2.3(b). In the case of Fig. 2.3(c), the gain separation is large enough so that the spectrum of the probe is almost completely within the region where the dispersion is anomalous (i.e. negative). This leads to advancement of the probe pulse, as can be seen in Fig. 2.3(c). The advance in this case is rather small due to the moderate steepness in the anomalous dispersion, and is evident only near the peak. Note also that a residual beat note is present at 2Hz, as expected. As the gain separation increases further (Fig 2.3(d)), the slope of the anomalous dispersion become smaller, thus reducing the pulse advancement, as seen in Fig. 2.4(d). It is evident from Figs. 2.3(c), (d) that
\( \frac{\partial^2 \Gamma_{\text{ph}}}{\partial \omega^2} \neq 0 \) for \( \omega \neq \omega_s \). The resulting group velocity dispersion\cite{27,28} causes pulse compression in this case, as can be seen clearly in Figs. 2.4(c), (d).

Fig. 2.4. Simulated output signal for the Gaussian input of \( \exp(-t^2/t_0^2) \) for the same parameters as set in Fig. 2.1. The frequency difference between the gains is given as (a) 0Hz, (b) 1Hz, (c) 2Hz, and (d) 4Hz. Dashed line indicates reference, solid line indicates signal output.

**E. Experimental set-up and results**

We carried out an experiment that corresponds closely to the simulations. The experimental set-up is illustrated schematically in Fig. 2.5. A collimated 532nm doubled Nd-YAG laser was split into a probe and two pump beams. The acousto-optic modulators (AOMs) were driven by frequency synthesizers (PTS’s). The IQ modulator (JCIQ-176M, Minicircuit) and the AOM generated the Gaussian probe pulse with a carrier
frequency of \( f_L + 110\text{MHz} + 4\text{Hz} \) corresponding to \( \omega_0/2\pi \), where \( f_L \) is the laser frequency. The Gaussian pulse with a temporal width of \( t_0 = 0.6\text{sec} \) was generated by a DAQ-Card (DAQCard−6036E, National instrument) with a repetition period of 20sec for \( \Delta\omega/2\pi = 0 \) and 0.5Hz and, 10sec for \( \Delta\omega/2\pi = 1 \) and 2Hz. The first and second pumps (P1 and P2) were shifted by \( \pm \Delta\omega/2\pi \) from \( f_L + 110\text{MHz} + 4\text{Hz} \), respectively, thereby producing two gain peaks with a separation of \( 2\Delta\omega/2\pi \). The incident angle of the probe is approximately 90° from the C-axis of the Ce:BaTiO₃ crystal used for this experiment. P1 and P2 were angular multiplexed at angles of 40° and 60° with respect to the probe. The probe was coupled to the two pumps over an interaction length of 0.5cm inside the crystal. The polarization direction of all the beams and the C-axis were parallel to the optical table. A part of the probe was split-off to provide a reference pulse. This probe was monitored simultaneously with the pulse that traveled through the crystal, and was used to determine the degree of pulse delay/advancement and compression.

Figs. 2.6(a)–(d) show the normalized reference and output signals as \( \Delta\omega \) increases. For \( \Delta\omega = 0 \) in Fig. 2.6(a), the two gains coincide to form a single gain which results in the time delay (1.3sec) of the Gaussian input pulse. As \( \Delta\omega/2\pi \) increases to 0.5Hz, parts of the probe spectrum shifted from \( \omega_0 \) by \( \pm 0.5\text{Hz} \) (the maximum gain and slowing region) become the primary frequency components which are amplified and delayed, thereby resulting in the beat frequency of 1Hz, corresponding to \( 2\Delta\omega/2\pi \), as shown in Fig. 2.6(b). In Fig. 2.6(c), since the gain doublet is sufficiently separated so that the spectrum of the probe exists entirely within the anomalous dispersion region, the
output is advanced by 0.28sec compared to the reference. As $\Delta \omega/2\pi$ increases up to 2Hz, the anomalous dispersion becomes very small. Hence, it is observed that the output has virtually no advancement, as can be seen in Fig. 2.6(d)

Fig.2.5. Schematic diagram of the experimental set-up; AOM : Acousto-optic modulator, B.S : Beam Splitter, B.C : Beam Collimator, H.P: Half waveplate, M: Mirror, PTS : Frequency synthesizer, Pump1: $f_L+110MHz+4Hz+\Delta \omega$, Pump2: $f_L+110MHz+4Hz–\Delta \omega$, Probe : Gaussian pulse with the carrier frequency of $f_L+110MHz+4Hz$
Fig. 2.6. Experimental results showing the group velocity variation of the signal output with setting the separation of the gain doublet to (a) 0Hz, (b) 1Hz, (c) 2Hz, and (d) 4Hz. Dashed line indicates reference, solid line indicates signal output.
A. Motivation for superluminal lasers

Ring Laser Gyroscopes (RLGs) have been used in inertial navigation systems since the 1970s for everything from aerospace and military guidance systems to oil prospection [29]. The basic principle is simple: the length of an optical cavity is perturbed under rotation by the Sagnac effect [30], so that the resonant frequency of the light with the cavity changes. The degree of this shift depends, as has been shown [16, 17] on the dispersion of the medium inside the cavity. In previous White Light Cavity (WLC) demonstration, it was realized by placing a fast-light medium inside a conventional cavity [15]. The WLC becomes highly sensitive to mirror displacement and rotation [13]. However, the concomitant broadening of the linewidth, by essentially the same factor, means that there is virtually no net improvement in the smallest measurable displacement or rotation. This constraint was eliminated if an active cavity is used, where the rotation (or any non-reciprocal change in the cavity length) is measured by monitoring the beat note between the counter-propagating lasing modes [13].

In this chapter, we develop the theoretical model for WLC behavior of such an active resonator: a superluminal ring laser. Since the laser operates under the condition where the gain per pass balances the loss per pass, the conventional model for WLC does not apply. However, we show that if the gain profile is flat over the region of interest, with a dip in the middle, the lasing mode centered at the dip behaves like WLC, with its frequency becoming highly sensitive to mirror displacement or rotation. The
enhancement factor can be as high as $10^7$, comparable to what is achievable in a passive cavity. We also present a physical interpretation of this behavior in terms of an effective Kramers-Kronig relation, and show that the process works with both homogeneously or inhomogeneously broadened gain media.

B. Signal from laser cavity and homogenous & inhomogeneous gain medium in laser cavity

First, we consider the light in a cavity which contains a dispersive medium. The resonant frequency can be described by [31]:

$$\nu + \phi = \Omega - \frac{1}{2} \nu \chi'(E,v)$$  \hspace{1cm} (3.1)

where $\chi'$ is a real part of the susceptibility, $\phi$ is a round-trip phase shift, $\nu$ is the resonant frequency of a medium-filled cavity, and $\Omega$ is that of the cavity in the absence of the medium so-called an empty cavity. $\Omega$ is equal to $2\pi mc^2/L$. The parameter $m$ is an integer and $c$ is the speed of light in free space. $L$ is the cavity length. Note $\chi'$ is itself frequency and intensity dependent. Let us assume that the resonance frequencies $\Omega$ and $\nu$ are both initially equal to $\nu_0$ when the cavity length is $L_0$. Let us further assume that the gain (or absorption) profile of the medium in the cavity is symmetric about $\nu_0$. For the sake of comparison, we define the normalized parameters, $\Delta = (\Omega - \nu_0)/\Gamma$, $\delta = (\nu - \nu_0)/\Gamma$ where $\Gamma$ is the linewidth of the absorption or gain line in radian. The derivatives $d\Delta/dL$ and $d\delta/dL$ represent the resonant frequency shifts induced by the perturbation of $L$ associated with the empty and the medium cavity, respectively. One can consider the
ratio, \( R = \left[ \frac{d\delta/dL}{d\Delta/dL} \right] \) to determine if the amount of the frequency shift is enhanced (\( R > 1 \)) or diminished (\( R < 1 \)) by the dispersive intracavity medium. In steady state \( \left( \dot{\phi} = 0 \right) \), after dividing Eq.(3.1) by \( \Gamma \) and using \( \nu/\Gamma = \delta + \nu_0/\Gamma \), the differentiation of the resultant with respect to \( L \) leads us to

\[
\frac{d\delta}{dL} + \frac{1}{2} \frac{d\chi'}{dL} \nu \frac{1}{\Gamma} + \frac{1}{2} \frac{d\delta}{dL} \nu \chi' + \delta = \frac{d}{dL} \left( \frac{\Omega - \nu_0}{\Gamma} \right) \tag{3.2}
\]

By substituting \((d\chi'/d\delta)(d\delta/dL)\) for \( d\chi'/dL \), the ratio \( R \) can be expressed as

\[
R = \frac{1}{1 + \frac{1}{2} \chi' + \frac{1}{2} \frac{d\chi'}{d\delta} \nu} \tag{3.3}
\]

By applying \( d\delta = 1/\Gamma d\nu \) and \( \nu_0 = \sqrt{1 + \chi'} = 1 + \chi'/2 \), one can easily prove that the denominator in Eq.(3.3) is the group index, \( n_g = 1 + \chi'/2 + [\nu/2][d\chi'/d\nu] \) of the dispersive intracavity medium. For normal dispersion (\( n_g > 1 \)) in active cavities, \( R(=n_g) \) becomes less than one so that resonance frequency change induced by the length variation \( dL \) is reduced. For \( n_g < 1 \), i.e. in the cavity filled with anomalous dispersion medium, the frequency shift is amplified to be equal to \( 1/n_g \) times the amount of the shift for the empty cavity. Eq.(3.3) can be applied to any intracavity medium. As such, if one can calculate the dispersion \( d\chi'/d\nu \) of given medium, then the resonance frequency shift of the medium cavity can be predicted compared to that of the empty cavity. \( \nu_0 \) and \( \Gamma \) are the medium property and thereby being independent from the length change \( dL \). Eq. (3.3) can be modified as \( (d\nu/dL)/(d\Omega/dL) = 1/n_g \). Recent study[13] showed \( \Delta \nu = \Delta \nu_{ec}/n_g \) where the resonant frequency shift of a passive cavity \( \Delta \nu \), an empty cavity \( \Delta \nu_{ec} \). Now,
note that the modified equation associated with an active cavity is consistent with the previous results. To interpret this result for the case of the active cavity, however, we must take into account the fact that the gain medium can be saturated. Indeed, the field inside a laser cavity increases until it saturates the gain medium such that the resulting gain exactly balances the loss within the cavity. This is the steady-state lasing condition. Field amplitude and phase in the laser cavity are described by a set of self consistency equations[31]. As such, the steady-steady gain condition can be expressed as:

\[ \dot{E} = -\frac{1}{2} Q E - \frac{1}{2} v E \chi''(E, \nu) = 0 \quad (3.4) \]

where \( E \) is the laser field amplitude, \( \chi'' \) is an imaginary part of the susceptibility. \( Q \) represents loss characteristics of the cavity. It leads to

\[ \chi''(E, \nu) = -\frac{1}{Q} \quad (\text{for } E, \nu \neq 0) \quad (3.5) \]

\( \chi'' \) implies the gain (or loss) characteristics and \( \chi' \) is relevant to refractive index. Note that \( \chi'' \) and \( \chi' \) depends on \( E \) as well as its frequency \( \nu \).

To understand the relationship between the gain saturation and the self-consistency equations, let us consider homogeneously broadened gain medium. Fig.3.1 illustrates the saturated gain corresponds to the cavity loss at steady state. We assume that a lasing mode frequency \( \nu \) exists within the gain profile but is apart from the gain center. The saturated gain \( G(\nu) \) and the non-saturated gain \( G_0(\nu) \) are expressed as \( G \sim \Delta N g(\nu) \) and \( G_0 \sim \Delta N_0 g(\nu) \), respectively where \( \Delta N \) is the population inversion in the presence of the lasing field, \( \Delta N_0 \) is the initial population difference without field and
g(ν) is a normalized Lorentzians lineshape function[32]. Note that G and G₀ have the same

![Diagram](image)

Fig. 3.1: Saturation of homogeneous gain profile to satisfy self-consistency equations at lasing frequency. Cold cavity represents a cavity before the gain reach to steady state. When the laser cavity reaches at steady state, the lasing beam experiences gain and index represented by red an blue points, respectively.

lineshape function. As such, the shape of profiles remains unchanged. The saturation effect by the lasing field is to deplete the population of the excited state i.e. reduction of ∆N. The magnitude of the homogeneous gain profile reduces but until the gain at the frequency of the lasing field becomes equal to the loss in the cavity. The question then becomes, what effect does this gain saturation have on the dispersion associated with the gain profile? One might at first think that the fact the steady-state gain is the same at every lasing frequency would mean that there could be no variation in the steady-state index of refraction as a function of frequency. This is not the case, however. In a homogenously broadened medium, the saturation effect is simply to decrease the magnitude of χ' over the whole profile[32, 33]. Note that the frequencies for which χ' is
plotted here are not in general resonant in the cavity and cannot be sustained in steady state. Only the single point at $\omega=\nu$, the lasing frequency, provides us information about the phase shift that would be seen by a resonating beam. However, if we draw this graph for a range of different choices of $\nu$, corresponding to different choices of cavity length, we will see that each choice determines a different index value at steady state. Thus, the index does indeed depend on lasing frequency. This variation is the primary reason that the resonant frequency of a laser cavity differs from that of a passive cavity of equal length, usually being closer to the center of the gain line, a phenomenon called mode-pulling.

Next, consider an inhomogeneously broadened gain medium. If the medium is inhomogeneously broadened, we may regard such inhomogeneous gain profile as the sum of many narrower, homogeneously broadened gain profiles individually centered at a different frequency. The saturation does not reduce the gain over the whole profile, but a localized frequency range around the lasing frequency. As such, the gain saturation occurs at the frequencies supported by the lasing mode. The lasing beam so-called hole burning laser bleaches a narrow band “hole” into the inhomogeneously broadened gain. This result is referred to as spectral hole burning[32, 33]. Next, let us imagine that a gain profile has four spectral holes in it separated approximately by the free spectral range (FSR) of the cavity. We assume that the inhomogeneous gain bandwidth is sufficiently broad so that it can sustain 4 cavity modes. The net gain can then be modeled as sum of a broad Lorentzian (or more accurately, a Voigt profile) and four narrow, inverted Lorentzians. The associated $\chi'$ at any frequency is then determined by the sum of the
phase shift due to such a broad Lorentzian gain and that due to each of the narrow Lorentzian “absorption” features, as illustrated in Fig.3.2 [34,35,36,37].

Fig. 3.2: Saturation of inhomogeneous gain profile to satisfy self-consistency equations at lasing frequency or frequencies, and index profile corresponding to the saturated gain profile.

The phase associated with a hole is zero at the center of that hole, but there are two sources of the phase contributions. The first is the phase from the dispersion within the absorption bandwidth of the hole. The second is phase from other burned holes at other frequencies. They may cause the phase to be different from that which would be associated with the broad gain alone. At any given frequency, in other words, the phase depends on the shape of the entire profile. Let us assume that the inhomogeneous gain support a single lasing mode. As such there is only one spectral hole. The dispersion at the lasing frequency will be unaffected by the saturation. As a result, the lasing beam experiences the single phase associated with the inhomogeneous gain. Again, the index of refraction depends on lasing frequency. This brief review is intended to make clear the distinction between the dispersion of a medium in free space, and the dependence on
frequency of the steady-state refraction index in a laser cavity. Though related, they are not same. The group index to which we refer here is related to the velocity of groups composed entirely of lasing frequencies. Superluminal pulses of this type have in fact been observed[38] in inhomogenously broadened media.

C. Absorption dip embedded in flat broad gain

We have established that the resonant frequency shift is induced by the change of the laser cavity length. It is inversely proportional to the group index of the material. Furthermore, we have shown that the dispersion inside a laser in steady state is not zero, though it does depend on the saturating field. We now ask how to achieve the dispersion profile we desire. Anomalous dispersion is required to maximize the dependence of lasing frequency on cavity length. The Kramers-Kronig relations[32] tell us that anomalous dispersion is usually associated with an absorption feature, or equivalently, a reduction in gain. In this section, let us consider the case that the cavity contains a medium with a narrow absorption as well as a medium with a saturable and infinitely broad gain. This configuration creates the absorption dip in the broad gain, resulting in anomalous dispersion within the absorption bandwidth. We then express the net susceptibility of the media as a sum[39]:

\[
\chi' = -\frac{N_i \hbar \Omega_n}{\epsilon_0 E^2} \left( \frac{2\Omega_n (v-v_a)}{2\Omega_n^2 + \Gamma_i^2 + 4(v-v_a)^2} \right)
\] (3.6a)

\[
\chi'' = -\frac{N_e \hbar \Omega_{re}}{\epsilon_0 E^2} \left( \frac{\Omega_{re} \Gamma_e}{2\Omega_{re}^2 + \Gamma_e^2} \right) + \frac{N_i \hbar \Omega_n}{\epsilon_0 E^2} \left( \frac{\Omega_n \Gamma_i}{2\Omega_n^2 + \Gamma_i^2 + 4(v-v_a)^2} \right)
\] (3.6b)
Here $\varepsilon_0$ is the permittivity of free space. $E$ is the field amplitude. $\hbar$ is plank’s constant. $N_e$ and $N_i$ represent the number of atoms per unit volume for the broadband gain medium and for the inserted narrow absorptive medium, respectively. We use the subscript “$e$” for the “envelope” gain profile and “$i$” for the narrower absorption profile, in order to remember that the absorption is “inside” the envelope of the gain. We assume $\Gamma_e > > \Gamma_i$. $\Gamma_e$ is the gain decay rate. $\Gamma_i$ denotes the absorption linewidth. Note, $\Gamma_i$ corresponds to $\Gamma$ in Eq.(3.3). $\nu_0$ is the center frequency of the absorption line. The field with Rabi frequency $\Omega_{re}$ couples the background gain medium. The absorption medium is driven by the field with Rabi frequency $\Omega_{ri}$. $\Omega_{ni}$ and $\Omega_{re}$ are equal to $\varphi_i E / \hbar$ and $\varphi_e E / \hbar$, respectively, where $\varphi_i$ and $\varphi_e$ are the dipole moments associated with the media. We now define $\xi_i$ and $\xi_e$ such that $\Omega_{ni}^2 = \Gamma_i E^2 \xi_i$ and $\Omega_{re}^2 = \Gamma_e E^2 \xi_e$.

The field intensity inside the medium is given by $I_{e,i} = E^2 / 2 \eta_{e,i}$ where $\eta_{e,i}$ is impedance. $I_{e,i}$ can be rewritten as $I_{e,i} = \Omega_{re,i}^2 \hbar^2 / 2 \eta_{e,i} \varphi_{e,i}^2$ by using the Rabi frequencies $\Omega_{re,i} = \Omega_{re,i}$. According to $\Omega_{re,i}^2 = \Gamma_i^2$, for the saturation, the saturation intensity is given by $I_{sat,e,sat,i} = \Gamma_i^2 \hbar^2 / 2 \eta_{e,i} \varphi_{e,i}^2$. As such, $I_{e,i} / I_{sat,e,sat,i}$ can be written as $\Omega_{re,i}^2 / \Gamma_i^2$.

Also, from $I_{e,i} = E^2 / 2 \eta_{e,i}$, we can derive $I_{e,i} / I_{sat,e,sat,i} = E^2 / 2 \eta_{e,i} I_{sat,e,sat,i}$. Using $I_{e,i} / I_{sat,e,sat,i} = \Omega_{re,i}^2 / \Gamma_i^2 = E^2 / 2 \eta_{e,i} I_{sat,e,sat,i}$ and substituting $\Gamma_i^2 \eta_{e,i}^2 \varphi_{e,i}^2$ for $\Omega_{re,i}^2$, one finds $\xi_{e,i} = 1 / 2 \eta_{e,i} I_{sat,e,sat,i}$. To understand physical characteristics of $\xi_{e,i}$, note that according to $I_{sat,e,sat,i} = \Gamma_e \hbar^2 / 2 \eta_{e,i} \varphi_{e,i}^2$, $\eta_{e,i} I_{sat,e,sat,i}$ consists of the medium-dependant
parameters such as absorption bandwidth and dipole moments in the absence of the lasing frequency-dependent parameters. Therefore, one recognizes that \( \xi_{e,i} (=1/2n_{e,i}I_{\text{sat}_{e,sat}_{i}}) \) is independent of the lasing frequency. For the sake of simplicity, we introduce \( G = N_e \hbar \xi_e \Gamma_e / \varepsilon_0, \quad H = N_i \hbar \xi_i \Gamma_i / \varepsilon_0, \quad \delta = (\nu - \nu_0) / \Gamma_i \) and \( B = 2E^2 \). Eq(3.6) is modified as

\[
\chi' = \frac{-2\delta H}{B\xi_i + 1 + 4\delta^2} \quad (3.7a)
\]

\[
\chi'' = -G \left( \frac{1}{B\xi_i + 1} \right) + H \left( \frac{1}{B\xi_i + 1 + 4\delta^2} \right) \quad (3.7b)
\]

We are now at the moment to evaluate the ratio \( R \) defined in Eq. 3.3. Before differentiating this \( \chi' \) with respect to \( \delta \), we should note that the steady-state field intensity inside the cavity is the frequency dependent function, i.e. \( B(\delta) \), so that the derivative of \( \chi' \) in Eq(3.7a) can be expressed as \( \frac{d\chi'}{d\delta} = \frac{\partial \chi'}{\partial \delta} + (\frac{\partial \chi'}{\partial B}\frac{dB}{d\delta}) \). \( \frac{\partial \chi'}{\partial \delta}, \frac{\partial \chi'}{\partial B} \) are assessed by partially differentiating Eq.(3.7a) with respect to \( \delta \) and \( B \), respectively.

To calculate \( dB/d\delta \), we must find \( B \) as a function of \( \delta \). Let us recall the field amplitude equation in the laser cavity. In a steady state, it gives \( \chi'' = -1/Q \). Inserting \(-1/Q\) into \( \chi'' \) of Eq(3.7b) and modifying it, we find a quadratic equation for \( B \),

\[
\xi_e \xi_i B^2 + [\xi_i + \xi_e (1 + 4\delta^2)] - QG\xi_i + QH\xi_i \] B + 1 + 4\delta^2 - QG(1 + 4\delta^2) + QH = 0 \quad (3.8)
\]

Keeping in mind that the intensity is always a positive number, we can choose the solution for “\( B \)” which is positive over the lasing bandwidth. In Fig. 3.3, we can now plot the real part of the steady-state susceptibility as a function of frequency. For
illustration, we consider $Q=3\times10^7$, $\Gamma_i=10\text{MHz}$ and $\lambda_0=780\text{nm}$, $H=5$, $G=10$ and 
$\xi_n/\xi_s = 1.1333333398$ ($\lambda_0=2\pi c/\nu_0$ where $c$ is speed of light in vacuum)

Fig.3.3. Real part of the steady-state susceptibility as a function of lasing frequency.

We are now able to find the partial derivatives $\partial\chi'/\partial\delta$, $\partial\chi'/\partial B$ from Eq(3.7a) and $B$ as function of $\delta$ from Eq(3.8).

The remaining procedure to calculate $R = [d\delta/dL]/[d\Delta/dL]$ is to express $\chi'$ and $d\chi'/d\delta$ in terms of $\delta$. $d\chi'/d\delta$ is obtained by differentiating $B$ with respect to $\delta$ and then inserting $\partial\chi'/\partial \delta, \partial\chi'/\partial B, dB/d\delta$ into $\partial\chi'/\partial \delta + (\partial\chi'/\partial B)(dB/d\delta)$. Substituting $B$ from Eq(3.8) into $\chi'$ in Eq(3.7a), one can find $\chi'$ as function $\delta$, as well. Inserting $\chi'(\delta)$, $d\chi'/d\delta$ into Eq(3.3), we obtain the ratio $R$ for the narrow absorption medium in the environment of the infinite broadband gain as a function of $\delta$. This $R$ is a measure of the degree to which the sensitivity of a laser cavity with these dispersive media is enhanced over that of an empty cavity. We note, however, the equation for $R$ is frequency dependent. This essentially tells us that once the lasing frequencies are outside of the
anomalous dispersion regime, there is no enhancement of their sensitivity to length changes. We find, with the numbers above, the profile of $R$ gives a peak value of $11 \times 10^6$ and reduces to zero as leaving such a dispersion regime.

We find furthermore that the peak value of $R$, i.e. $R_p$ varies with the ratio $\xi_i / \xi_e$. Fig. 3.4 displays the normalized $R_{p,nor}$ by its maximum value for each case of the different $H$. For illustration, we use the same parameters as in Fig.3. We consider more $H$’s corresponding the absorption dip. $R_{p,nor}$ changes very rapidly with $\xi_i / \xi_e$. Note, $\xi_i / \xi_e$ corresponds to $\Gamma_i^2 \varphi_i^2 / \Gamma_e^2 \varphi_e^2$. Therefore, in order to significantly enhance the frequency shift compared to the empty cavity, one must carefully choose the media. Its $\varphi_i^2 / \varphi_e^2$ can yield the appropriate $\xi_i / \xi_e$ for maximum $R_p$. Fig. 3.4 suggests that the optimal values of $\xi_i / \xi_e$ for maximum of $R_p$ varies with the depth of the absorption dip.

![Graph showing normalized R vs. xi ratio](image)

**Fig. 3.4:** Maxima of the ratio for different absorption depth. (red: $H=2$, green: 5, blue: $H=8$, black: $H=10$) as the function of $\xi_i / \xi_e$. The graphs are normalized by maximum for each case.
The variation of the absorption, “H,” also has the effect of shifting the frequency of the maximum sensitivity and altering the bandwidth of that sensitivity. Fig. 3.5 displays R for different absorption dip. For illustration, we adjust $\xi_i/\xi_e$ such that maximum R’s for H=2, 5, 8, 10 become equal to $1.1 \times 10^7$. The graph indicates that the peak of R is shifted from $\delta=0$, and the bandwidth of R broadens as increasing H from 2 to 10. We have carried out similar simulations for a wide variety of gain and absorption media, varying the bandwidth and depth of the gain and the “hole” in the gain, for homogeneous and inhomogeneously broadened media. The results in all cases support those we have seen in this presented example.

![Graph showing R as function of $\delta$. $\xi_i/\xi_e$ is equal to 0.4533333530 for H=2 (red), 1.1333333938 for H=5 (green), 1.8133334374 for H=8 (blue), and 2.2666668013 for H=10 (black).]

**D. Brief discussion about noise**

These results imply that we may achieve the same sensitivity in lasers, whether based on homogeneously or inhomogeneously broadened gain media, which we have seen in passive cavities. Of course, an increased sensitivity is worthless if the noise is also
increased. We will not go into detail here about the affects of the fast light medium on laser frequency noise. These details can be found in ref.\textsuperscript{40}. However, we do wish to make one important point. The primary source of frequency noise for most lasers is mirror jitter, and we might expect that the fast light medium would increase the sensitivity of lasers to such jitter, thus, increasing the noise by the same factor as the sensitivity. However, laser gyroscopes do not involve direct frequency measurements, but rather measurements of the beat note between counter propagating modes, as illustrated in Ref.[29]. Mirror jitter, thermal effects, and other macroscopic noise sources affect both beams, and this common-mode noise cancels out, when one measures the beat frequency. The only noise on such a measurement is quantum noise, and Ref.[40] shows that this is unaffected by the fast light material. Ultimately the details of real, physical systems will determine how well the approach works for gyroscope applications. These theoretical results, however, are sufficiently promising to justify significant further experimental investigation.
IV. APPLICATION OF FAST LIGHT INTRACAVITY MEDIUM TO DATA BUFFER SYSTEM

A. Motivation for data buffer via white light cavity

Due to significant applications to optical buffers, optical memories, the group index ($n_g$) has been controlled through manipulation of atomic resonances [1, 3] or light interactions in non-linear materials such as Stimulated Brillouin scattering (SBS) in optical fibers[6789], two-wave mixing in photorefractive crystal[5,41]. So far, in slow-light demonstration ($n_g > 1$), a light pulse was delayed less than or several times the pulse duration [1], [3], [5], [6], [8,9]. SBS generated by pulsed Brillouin pump in fibers[42] was used to store a optical pulse with the constraint on the storage time that should be less than the acoustic lifetime of nanoseconds in optical fibers. In stored-light based on electromagnetically induced transparency (EIT), a photon wave packet is transferred to atomic coherence in collective atomic system so that the storage time corresponds to the coherence time of hyperfine transitions [43]. In low temperature environment, 20µs pulse was recorded as the coherence of the ground hyperfine states in Pr:Y$_2$SiO$_5$ for time scales of several seconds[44].

A material that is anomalously dispersive over a limited frequency range is known as a fast-light medium because it can support group velocities greater than speed of light in free space. Optical cavities which contain such a fast light medium have a broadened linewidth without a reduction of the cavity buildup so-called White Light Cavity (WLC)[15]. It has been implemented in a cavity filled with rubidium vapor
where Raman gain lines were produced around the cavity resonance to achieve the fast-light condition \((n_g<1)\). The pump intensity and the gain separation were controlled to adjust a negative dispersion slope so that the linewidth of the WLC was tunable. In the absence of the pumps, the WLC was converted to an ordinary cavity.

Utilizing such features of the WLC, in next section we propose the details of a data pulse delay system. Due to the tunable bandwidth of the WLC, a data pulse is successfully delayed for sufficiently longer time than the pulse duration without distortion. Furthermore, the delay time is adjustable by switching the pumps to control WLC effect. Unlike the previous works [44], this system can be possibly realized by simpler experimental configurations in room temperature.

**B. Data buffering system via Fabry-Perot cavity in free space**

Before presenting a distortion free pulse delay system, we briefly review a theoretical model to explain the aspects of a WLC. Consider a tunable-bandwidth WLC. Fig.4.1 displays the WLC where two partial reflectors form into a conventional Fabry-Perot cavity. For simplicity, the cavity is assumed to be completely filled with a dispersive medium.

**Fig. 4.1** Schematic of a tunable-bandwidth WLC; Two partial reflectors with reflectivity \(R\) enclose the intracavity medium and form into the cavity.
To calculate the output of the WLC with a Gaussian data pulse, note the transfer function analysis of a standard cavity[45] and adapt the complex amplitude transfer function for the WLC. A monochromatic optical wave \( E_0(\omega) = E_0 e^{i\omega t} \) is incident on the WLC in Fig.4.1. The input wave can be related to the WLC output as

\[
E_{\text{out}} = E_0 e^{i\omega t} \times \frac{t^2 e^{-i\omega L}}{1 - r^2 e^{-2i\omega L}} \quad (4.1)
\]

where \( E_0 \) is the field amplitude, \( L \) is the cavity length, \( t \) and \( r \) are the amplitude transmission and reflection coefficient, respectively. (For intensity, \( R = r^2 \), \( T = t^2 \) and \( R + T = 1 \)) and \( \omega \) is an angular frequency. \( k \) is a wave number and is expressed as

\[
k = n\omega/c \quad (\text{Here, } n \text{ is the refractive index of the intracavity medium, and } c \text{ is speed of light in vacuum.})
\]

For a cavity in the absence of medium i.e. empty cavity, \( n \) is simply equal to the index of vacuum (=1). For our case that the cavity contains the dispersive medium, \( n \) is a function of \( \omega \). As such, the wave number for the WLC can be modified as

\[
k(\omega) = n(\omega)\omega/c \quad (n(\omega) \text{ is expressed by Taylor expansion around } \omega_0 \text{ as})
\]

\[
n(\omega) = n_0 + (\omega - \omega_0)n_1 + (\omega - \omega_0)^3 n_3 \quad n_1 = \frac{dn}{d\omega}\bigg|_{\omega=\omega_0} , \quad n_3 = \frac{(1/6)dn^3}{d\omega^3}\bigg|_{\omega=\omega_0}
\]

where \( n_0 \) is the mean index of the medium. \( \omega_0 \) is the resonance frequency of the cavity before creating dispersion in the medium. More specifically, it is equal to the center frequency of a gain doublet so that the medium can obtain a negative dispersion asymmetrically around \( \omega_0 \)[15]. In the previous WLC experiment, the bi-frequency pumps provided two gain lines. The negative dispersion was created in the intermediate region between the two gains. \( \omega_0 \) was in the middle of the gain doublet such that the dispersion was
asymmetric around $\omega_0$. Due to the asymmetrical dispersion, we can consider \( n_2 = \frac{1}{2} \frac{d^2 n}{d\omega^2} \bigg|_{\omega=\omega_0} \) as a null value and thus eliminate \( n_2 \) in the Taylor expansion of \( n(\omega) \). A cavity response for the tunable-bandwidth WLC is obtained by inserting the Taylor expansion of \( n(\omega) \) into \( k \) of \( t^2 e^{-\text{j}kL}/(1 - t^2 e^{-2\text{j}kL}) \) in Eq(4.1). If the cavity represents a general system, then one can regard the cavity response as a transfer function of the system. For the case of WLC, the transfer function can be expressed as

\[
H_{\text{WLC}}(\omega) = t^2 e^{-\text{j}kL}/(1 - t^2 e^{-2\text{j}kL})
\]

where \( k = \{[n_0 + (\omega - \omega_0)n_1 + (\omega - \omega_0)^2 n_2] \omega \}/c \). For the empty cavity, the transfer function can be written as \( H_{\text{EC}}(\omega) = t^2 e^{-\text{j}k_0L}/[1 - t^2 e^{-2\text{j}k_0L}] \) where \( k_0 = \omega/c \).

Next, we extend a monochromatic input to an arbitrary pulse. Consider the Fourier transform of the pulse, \( S(\omega) \). By convolution theorem, the WLC output is the inverse Fourier transform of the product of \( S(\omega) \) and \( H_{\text{WLC}}(\omega) \) and thus one can obtain the output intensity \( \left| S_{\text{WLC}}(t) \right|^2 \) where \( S_{\text{WLC}}(t) = 1/\sqrt{2} \int_{-\infty}^{\infty} S(\omega) H_{\text{WLC}}(\omega) \exp(j\omega t) d\omega \). We choose the input pulse to be the form of \( S(t) = \exp(-t^2/t_0^2) \exp\left[ j(\omega_0 + \xi) t \right] \), whose Fourier transform is written as \( S(\omega) = t_0/\sqrt{2} \exp\left[ j(\omega - \omega_0 - \xi) t_0 \right]^2 /4 \). The carrier frequency of the pulse is upshifted as much as \( \xi \) from the empty cavity resonance \( (\omega_0) \). Likewise, for a reference pulse which propagates free space as much as the cavity length \( L \), the resultant pulse after traveling is \( S_{\text{free}}(t) = 1/\sqrt{2} \int_{-\infty}^{\infty} S(\omega) H_{\text{free}}(\omega) \exp(j\omega t) d\omega \) where a transfer function \( H_{\text{free}}(\omega) \) is the phase change \( \exp(-j\omega L/c) \) due to the free space.
propagation. Let us next consider the group velocity of the pulse in the WLC. The phase \( \phi \) for an individual frequency wave after the propagation distance of \( L \) can be written as

\[
\phi = \omega t - \frac{\omega n_{\text{eff}} L}{c} \tag{4.2}
\]

where \( n_{\text{eff}} \) is the effective refractive index induced by the WLC. For simplicity, assuming that the pulse propagates the WLC without distortion, all frequency components are added in phase \([12]\) at the exit of the WLC thereby \( d\phi/d\omega = 0 \). We can define the group velocity as \( v_g = L/t \). After differentiating Eq(4.2) with respect \( \omega \), \( L/t(=v_g) \) can be expressed as \( c/[n_{\text{eff}} + \omega (dn_{\text{eff}}/d\omega)] \). From \( v_g = c/[n_{\text{eff}} + \omega (dn_{\text{eff}}/d\omega)] \), we obtain the group index as \( n_g = n_{\text{eff}} + \omega (dn_{\text{eff}}/d\omega) \). Next consider \( n_g \) in terms of the phase angle of \( H_{\text{WLC}} \). If the complex transfer function \( H_{\text{WLC}} \) is written as \( |H_{\text{WLC}}| \exp(i\angle H_{\text{WLC}}) \) in terms of the amplitude \( |H_{\text{WLC}}| \) and the phase angle \( \angle H_{\text{WLC}} \), the WLC output in frequency domain can be written as \( S_{\text{WLC}}(\omega) = |H_{\text{WLC}}| \exp(i\angle H_{\text{WLC}})S(\omega) \). One can note that \( \angle H_{\text{WLC}}(\omega) \) is the phase resulting from the propagation inside the WLC thereby being equal to the second term in Eq(4.2). From \( \angle H_{\text{WLC}}(\omega) = -(\omega n_{\text{eff}} L)/c \), we can derive \( n_{\text{eff}} = -(c \angle H_{\text{WLC}})/(\omega L) \). Inserting \( n_{\text{eff}} \) into \( n_g = n_{\text{eff}} + \omega (dn_{\text{eff}}/d\omega) \), one can obtain \( n_g \) as

\[
n_g = -\frac{c}{L} \frac{d\angle H_{\text{WLC}}}{d\omega} \tag{4.3}
\]

In the treatment of pulse distortion, it is convenient to consider the group index dispersion i.e. group velocity dispersion. The degree of pulse stretch (or compression) in
time domain after propagation through a medium of length L is given by
\[ \Delta t \approx (L/c)(dn_3/d\omega)\Delta \omega \] where \( \Delta \omega \) is that pulse bandwidth [12]. By inserting Eq(4.3) into \( \Delta t \), we obtain
\[ \Delta t \approx -(d^2 \angle H_{WLC}/d\omega^2)\Delta \omega \] and thus confirm that the pulse maintains its original shape after propagation on the condition that its spectrum belongs to the spectral region where \( d^2 \angle H_{WLC}/d\omega^2 = 0 \).

Fig.4.2~3 display the transfer functions \(|H_{WLC}|^2, |H_{EC}|^2\), the phases \( \angle H_{WLC}, \angle H_{EC} \), the frequency spectrum of the input pulse \(|S|^2\) and the output pulses \(|S_{WLC}|^2 \) and \(|S_{free}|^2\). For illustration, we choose the parameters as cavity length \( L \) =5cm, Finesse=999(R=0.999), and Full Width Half Maximum (FWHM)=2.9MHz. Assuming \( n_3=0 \), \( n_1 \) is adjusted to satisfy the ideal WLC condition[15] where the WLC linewidth is infinite and the group index \( n_g \) is 0 for our case that the cavity is completely filled with a medium. Next, we discard the assumption and consider the case that the dephasing away from \( \omega_0 \) due to \( n_3 \) results in a cavity response drop, i.e. a finite WLC linewidth. For the Gaussian input pulse, the width is \( \Delta \nu_{\text{pulse}}=29\text{MHz} \) \( (t_0=34\text{ns}) \) and the carrier frequency is shifted from \( \omega_0 \) by \( \xi=1.5 \times \Delta \nu_{\text{pulse}} \). Therefore, the frequency spectrum of the pulse is sufficiently separated from the spectral region of an empty cavity resonance. Later, we will discuss about the necessity for the shifted pulse in our delay system.

Fig. 4.2(a) indicates that for a WLC associated with \( n_3=5.223 \times 10^{-35} \), the pulse spectrum \( S(\omega) \) belongs to the spectral region where the constant cavity response \(|H_{WLC}|=1\) ends and subsequently begins to reduce. Since the amplitude of the transfer
function decreases within the pulse spectrum, we can observe that the intensity of output pulse (red) is reduced as indicated in Fig.4.2(c). Fig. 4.2(b) illustrates $\frac{d\angle H_{WLC}}{d\omega} < 0$ in the pulse spectrum and thus $n_5 > 1$. Due to $v_p < c$, the pulse slows down compared to the reference which propagates the free space as much as $L$. With the consideration of $d^2\angle H_{WLC}/d\omega^2$, the pulse spectrum partially exists in the spectral region of $d^2\angle H_{WLC}/d\omega^2 \neq 0$ and thus one can observe the pulse compression in Fig.4.2(c).

Fig.4.2 (a) Transfer functions for empty cavity(blue) and for WLC(red), and the Fourier Transform of Gaussian input(green) (b) Phase of $H(\omega)$ for empty cavity(blue) and for WLC(red).(c) $|S_{free}|^2$ (blue) and $|S_{WLC}|^2$ (red). The parameters of the intracavity medium are $n_1 = -8.223 \times 10^{-16}/\text{rad}$ $n_3 = 5.223 \times 10^{-35}/\text{rad}^3$

Fig.4.3(a) suggests that the WLC associated with $n_3 = 1.723 \times 10^{-36}$ resonates over broader spectral range than the WLC of $n_3 = 5.223 \times 10^{-35}$. As indicated in Fig.4.3(b), the input pulse spectrum mostly belongs to the spectral region of WLC where $d\angle H_{WLC}/d\omega = 0$, $d^2\angle H_{WLC}/d\omega^2 = 0$, minor frequency components are in $d^2\angle H_{WLC}/d\omega^2 \neq 0$. Hence, in the insets of Fig.4.3(c) it is observed that $S_{WLC}(t)$ exhibits no time delay or advancement but the minor pulse compression.
Fig. 4.3 For the medium with $n_1 = -8.223 \times 10^{-16}/\text{rad}$, $n_3 = 1.723 \times 10^{-36}/\text{rad}^3$ (a) Transfer functions for empty cavity (blue) and for WLC (red), and the Fourier Transform of Gaussian input (green) (b) Phase of $H(\omega)$ for empty cavity (blue) and for WLC (red). (c) $|S_{\text{free}}|^2$ (blue) and $|S_{\text{WLC}}|^2$ (red).

In order to design a pulse delay system, we utilize the characteristics of the WLC in Fig. 4.3 and choose the shifted Gaussian pulse $S(\omega) = t_0/\sqrt{2} \exp\left\{((-\omega - \omega_0 - \xi) t_0)^2/4\right\}$ as an input pulse. Fig. 4.4 displays that the proposed system consists of two WLCs. The intermediate region between the WLCs is surrounded by the partial reflector (PR) on right hand side (RHS) of the left WLC (LWLC) and the PR on left hand side (LHS) of the right WLC (RWLC).

Fig. 4.4 Diagram of the proposed pulse delay system. Two identical WLC is separated by $L_2$. 

\[ \begin{array}{c}
\text{LWLC} \\
L \\
L_2 \\
\text{RWLC}
\end{array} \]
In the previous WLC demonstration, the interaction of bi-frequency pumps with the intracavity medium created a negative dispersion[15]. In the absence of the pumps, the dispersion vanished and thus the WLC was converted to an ordinary cavity. In the system of Fig.4.4 one can turn off the pumps to eliminate WLC effect on LWLC(or RWLC). Without WLC effect, the transfer functions of the two WLCs are equal to that of an empty cavity as illustrated in Fig. 4.3(a). The separation of the pump frequency as well as the pump intensity determines the slope of the dispersion. One can control the parameters to change \( n_1, n_3 \) and thus to manipulate the linewidth of LWLC(or RWLC).

We use such WLC characteristics in the operation of the delay system. Here, consider an appropriate operation scheme to delay the pulse without distortion. Imagine a Gaussian data pulse whose bandwidth is broader than the linewidth of RWLC without WLC effect. The carrier frequency is shifted by \( 1.5 \times \Delta \nu_{\text{pulse}} \) from the cavity resonance (\( \omega_0 \)). Before the pulse enters into RWLC, one activates WLC effect. As illustrated in \( S_{\text{cavity}}(t) \& S_{\text{free}}(t) \) of Fig.4.3(c), the pulse appears at the exit of LWLC without delay and any distortion compared to the reference pulse which propagates in free space. We assume that the intermediate zone between LWLC and RWLC is long enough to spatially confine the data pulse. Afterward, the pulse propagates in the intermediate zone of the two WLCs and reaches at the LHS PR of RWLC. Remind that RWLC in the absence of WLC effect converts to an empty cavity. The pulse spectrum within the FWHM(\( \Delta \nu_{\text{FWHM}}=2.9\text{MHz} \)) of the empty cavity can propagate through RWLC and thus pulse is leaked out of RWLC. As such, we loose all the frequency components of the pulse which are inside the FWHM. Since the pulse is sufficiently shifted from the resonance frequency of \( \omega_0 \),
however, RWLC becomes a simple reflector with the reflectivity of R. The pulse is reflected with minor loss due to the transmission of the PR. Likewise, before the reflected pulse returns to the RHS PR of LWLC, we deactivate WLC effect so that the pulse is reflected again without the leaking. It will travel multiple round trips between the WLCs until we activate WLC effect on RWLC. As soon as RWLC is activated, the pulse passes through RWLC without frequency component loss and thus being free from distortion. Hence, the signal will be delayed as much as time elapse inside the intermediate zone, compared to the reference. Next, let us find a transfer function for the entire buffer system which is composed of LWLC, RWLC and the region in the middle of the two WLCs. First, consider a transfer function in the intermediate zone. As illustrated in Fig. 4.5, a monochromatic wave begins to propagate from RHS PR of LWLC and arrives at LHS PR of RWLC after N round trips. The wave before passing through the LHS PR is written as $E_0 e^{j\omega t} R^N e^{-j\omega(2N+1)\frac{L_2}{c}}$ where $L_2$ is distance between the two PRs. Since $E_0 e^{j\omega t}$ represents the input wave, one can write the transfer function as

$$H_i(\omega) = R^N e^{-j\omega(2N+1)\frac{L_2}{c}} \quad (4.4)$$

Fig. 4.5 Monochromatic wave travels N round trips between two reflectors.
Since $H_{WLC}(\omega)$ describes the response of the WLCs, Eq(4.4) together with $H_{WLC}(\omega)$ leads us to a transfer function $H_{total}(\omega)$ of the pulse delay system in Fig. 4.4. $H_{total}(\omega)$ can be expressed as $H_{total}(\omega) = H_L(\omega)H_R(\omega)$ where $H_R(\omega)$ and $H_L(\omega)$ are the transfer functions of RWLC and LWLC, respectively. Remember that the WLCs are identical thereby $H_R(\omega) = H_L(\omega) = H_{WLC}(\omega)$. Again, by convolution theorem, the output of the delay system can be written as

$$S_{system}(t) = \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} H_{total}(\omega)S(\omega)\exp(j\omega t)d\omega \quad (4.5)$$

Likewise, the reference pulse propagates as amount of the length of the system and thus $S_{free}(\omega)$ is

$$S_{free}(t) = \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} S(\omega)H_{free}(\omega)\exp(j\omega t)d\omega$$

where $H_{free}(\omega) = \exp(-j\omega(2L + L_2)/c)$. Fig.4.6 graphically illustrates $S_{free}(t)$ and $S_{system}(t)$.

Fig.4.6 Series of pulses in time domain. At $t=0$, the reference and the data pulse are launched at the entrance of the RWLC. Blue is the reference pulse ($S_{free}(t)$). It propagates the optical path of $2L+L_2$ in free space and the center of the pulse appears at $t = (2L + L_2)/c \approx 8.55 \times 10^{-6}$ second. The data pulse is observed at $t = (2L + 3 \times L_2)/c \approx 2.56 \times 10^{-5}$ second for one round trip ($N=1$) and $t = (2L + 201 \times L_2)/c \approx 1.71 \times 10^{-3}$ second for $N=100$. 
Remind that we choose the input pulse equal to the Gaussian pulse in Fig.4.3(a) and use the WLC in Fig.4.3(b) for RWLC and LWLC. For illustration, we consider $L_2=2557\text{m}$.

The numerical simulation suggests that for one round trip ($N=1$), the data pulse is delayed as much as $2L_2/c = 1.7\times10^{-5}\text{sec}$. For one hundred round trips ($N=100$), the delay time is observed as $200L_2/c = 1.7\times10^{-3}\text{sec}$ with an intensity attenuation due to the transmission of the two PRs. It is equal to approximately 5000 times input pulse duration ($=1.7\times10^{-5}/t_0 = 5000$). The bandwidth of a delay system can be defined as the maximum frequency spectrum width of a data pulse that the system can delay without distortion. One can note that $\Delta v_{\text{FWHM}}$ of WLC is an important parameter to determine our system bandwidth and it increases up to 340MHz in the presence of the WLC effect as shown in Fig.4.3(a). The data pulse of $\Delta v_{\text{pulse}}=29\text{MHz}$ is loaded to the intermediate zone through LWLC without distortion. The carrier frequency of the pulse is deviated from the empty cavity resonance as much as $\nu_{\text{pulse}} = \nu_{\text{FWHM}} = 33.5\text{MHz}$. Since $\Delta v_{\text{FWHM}}$ of the WLCs in the absence of WLC effect corresponds to 2.9MHz, the pulse is non-resonant with the cavities. We eliminate WLC effect on LWLC after loading to the intermediate zone. The pulse can be spatially confined inside the zone to delay until we activate RWLC. One can either downshift or upshift an input data pulse from the empty cavity resonance. To calculate the system bandwidth, Note the spectral region which the empty cavity spectrum excluded from the WLC. We can regard such a region as available spectral range where the pulse spectrum is placed. Therefore, our system bandwidth $\Delta v_{\text{system}}$ is approximately equal to $(340-2.9)/2=168\text{MHz}$. Due to the tunable
linewidth of WLC, it is easily expandable and thus one can obtain wider bandwidth than the current system. Another remarkable feature is that the delay time is independent parameter of $\Delta \nu_{\text{system}}$ and simply corresponds to time elapse inside the region between LWLC and RWLC. Therefore, our system provides a solution to delay time-bandwidth product encountered by delay systems in current optical communication.
V. DATA BUFFER SYSTEM IN FIBER RESONATORS

A. Motivation for data buffer in fiber optics

Slow- and fast-light demonstration in optical fibers has been attracted due to applicability to current optical devices such as optical buffers, optical delay lines and fast memory access which are necessary for fiber optic communication system[6, 7, 8, 9]. A nano-second data pulse was delayed by the interaction of the pulse with a single Brillouin pump, so-called slow light. However, the delay time was less than the pulse duration which is short delay from the view of practical points. A data pulse was stored by the writing pulse (Pulsed Brillouin pump) as an acoustic excitation and retrieved by the read-out pulse converting such excitation to the original data pulse[42]. The storage time was several times data pulse duration. Since the pulse was stored in the form of the decaying acoustic disturbance, however, the retrieved pulse was decayed and thus the storage time in this scheme was limited by the acoustic lifetime. In chapter IV, we have shown that paradoxically fast-light technique can be employed to build a data buffering system. In a so called White light cavity (WLC), a cavity contains fast-light medium whose negative dispersion compensates for wavelength change. It resonates broader spectral range compared to an empty cavity of equal length and finesse without loss of cavity build-up[15]. The buffering system was composed of two WLCs in the version of Fabry-Perot (FP) and the intermediate delay zone. In theoretical analysis, a data pulse was delayed several thousands times pulse duration with minor attenuation. However,
the intermediate zone was required to be sufficiently long for such a long delay time so that it is hard to be constructed in free space optics.

Coupling characteristics between a ring resonator and a waveguide has been an important research area in optical communication [46,47,48,49,50]. Their physical aspects such as power transfer and phase are consistent with that of a fiber ring resonator [51, 52, 53, 54]. Its coupling coefficient is represented by the transmission in FP while transmission coefficient corresponds to the reflection. In fast-light demonstration based on Stimulus Brillouin Scattering (SBS), negative dispersion was produced by bi-frequency Brillouin pumps in optical fibers. Therefore, we can expect that the combination of the fiber resonator with the fast-light technique possibly provides a solution to created WLC effect in fiber resonator.

For a more practical application to existing telecommunication, in this chapter we design a fiber white light cavity and employ it to build a fiber-based data buffering system. Another noticeable feature of the system is that the broadband negative dispersion (GHz) is obtained by using two broadened gains and thus we can extend the system bandwidth to multiple times Brillouin frequency ($v_B$: 8~12GHz [55, 56]).

**B. Application of fiber resonators to data buffering system**

First, let us review an optical fiber resonator [46, 47] in Fig.5.1(a). We are interested in the field amplitude transfer characteristics $b_2/a_1$ of this configuration. On the condition that a 2 by 2 coupler is internally lossless, the relationship between the complex amplitudes $a_i$ and $b_i$ can be described by the matrix for a 2 by 2 coupler
where $k$ is the intensity coupling coefficient. In addition, the transmission within the resonator can be described as $a_2 = \alpha e^{i\theta}b_2$ in terms of the internal transmission factor $\alpha$ and the round trip phase shift $\theta$. For $\alpha=1$, the coupled field $a_2$ travels one round trip without attenuation. $\theta$ is written as $-\text{tan}(\omega)L/c$ where $L$ is the circumference of the ring resonator. $n(\omega)$ is the refractive index of the fiber. In order to explain dispersion induced by SBS, it is expressed by Taylor expansion about the cavity resonance frequency $\omega_0$. $\omega_0$ is close to the stokes frequency $\omega-\nu_B$ which downshifted from Brillouin pump $\omega$ as much as the Brillouin frequency $\nu_B$ of the fiber. Using Eq.(5.1) together with $a_2 = \alpha e^{i\theta}b_2$, we can derive the ratio of the complex transmission amplitude to the input,

$$\frac{b_1}{a_1} = \frac{\sqrt{1-k} - \alpha e^{i\theta}}{1 - \alpha \sqrt{1-k} e^{i\theta}}$$  \hspace{1cm} (5.2)$$

Likewise, the input and the circulating field are related by

$$\frac{a_2}{a_1} = \frac{j\alpha \sqrt{k} e^{i\theta}}{1 - \alpha \sqrt{1-k} e^{i\theta}}$$  \hspace{1cm} (5.3)$$

Next, we consider a fiber resonator coupled to a fiber. Fig. 5.1(b) displays that the input port of a second coupler is connected to that of the first coupler to construct a semicircle and the closed loop is formed by likewise connecting the output ports. It can be regarded as the microresonator coupled to a second waveguide in Ref.46 and will be used as a building block for our fiber-based data buffering system. In the presence of the second coupler, we define the intensity coupling coefficients $k_1$ and $k_2$ (the subscripts ‘1’ and ‘2’
Fig. 5.1 Schematics of (a) fiber ring resonator, (b) ring resonator coupled to a fiber

represent ‘first’ and ‘second’ couplers, respectively) and modify the internal transmission from \( \alpha \) to \( \alpha \sqrt{1-k_2} \). Substituting \( k_1 \) and \( \alpha \sqrt{1-k_2} \) into \( k \) and \( \alpha \) in Eq (5.3) respectively, the ratio of the circulating amplitude to the input in the new configuration is written as

\[
a_{21} = \frac{i \alpha \sqrt{k_1 \sqrt{1-k_2}} e^{i \theta}}{1 - \alpha \sqrt{1-k_1} \sqrt{1-k_2} e^{i \theta}} \quad (5.4)
\]

The transmitted amplitude is related to \( a_1 \),

\[
b_2 = \sqrt{1-k_1} - \alpha \sqrt{1-k_2} e^{i \theta} \quad \frac{a_1}{1 - \alpha \sqrt{1-k_1} \sqrt{1-k_2} e^{i \theta}} \quad (5.5)
\]

Likewise, we also change from \( a_2 = \alpha e^{i \theta} b_2 \) to \( a_2 = \alpha \sqrt{1-k_2} e^{i \theta} b_2 \). With the consideration of the power transfer from \( b_2 \) to \( a_r \) by the second coupler, we write \( a_r \),

\[
a_r = \sqrt{\alpha j \sqrt{k_2}} e^{i \theta} b_2 \quad (5.6)
\]
Inserting $a_2 = \alpha \sqrt{1-k_2} e^{\alpha} b_2$ into Eq.(5.4) and substituting the resulting $b_2$ from such insertion into Eq.(5.6), one can obtain $a_r/a_1$,

$$\frac{a_r}{a_1} = \frac{-\sqrt{\alpha k_1 k_2} e^{i \omega}}{1 - \alpha \sqrt{1-k_1 \sqrt{1-k_2} e^{i \omega}}}$$  \hspace{1cm} (5.7)$$

where the round trip phase shift($\theta$) is expressed as $-\omega n(\omega)L/c$ and $L$ is the circumference of the closed loop. Dispersion in the fiber loop resulting from SBS can be considered by using Taylor expansion $n(\omega)$ as $n(\omega) = n_0 + (\omega - \omega_0)n_1 + (\omega - \omega_0)^2 n_3$,

$$n_1 = \left. \frac{dn}{d\omega} \right|_{\omega=\omega_0}, \hspace{0.5cm} n_3 = \left. \frac{1}{6} \frac{d^3 n}{d\omega^3} \right|_{\omega=\omega_0}$$

where $n_0$ is the mean index of the fiber, and $\omega_0$ is the cavity resonance frequency.

Fig.5.2 displays $|b_1|/|a_1|^2$ and $|a_r|/|a_1|^2$ in the absence of dispersion (blue line, $n_1=0$, $n_3=0$) and in the presence of negative dispersion (red line, $n_1<0$, $n_3\neq0$). We consider $|a_1|=1$ so that $|b_1|^2$ and $|a_r|^2$ are normalized to the input and assume that the internal loss is negligible ($\alpha=1$). We choose $n_1$ carefully so that regardless of the frequency change the cavity resonance occurs i.e. ideal White Light Cavity (WLC) Condition. For our case that the length of the dispersive medium is equal to the cavity circumference, the ideal WLC condition is that $n_g=0$ where $n_g$ is the group index of the dispersive fiber [15]. Next, $n_3$ is adjusted to reduce the infinite linewidth of the ideal WLC to finite value. For other parameters, we choose $k_1=k_2=0.01$, $\ell = 1m$, $n_0=1.45$ and $L=10m$ where $L$ is the circumference of the closed loop. For the ordinary ring resonator ($n(\omega)=n_0$), only at resonance the input completely transfer to the output. For the ring cavity with the WLC effect, the resonance spectral region is expanded without the build-
up loss so that full power of the input transfers to the output over broader frequency range than the ordinary resonator.

Fig. 5.2 Power transfer of the configuration in Fig. 5.1(b). (a) $|b_1|^2 / |a_1|^2$ (b) $|a_1|^2 / |a_0|^2$ for non-dispersive fiber (blue) and for negative dispersion (red) ($n_1 = -1.192 \times 10^{-15}/\text{rad}, n_3 = 1.223 \times 10^{-32}/\text{rad}^3$)

Next, we take into account Eq.(5.5) and Eq.(5.7) as the transfer functions of the configuration in Fig. 5.1(b). $H_{b1,a1}$ corresponds to Eq.(5.5). $H_O$ and $H_{WLC}$ are Eq.(5.7) with $n(\omega) = n_0$ and with $n(\omega) = n_0 + (\omega - \omega_0) n_1 + (\omega - \omega_0)^3 n_3$, respectively. For $H_{WLC}$, we choose $n_1, n_3$ to be equal to the values used in Fig. 5.2. In previous Fabry-Perot (FP) analysis, we derived the group index ($n_g$) for a pulse propagating through FP configuration in term of $\angle H_{WLC}$. Now, we are interested in $n_g$ associated with the configurations in Fig. 5.1. $\angle H_i(\omega)$ is the ring resonator contribution to the phase (the subscripts ‘i’ denotes O or WLC or b1,a1) and thus the phase shift associated with the propagation from the inputs to the outputs of the resonators is expressed by $(\omega n_{\text{eff}} / c) = (n_0 \omega / c) - \angle H_i$ in terms of the mean index of the fiber $n_0$ and the effective
refractive index of the resonators $n_{\text{eff}}$. Deriving $n_{\text{eff}}$ as a function of $\angle H_i$ and inserting $n_{\text{eff}}$ into $n_g = n_{\text{eff}} + \omega (dn_{\text{eff}}/d\omega)$, one can obtain $n_g$ as

$$n_g = n_0 - \frac{c}{\ell} \frac{d\angle H_i}{d\omega} \tag{5.8}$$

Pulse distortion is characterized by $\Delta T = -\left( d^2 \angle H_i / d\omega^2 \right) \Delta \omega$ as done in chapter IV.

Fig.5.3 (a) Phases associated with the transfer functions of the system in Fig.5.1(b), $\angle H_{bl,al} = \text{arg}(b_i/a_i)$, $\angle H_O = \text{arg}(a_i/a_i)$ in the absence of WLC effect, $\angle H_{WLC} = \text{arg}(a_i/a_i)$ for $n_i = -1.192 \times 10^{-15} / \text{rad}$, $n_3 = 1.223 \times 10^{-32} / \text{rad}^3$ (b) Reference pulse after propagating $\ell$ (black), system output associated with $H_O$ (blue), $H_{WLC}$ (red). Output pulse in the presence of WLC effect is overlapped with the reference.

Fig.5.3(a) exhibits $\angle H_{bl,al}$, $\angle H_O$ and $\angle H_{WLC}$. Fig.5.3(b) presents the pulse output after propagating distance $\ell$ through free space (black) and the outputs of the systems associated with $H_O$ (blue) and $H_{WLC}$ (red). The cavity parameters are the same as used in Fig.5.2. For illustration, we choose the input to be Gaussian pulse in the form of $S(t) = \exp(-t^2/t_0^2) \exp(j\omega dt)$ and the pulse bandwidth $\Delta \nu_{\text{pulse}} (= 1/t_0)$ is 10 times the Full.
Width Half Maximum (FWHM) of the resonator $\Delta\nu_{\text{cavity}}$. By convolution theorem, we obtain the output amplitude as

$$S_{\text{out}}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S(\omega) \exp(-j k_0 \ell) H_1(\omega) \exp(j \omega t) d\omega$$

where $k_0 = n_0 \omega \ell / c$. Since the amplitude transmission factor $\alpha \sqrt{1 - k_2}$ is equal to the transmission coefficient of the 1st coupler $\sqrt{1 - k_1}$, the system can be regarded as a critically coupled fiber ring cavity. Therefore, $\angle H_{b1,a1}$ is discontinuous at resonance and leaps as much as $\pi$. Such discontinuity and phase jump correspond to the case of a critically coupled microresonator as indicated in ref.47 (Since we define the phase shift as $\phi = \omega t - \omega n_{\text{eff}} \ell / c$ but ref.47 uses $\phi = \omega n_{\text{eff}} \ell / c - \omega t$, there is sign difference.) Due to the additional phase shift $\pi$ resulting from the second coupler, at resonance $\angle H_O$ and $\angle H_{WLC}$ jump $\pi$ more than $\angle H_{b1,a1}$ thereby the phase leap being $2\pi$. $\angle H_O$ indicates $n_e > n_0$ inside the dotted circle. Since the pulse carrier frequency $\omega_0$ corresponds to the cavity resonance, and $\Delta \nu_{\text{pulse}} = 10 \times \Delta \nu_{\text{cavity}}$, its spectrum completely covers such slow light zone ($d\angle H_O / d\omega < 0, d^2 \angle H_O / d\omega^2 \neq 0$) in $\angle H_O$ and mainly belongs to the spectral region of $|H_O|=0$. The pulse output associated with $H_O$ is delayed with serious distortion (attenuated) as displayed in Fig.5.3(b). $\angle H_{WLC}$ in Fig.5.3(a) suggests that due to the negative dispersion the system resonance (i.e. $|H_{WLC}| = 1, d\angle H_{WLC} / d\omega = 0, d^2 \angle H_{WLC} / d\omega^2 \approx 0$) occurs sufficiently broad to cover the pulse spectrum. The output (red) associated with $H_{WLC}$ in Fig.5.3(b) indicates the pulse propagates without time advancement or delay but maintains its original shape.
Now, we employ the fiber ring cavity investigated in Fig.5.1~3 to design a fiber-based buffering system. In the previously proposed system, the FP based buffering system is composed of Right White Light Cavity (RWLC), intermediate delay zone and Left White Light Cavity (LWLC). Fig. 5.4 displays our fiber-based buffering system.

![Fig.5.4 Schematics of the fiber-based buffering system](image)

Two ring cavities serve as RWLC and LWLC. The ports at right side of the right ring resonator ($\mathcal{R}rr$) are connected to the ports at left side of the left ring resonator ($\mathcal{S}rr$) through the fiber spools to construct the closed loop delay zone. Let us assume that the bandwidth of the input pulse is much broader than FWHM of the resonators. We consider the operating scheme to delay such broad pulse without distortion. At the stage of pulse loading to the intermediate delay loop, we turn on bi-frequency Brillouin pumps to activate WLC effect on the right resonator. The WLC bandwidth is sufficiently wide
to resonate over the pulse spectrum. As such, the pulse is successfully loaded remaining its original shape as indicated in Fig 5.3(b). After loading, we turn off the pumps so that the pulse is trapped and circulates inside the delay zone. At this trapping stage, it is necessary to consider the constraint on the carrier frequency of the pulse. If the carrier frequency corresponds to the resonance (ω₀) of the fiber ring cavities in the absence of WLC effect (i.e. ordinary resonator), the major spectral components of pulse spectrum around ω₀ are located inside the ordinary resonator transmission in Fig.5.2(b). At every passage through the couplers inside the delay loop, such components leak out through RR and RR causing pulse distortion. To avoid such spectrum loss, we shift the carrier frequency from the resonance by several times Δνcavity so that the pulse circulates the loop with minor loss associated with the coupling coefficients \(\sqrt{k_1}, \sqrt{k_2}\). Activating WLC effect on RR, we can extract the data pulse from the delay loop. Let us assume that RR and RR has same physical parameters (\(\sqrt{k_1} = \sqrt{k_2}\), α and Δνcavity). The amplitude transfer characteristics through the resonators (\(H_{1r} = a_r/a_1, H_{1r}' = a'_r/a'_1\)) are given by Eq(5.7). Next, after N multiple round trip \(a_r\) is related to \(a'_1\) by

\[
H_{1r} = \left(\sqrt{1 - k_1}, \sqrt{1 - k_2}\right)^N e^{-jk_0 L_2/2} (5.9)
\]

where \(L_2\) is the delay zone length, \(k_0 = \omega n_0/c\). Fig. 5.5 displays data pulse outputs after propagating distance \(ℓ\) (reference), 5 round trips and 10 round trips. For illustration, we choose a Gaussian pulse with the shifted carrier frequency in the form of

\[
S(ω) = t_0/\sqrt{2} \exp\left[(-ω - ω_0 - ξ)t_0^2/4\right]
\]

where ξ = 1.5 × Δνpulse. For reference after
Fig. 5.5 Output pulse sequence of the fiber-based buffering system (a) Reference pulse, (b) 5 (c) 10 times round trips inside the delay zone

\[ S_{\text{ref}}(t) = \int_{-\infty}^{\infty} \exp(-jk_0/\ell)S(\omega)\exp(j\omega t)d\omega. \]

The system output is written as

\[ S_{\text{out}}(t) = \int_{-\infty}^{\infty} \exp(-jk_0/\ell)H_{rr}(\omega)H_{\text{ref}}(\omega)S(\omega)\exp(j\omega t)d\omega \quad (5.10) \]

We use the resonator investigated in Fig.5.2 for \( \mathcal{rr} \) and \( \mathcal{rr} \). We choose \( \ell = 1\text{m}, L_2 = 6.7\text{Km} \). The output pulses lag behind the reference as much as \( 1.78 \times 10^{-4} \) sec for \( N=5 \) and \( 3.40 \times 10^{-4} \) sec for \( N=10 \) which corresponds to \( [n_0(2N+1)L_2]/c \). One can observe the attenuation resulting from the power coupling to \( \mathcal{rr} \) and \( \mathcal{rr} \) during the propagation inside the delay zone.

**C. GHz bandwidth negative dispersion**

The numerical simulations in Fig.5.2 suggest that a half of the WLC linewidth \( (\Delta\nu_{\text{WLC}}) \) corresponds to the dynamic range of our system where a data pulse can be delayed without distortion. The WLC linewidth coincides with the spectral region where negative dispersion occurs. Since the negative dispersion is produced in the middle of
the two gain lines, in gain doublet system, the achievable $\Delta \nu_{WLC}$ was determined by the gain separation $\delta$ and was estimated proportionally to $\delta^{2/3}$ [15]. If the gain separation increases too broad compared to the gain bandwidth, however, the dispersion slope around center of the two gains is not negative. The cavity looses the WLC effect. In the broader gain bandwidth system, one can more separate the two gains and thus obtain broader spectral range of negative dispersion. As such, $\Delta \nu_{WLC}$ increased by broadening $\Delta \nu$. The Brillouin pump spectrum was broadened by superposing Gaussian white noise on laser diode dc current. Such Brillouin pump was widely used to produce broadband gain profile [57, 58, 59, 60]. When Double Brillouin pumps were separated by $2\nu_B$ ($\nu_B$: Brillouin frequency) and had equal power, gain width was expanded equal to $2\nu_B$ [60].

Now we adopt the Double pumps technique in ref.60 to produce broadband negative dispersion. We use two groups of the double equal power pumps. Pump1 and 1′ (2 and 2′) are separated by $\nu_B$, and pump 1 and 2(1′ and 2′) by $\delta$. Due to the separation by $\nu_B$ and the equal power, the loss induced by pump 1(pump 2) is completely cancelled by the gain line of pump1′ (pump2′). Therefore, we can eliminate the overlapping between the gain and the loss of pump1 (pump2) and thus avoid the gain spectrum distortion. We increase the gain bandwidth ($\Delta \nu_B$) to $2\nu_B$ and stop before the gain of pump1 (pump2) meets the loss of pump1′ (pump2′). In Fig.5.6 (c), finally only two broadened gain lines survive separated by $\delta$. In the two groups of the double pumps configuration, however, Fig.5.6 (d) indicates that the loss spectrum of pump1′ extends beyond the gain of pump2 (Note inside the dotted area). Such overlapping causes the gain distortion. To prevent it,
Fig.5.6 (d) suggests that $\delta$ is related to $\nu'$ by $\delta + \nu'/2 = 2\nu_B$, where $\nu'$ is the width measured along the bottom of the gain. The single gain bandwidth can be more expanded if we employ more pumps in the individual group.

Fig.5.6 Scheme to create negative dispersion in the scale of several GHz. (a) Double pumps (red, green) in group I,II. The pumps in green provide gain spectrums to compensate for the loss spectrums of the pump in red. (b) Pump spectrum is broadened by Gaussian white noise to create the broadband gain equal to Brillouin shift ($\nu_B$). (c) Gain bandwidth of $2\nu_B$ (d) Due to the overlapping of the gain in group II with the loss in group I, $\delta + \nu'/2 = 2\nu_B$ should be observed to avoid gain distortion resulting from such overlapping.
More generally speaking, N pumps create a single gain with the maximum bandwidth of \( N \nu_B \) [60]. In the configuration of the two groups, the achievable gain separation with the broadened gain-doublet of the bottom width \( \nu' \) is written as \( \delta + \nu'/2 = N \nu_B \). Since the Brillouin frequency of optical fibers is 8~12GHz, \( \delta \) in the range of GHz can be easily obtained. Negative dispersion occurs in such GHz spectral area between the two gains and thus one can increase WLC linewidth up to GHz.
VI. PRELIMINARY EXPERIMENT

A. Motivation

When Laser light propagates through optical fibers, the incident light is back-scattered by the refraction index modulated gratings associated with a sound wave of Brillouin shift frequency \( \nu_B \)[61]. This non-linear process, so-called stimulated Brillouin scattering (SBS) has been investigated in conventional silica fibers [62,63] or fibers with enhanced Brillouin coefficient [56,64]. It leads to many applications such as Brillouin amplifier, lasing, and optical pulse delay[6, 65, 66, 67, 68]. Using Brillouin pump to induce group index change, the data pulse was delayed less than the pulse duration of 20~50ns [68].

In chapter V, white light cavity (WLC) in a fiber ring resonator was proposed to build a data buffering system to trap the data pulse in long fiber loop. WLC is a cavity that contains anomalously dispersive medium. Such dispersion compensates for the frequency-dependent wavelength change and thus WLC resonate without reduction of build up factor over broader frequency range than ordinary cavities. This proposed buffering system overcame the delay-bandwidth constraint imposed on current slow light technique.

In this chapter, we build a fiber ring cavity based on the Brillouin characteristics of fiber in use. With a resonantly enhanced single pump, 1\(^{st}\), 2\(^{nd}\), and 3\(^{rd}\) order of Brillouin threshold are evaluated as 20, 60 and 100mW. For bi-frequency pumps, the pumps are simultaneously locked to different longitudinal modes of the ring cavity. Two
Brillouin lasing modes at frequencies corresponding to the dual gain peaks show equal intensity without any noticeable cross-talks between two resonant pumps. For the application to WLC in fiber resonators, our analysis of cavity modes, pump beams and dual gains offer proper gain separations as well as free spectral range (FSR) of the cavity.

**B. Theoretical background**

Before constructing a fiber ring resonator, we review the theoretical model of the complex Brillouin gain factor produced by SBS process. Afterward, we will show the experimental demonstration to examine the Brillouin characteristics of a fiber. Material density variation induced by optical fields is a source of SBS process. The material system responds to an incident laser light and has the refractive index variations (gratings). The gratings move with a sound wave of frequency $\Omega$ away from the incident light. The laser with the frequency of $\omega_L$ is scattered by the gratings. Since the acoustic wavefronts are moving with the frequency of $\Omega$ from the laser, the counterpropagating scattered light becomes downshifted as much as $\Omega$ and has the stokes frequency of

![Fig.6.1 Schematic representation of (a) Stimulated Brillouin Scattering(SBS), (b) an SBS amplifier](image-url)
\( \omega_s (= \omega_L - \Omega) \) indicated in Fig.6.1(a). In such circumstances, the interference of the laser and stokes wave contributes to amplifications of the sound wave, or beating of the sound wave with the laser strengthens the stokes wave. Individual material system exhibits the different sound wave frequency so-called the Brillouin shift frequency \( \Omega_B \). It is an important parameter to be considered in an SBS amplifier. Fig.6.1(b) displays the configuration of an amplifier. In SBS amplifier, the Stokes and the laser are externally applied and counterpropagate, and \( \omega_s \) is required to be equal to \( \omega_L - \Omega_B \) for maximum amplification.

The information about the Brillouin characteristics of fibers will be useful to build the ring resonator. Let us consider the coupling of the stokes field (probe) at frequency \( \nu \) to the pump field via SBS process. The stokes field propagates in the fiber with the length of \( L \) where the pump counterpropagates. Assuming that the pump is non-depleted, the slowly varying amplitudes of the stokes field at steady state after the interaction with the pump field is described by

\[
E_S = E_{s0} e^{(\alpha + \beta)L} \tag{6.1}
\]

where \( E_s \) is the amplitude after the propagation along the fiber with length of \( L \), \( E_{s0} \) is the input field amplitude (weak stokes seed), \( \alpha \) and \( \beta \) are expressed as

\[
\alpha = \left( g_0 I_p / 2(1 + 4(\nu - \nu_B)^2 / \Gamma_B^2) \right) \quad \text{and} \quad \beta = \left( g_0 I_p (\nu - \nu_B) / \Gamma_B \right) / \left( (1 + 4(\nu - \nu_B)^2 / \Gamma_B^2) \right)
\]

in terms of the Brillouin frequency \( \nu_B \), gain coefficient \( g_0 \), gain linewidth \( \Gamma_B \) and pump intensity \( I_p \). For the case that one uses bi-frequency pumps to produce Brillouin gain-doublet, \( \alpha \) and \( \beta \) are modified as
\[ \alpha = \frac{1}{2} \left[ \frac{g_0I_{P1}}{1 + 4(v - v_B - \Delta)^2 / \Gamma_B} + \frac{g_0I_{P2}}{1 + 4(v - v_B + \Delta)^2 / \Gamma_B} \right] \] (6.2a)

\[ \beta = \frac{g_0I_{P1}(v - v_B - \Delta)/\Gamma_B}{1 + 4(v - v_B - \Delta)^2 / \Gamma_B^2} + \frac{g_0I_{P2}(v - v_B + \Delta)/\Gamma_B}{1 + 4(v - v_B + \Delta)^2 / \Gamma_B^2} \] (6.2b)

provided the pumps are separated as amount of \(2\Delta\). \(I_{p1}, I_{p2}\) are the intensities of the bi-frequency pumps. In such gain-doublet system, the gain separation corresponds to \(2\Delta\).

We define the parameter \(G\) as \(g_0I_{pj}/2\) (j=1,2) associated with the amplitude gain peak.

**C. Brillouin characteristics measurement**

Fig.6.2 Experimental set-up to measure Brillouin spectrum: FPC, fiber polarization controller; EDFA, Er-doped fiber amplifier, EOM, Electro optic modulator

Next, we measure Brillouin frequency, gain linewidth and \(G\) factor as increasing the pump power. Fig.6.2 displays the schematics of the experimental set-up for Brillouin gain spectrum measurements. The light source is a 1550nm CW laser diode (LD), (linewidth<1MHz). The LD output is divided by a 90:10 coupler. A 3dB coupler is used
to build a fiber loop mirror [69] and the 10% of LD’s light is inserted into the port of 1 m fiber pigtail of the coupler. Hence, the 1 m external cavity is implemented by the loop mirror so that it provides optical feedback to reduce the linewidth of LD. The 90% output is divided again by another 3dB coupler. One is inserted into an Electro-optic modulator (EOM-1) and is augmented by an Er-doped fiber amplifier (EDFA) to serve as the Brillouin pump. The pump from EOM-1 without modulation is used to produce a single gain. For the generation of the double peaks, the D.C voltage on EOM-1 is adjusted to eliminate the fundamental and thus the gain separation corresponds to $2f_M$ ($f_M$: modulation frequency). The modulation frequency of the EOM-2 is swept $\Delta v$ around $\nu_B$ to provide a probe.

Fig.6.3 illustrates the Brillouin spectrum of 88m Corning SMF-28e fiber. In Fig. 6.3(a), the single gain is characterized by scanning around $\nu_B$ as much as $\Delta v=40\text{MHz}$ in the step of 0.5MHz. We increase the pump power until the gain is saturated. The Brillouin shift frequency and gain width are measured as 10.867GHz and $\sim 10\text{MHz}$, respectively. $\nu_B$ is 50 MHz higher and $\Gamma_B$ is a factor of 3 less than the previous measurement for SMF-28e [62]. In ref.62, they used the long fiber (6.4km) and the low power pump in the scale of mW compared to ours. Hence, the Brillouin interaction length ($L$) for the given pump is long and thus the pump depletion occurs to broaden the linewidth more than our case [63] Fig.6.3(b) illustrates the gain factor, $G$ ( /m) associated with the gain spectrum displayed in Fig.6.3(a). Assuming the absence of saturation, we plot the linearly increasing graph (Purple line). The slope is calculated from the experimental data for the pump powers of 600 and 800mW. $G$ for the higher power (1,
1.5, 2W) is plotted with the calculated slope. In the comparison to the experimental data displayed as the yellow graph, we can confirm that the gain factor, G (/m) starts to saturate at the pump of 1.5W.

Fig.6.3 (a) Brillouin gain spectrum of a 88m single mode fiber, Y axis in voltage on photodiode (Volt. on PD) (b) G (Gain peak amplitude at resonance: /m) for linear increases without saturation and for real experimental data.

Fig.6.4 Dual Brillouin gain spectrum for the gain separation of 34.64MHz ($v_M = 17.32$MHz) and 51.96MHz($v_M = 25.98$MHz)
In Fig.6.4, the double gain peaks are displayed for the case that the modulation frequencies of EOM-1 are $\nu_M = 17.32\text{MHz}, 25.98\text{MHz}$. The scan range of the probe is $\Delta \nu = 100\text{MHz}$. The anti-symmetry of the double gains results from unbalanced sideband output of EOM-1. The peak of the individual gain approximately corresponds to that of the single gain associated with the single pump power which is equal to a half of the bi-frequency pump power. $\Gamma_B$ is measured as $\sim 12\text{MHz}$.

Fig.6.5 displays the schematics of the experimental setup to demonstrate Brillouin fiber laser. 90% of LD output is divided by a 3dB coupler. One is inserted into a Fabry-perot (FP) with FSR of 37GHz and finesse of 30. It provides a reference spectrum which corresponds to a laser frequency ($\nu_L$). The other is modulated by EOM while suppressing the fundamental ($\nu_L$) and only two sidebands are amplified by EDFA to generate bi-frequency pump. For a single pump, we remove the EOM. The coupling ratio of a variable coupler (VC) is set to critical coupling condition[54]. At such condition, a full width half maximum (FWHM) of our fiber resonator is measured as 0.23MHz. A lock-in amplifier (LIA) and A.C. servo lock the cavity length at the pump resonance. First, we observe cascaded Brillouin lasing while the single pump is resonant with the cavity. We adjust the coupling ration of V.C. as well as increase pump power to attain the higher order Brillouin. Fig.6.6 shows Brillouin lasers monitored by the F.P. spectrum analyzer. Fig.6.6(a) indicates that in the presence of 20mW pump, the smaller peak is a backreflection of the pump and the larger peak is the $1^{st}$ order Brillouin. The $2^{nd}$ order and $3^{rd}$ order Brillouin lasers are observed at 60mW and 100mW, respectively as displayed in Fig.6.6(b),(c). The separation between adjacent orders of the Brillouin
spectrums is equal to $v_B (10.867 \text{GHz})$. The clockwise propagating 2\textsuperscript{nd} order Brillouin produces small spectrum peak compared to the odd orders due to high isolation between the port 1 and 2 of V.C.

![Diagram of Brillouin fiber lasers](image)

**Fig.6.5 Schematic of cascaded and bi-frequency Brillouin fiber lasers**

**(a)**

**Fig.6.6** (a) 1\textsuperscript{st} order (Pump power: $P = 20 \text{mW}$) (b) 1\textsuperscript{st} and 2\textsuperscript{nd} order (60mW) (c) 1\textsuperscript{st}, 2\textsuperscript{nd} and 3\textsuperscript{rd} order (100mW) Brillouin laser
**D. Cavity modes, pump spectrums and dual gains under Bi-frequency pumps resonance**

Based on the measured $v_B$ and $\Gamma_B$ in Fig.6.3 and 6.4, a fiber ring cavity is designed to obtain $N \times \text{FSR} = v_B$ where $N$ and FSR are an arbitrary integer and the free spectral range of the cavity, respectively. The FSR of our cavity is 17.3262MHz which is broader than $\Gamma_B$. Due to $N \times \text{FSR} = v_B$, one longitudinal cavity modes is present close to the center of the gain. Provided FSR is broader than $\Gamma_B$, the Brillouin single gain supports only one longitudinal cavity mode at the peak of the gain profile and thus obtaining a single mode Brillouin laser. Next, for the future applications such as white light cavity (WLC) demonstration, let us consider the case that the bi-frequency pump are simultaneously resonant with the cavity. Fig.6.7(a) and (c) illustrates EOM outputs monitored by the fiber cavity. It consists of two sidebands ($v_L \pm v_M$) and the fundamental ($v_L$). The two sidebands share the equal cavity length to $L$ for $v_M = (2k + 1)/2 \times \text{FSR}$, and $L^*$ for $v_M = k \times \text{FSR}$ where $k$ is 0 or positive integer (For $L^*$, $k=0$ implies that EOM provides the fundamental). However, the sidebands occupy the different modes. For example, $(N + 2)\lambda_1 = L$, $(N + 3)\lambda_2 = L$ where $\lambda_1$ and $\lambda_2$ are wavelength associated with $v_L - v_M$ and $v_L + v_M$ ($v_M = 8.6631\text{MHz}$) respectively, $N$ is an arbitrary positive integer. After eliminating $v_L$, the cavity is locked at $L$ or $L^*$ so that $v_L \pm v_M$ simultaneously resonate in the cavity. Fig.6.7(b), (d) display the configurations of cavity modes (thick bars), double gains and pump spectrums (color arrows). The bi-frequency pumps create the gain-doublet which is represented by the same color as the pumps’ in use. For the cavity locking at $L$, fig.6.7(b) illustrate that the even number of cavity modes appears
between the two gains and thus no cavity mode exists in the middle of two gains. If the length is changed to \(L^*\), the cavity modes exist displaced from the modes supported by \(L\).

![Image](image-url)

**Fig. 6.7** (a) Pump resonance peaks from the fiber ring cavity. The pump is modulated at \(\nu_m = (2k + 1)/2 \times \text{FSR} \). (b) Bi-frequency pumps (color arrows) are resonant with the cavity modes (black bars). The pumps produce dual gains which is downshifted from the pump spectrums as much as \(a, b, c\) equal to \(\nu_B=10.867\text{GHz}\). The gains contain two modes at the center of each gain profile. For \(\nu_m = \text{FSR}/2\), the resonant pumps (red arrows) produce the two gains (red). As increasing \(\nu_m\), the gain separation are expanded to 3FSR for \(\nu_m = 3/2 \times \text{FSR} \) (green) and 5FSR for \(\nu_m = 5/2 \times \text{FSR} \) (blue). (c) \(\nu_m = k \times \text{FSR}\), (d) The gain separation is even number times FSR. For example, 2FSR for \(\nu_m = \text{FSR}\) and 4FSR for \(\nu_m = 2 \times \text{FSR}\). The single pump and gain are represented by the brown. The mode under the gain profile agrees with that of the 1st order Brillouin laser presented in Fig. 5(a).

As amount of \(\delta = 1/2 \times \text{FSR}\) indicated in Fig. 6.7(d). Therefore, the center of the two gains coincides with the cavity mode corresponding to \(\nu_L - \nu_B\). To explain the mode of
the lasing in Fig.6.6, note that the single cavity mode supports the center of the gain. As such, the Brillouin laser in Fig.6.6 is a single mode. For WLC demonstration, the cavity mode requires being equal to $\nu_L - \nu_B$ or the deviation from $\nu_L - \nu_B$ is small in comparison to gain separation. The gain doublet is symmetrically arranged around $\nu_L - \nu_B$. As we can scan a probe around $\nu_L - \nu_B$ to observe WLC effect, the configurations illustrated in Fig.6.7(d) is more suitable. However, it can have the residual fundamental after suppression thereby the residual is possibly resonant with the cavity to distort the double gain profile. Since the sidebands are locked onto different position from the fundamental illustrated in Fig.6.7(b), the dual gains can be generated without distortion. In WLC demonstration, we choose the cavity mode nearest by the center of the two gains and scan the probe frequency around it. Negative dispersion is created in the middle of the separated gains. Note that the center of the dual gains ($\nu_L - \nu_B$) is separated from its neighboring cavity modes by FSR/2. Provided that the gain separation is sufficiently wider than FSR/2, the scanning range completely belongs to the spectral range of negative dispersion. As such, the probe experiences constant negative slope even though the probe scanning mode is deviated from $\nu_L - \nu_B$. Wavelength change due to the scanning is compensated by the negative dispersion slope associated with the group index close to zero for our case, so-called WLC effect. The presented techniques in Fig.6.7 force the gain profile to support the cavity mode at the gain peak, allowing the single mode Brillouin-shifted wave to resonate and eventually to be lasing. Since the probe can be embedded in the lasing signal, the Brillouin laser becomes the main source of background noises.
For WLC operation below Brillouin threshold, we could propose alternative FSR to fulfill $(N+1/2)\text{FSR} = \nu_B$. Fig.6.8 illustrates the case that FSR is 17.4016MHz. Once the cavity length is fixed at $L$ or $L^*$, the resonant pumps produce the gain profiles downshifted from the pump frequencies as much as $\nu_B$. Due to the given condition $((N+1/2)\text{FSR} = \nu_B)$, the gain peaks appear in the middle of the two neighboring cavity modes illustrated in Fig.6.8. Since the Brillouin linewidth is less than the FSR, the resonant modes are present sufficiently apart from the gain peak. Hence, we can achieve higher gain peaks than the Brillouin resonating cavity, while staying below Brillouin threshold.

Fig.6.8 FSR is modified equal to 17.4016MHz so that $(N+1/2)\text{FSR} = \nu_B$. (a) Pump resonance peaks for $\nu_M = (2k+1)/2 \times \text{FSR}$, (b) Gain profiles, pump spectrums and cavity modes associated with the cavity locking on $L$, (c) $\nu_M = k \times \text{FSR}$, (d) Same diagram as (b) when $\nu_M = k \times \text{FSR}$. 
To demonstrate dual frequency Brillouin laser, the pumps are modulated at \( v_M = 7.9932 \text{GHz} \) i.e. \( v_M = (2k + 1)/2 \times \text{FSR} \), \( k=461 \). From the standpoint of the Brillouin threshold, the cavity condition presented in Fig.6.8 is useful for WLC application in principle. To evidently prove the generation of dual gains in the cavity, however, we use the cavity with FSR of 17.326MHz. As such, we observe two different Brillouin lasing modes at the frequencies associated with dual gain peaks. Fig.6.9(a) exhibits the EOM output observed by a photodetector (PD) at the port 3 of V.C. with scanning the ring cavity. Fig.6.9(b) displays the dual frequencies Brillouin lasers. According to the investigation in Fig.6.7, the lasing frequencies correspond to the longitudinal cavity modes, \( \nu_L - \nu_B \pm \nu_M \). From the equal intensity for the lasing peaks, one can determine that the two resonant pumps have same intensity and produce the gains without causing noticeable cross-talks.

Fig. 6.9 (a) Pump resonance peaks, (b) Dual frequency Brillouin lasing. The two lasing peaks are separated by \( 2v_M = 17.326 \text{GHz} \).
VILEXPERIMENT AND PARAMETER CONSTRAINTS

A. Motivation

Techniques to create negative dispersion over a limited frequency range have been performed through stimulated Brillouin scattering (SBS) [7, 8, 70]. In a so-called fast light demonstration, optical pulse propagates through materials with such an anomalous dispersion faster than free space. Fast light in optical fibers via SBS process have attracted much attentions, due to availability in current communication and potential applications. In chapter V, data buffering systems based on fast-light technique in fibers was proposed to exceed the delay-bandwidth constraint encountered in a typical data buffer by means of slow light. Also, it can be employed to enhance the rotational sensitivity of a passive resonator gyroscope [13] and to realize Superluminal ring laser as a versatile hypersensitive sensor.

White Light Cavity (WLC) is fundamental to implement all these devices. WLC is a cavity that resonates over a wider frequency range than an ordinary cavity without a drop in the field build-up factor. It has been implemented in a cavity filled with rubidium vapor where dual Raman gains were produced around the cavity resonance to achieve the fast-light condition ($n_g<1$) [15]. In chapter VI, primary analysis and experiments have been carried out to find out the proper free spectral range (FSR) as well as the optimal pump power level for WLC demonstration in fiber. FSR was adjusted to ensure that the cavity mode exists at the center of dual Brillouin gains. The Brillouin pump power was controlled to maintain WLC below Brillouin lasing threshold.
In this chapter, we investigate physical constraints in realizing such a WLC via SBS in conventional single mode fibers. In the experiment, Brillouin pump is resonantly enhanced in the cavity. To create negative dispersion, SBS of a single pump induces the probe depletion (Brillouin absorption) or bi-frequency pumps provide dual gains. In both cases, the cavity resonance is observed to be broadened by a very small amount due to small WLC effect. Numerical simulation confirms that under given experimental conditions the group index change is small ($\Delta n_g = -0.01 \sim -0.03$). As such, negative dispersion is not sufficient to induce large WLC effect. Theoretical analysis shows that major restraints on WLC based on conventional optical fibers are the requirements of extremely high pump power as well as the power damage threshold of optical elements. Key solution to these problems is higher Brillouin coefficient and low transmission loss fiber. To our knowledge, however, current fabrication technique can provide fibers with high Brillouin coefficient but high transmission loss in comparison to conventional silica fibers. We propose alternative fiber resonator to realize WLC with such high Brillouin coefficient but lossy fibers.

B. Theoretical background for WLC effect in fiber resonators

First, let us consider the optical field propagating an optical fiber resonator. Fig.7.1 displays that a fiber ring resonator is build by a 2 by 2 coupler. Provided that the coupler is internally lossless, the complex field amplitudes is explained by the matrix

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \sqrt{1-K} & j\sqrt{K} \\ j\sqrt{K} & \sqrt{1-K} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$ (7.1)
where $K$ is the intensity coupling coefficient.

![Fig.7.1 Schematics of fiber ring resonator](image)

Next, we include the internal loss in the resonator. The relationship between $a_2, b_2$ is expressed in terms of the loss characteristics of the resonator ($\alpha$) and the round trip phase shift ($\theta$)

$$a_2 = \alpha e^{i\theta} b_2 \quad (7.2)$$

Using Eq(7.1, 2), we can derive the power transmission ratio $|b_1/a_1|^{2}$ and the inserted power into the resonator $|b_2/a_1|^{2}$ with respect to the input $|a_1|^{2}$

$$\frac{|b_1|^{2}}{|a_1|^{2}} = \frac{\alpha^2 + |t|^2 - 2\alpha |t| \cos \theta}{1 + \alpha^2 |t|^2 - 2\alpha |t| \cos \theta} \quad (7.3a)$$

$$\frac{|b_2|^{2}}{|a_1|^{2}} = \frac{1 - |t|^2}{1 - 2\alpha |t| \cos \theta + \alpha^2 |t|^2} \quad (7.3b)$$

where $t$ is complex amplitude transmission coefficient ($K + T = 1, T = |t|^2$). For simplicity, we will choose $|a_1|^{2} = 1$. Next, consider $\theta$ in the presence of bi-frequency Brillouin pumps. The probe at frequency $\omega$ propagates the ring resonator whose
circumference is L. The phase shift resulting from the propagation is written as kL. k is the wave vector amplitude of the probe. To observe WLC effect, the bi-frequency pumps produce dual gains via SBS process in the fiber resonator and negative dispersion is created between the gains. For our case that Brillouin gain medium length corresponds to L, the group index, \( n_g = 0 \) is required to achieve WLC condition. The Brillouin dual gain \( \alpha_{Br} \) and the resultant phase \( \beta \) are given by

\[
\alpha_{Br} = \frac{1}{2} \left[ \frac{g_0 I_{p1}}{1 + 4(v - v_B - \Delta)^2 / \Gamma_B^2} + \frac{g_0 I_{p2}}{1 + 4(v - v_B + \Delta)^2 / \Gamma_B^2} \right] \tag{7.4}
\]

\[
\beta = \frac{g_0 I_{p1} (v - v_B - \Delta) / \Gamma_B}{1 + 4(v - v_B - \Delta)^2 / \Gamma_B^2} + \frac{g_0 I_{p2} (v - v_B + \Delta) / \Gamma_B}{1 + 4(v - v_B + \Delta)^2 / \Gamma_B^2} \tag{7.5}
\]

where \( v_B, g_0, \) and \( \Gamma_B \) are Brillouin frequency, gain coefficient, linewidth, respectively. \( I_{p1}, I_{p2} \) are pump intensities per unit area. We consider the equal intensity, i.e. \( I_{p1} = I_{p2} = I_p \) (W/m²). \( 2\Delta \) corresponds to gain separation. The probe propagates the cavity where negative dispersion is created by SBS. The phase shift after the one round trip can be written as

\[
kL = n_f \omega L / c + \beta L .
\]

\( n_f \) and \( c \) is the mean index of the fiber and speed of light.

The loss characteristics of the resonator in the presence of the dual gains can be given by \( \alpha \exp(\alpha_{Br} L) \). Let us call \( \alpha \exp(\alpha_{Br} L) \) as the modified gain. \( k \) is expressed by Taylor expansion around the cavity resonance frequency, \( \omega_0 \) as

\[
k = k_0 + k_1(\omega - \omega_0) + k_2(\omega - \omega_0)^2 + k_3(\omega - \omega_0)^3 \quad \text{where} \quad k_0 = n_f \omega_0 / c ,
\]

\[
k_1 = n_f / c + d\beta / d\omega \bigg|_{\omega_0}, \quad k_3 = (1/6) d^3\beta / d\omega^3 \bigg|_{\omega_0} .
\]

Assuming \( \omega_0 = v_B \), the second order term is \( k_2 = 0 \) due to the antisymmetrical dispersion profile. Finally, we obtain the transmitted power spectrum of the fiber-based WLC by inserting \( kL \) of Taylor expansion and
\[ \alpha \exp(\alpha_{\text{B}}, L) \] into \( \theta \) and \( \alpha \) in Eq.(7.3a), respectively. In the experiment, we control the cavity length with a lock-in detection circuit so that the pumps resonate with the ring cavity. Eq.(7.3b) at resonance (\( \theta = 2m\pi, m \) is integer) describes the relationship between the input pump intensity at port 1 and the resonant pump intensity inside the cavity. Suppose that the resonant pumps are non-depleted, the Brillouin pumps in the cavity is

\[ I_j \left( 1 - |t|^2 \right)^2 / (1 - \alpha |t|^2)^2 \]

where \( I_j \) (W) is the individual pump power at port 1, \( j = 1, 2 \) denotes 1\(^{\text{st}}\) and 2\(^{\text{nd}}\) pump. This yields \( I_p \) using the effective area \( A_{\text{eff}} \) (m\(^2\)) of a fiber. For the case that a single pump produce Brillouin absorption to create negative dispersion, all the expressions and the arguments given above apply provided we change

\[ \alpha_{\text{B}} \rightarrow -1/2 g_0 I_p / \left( 1 + 4(v - v_B)^2 / \Gamma_B^2 \right), \quad \beta \rightarrow - \left( g_0 I_p (v - v_B) / \Gamma_B \right) / \left( 1 + 4(v - v_B)^2 / \Gamma_B^2 \right). \]

Let us briefly estimate the WLC linewidth in comparison to an ordinary cavity. The wave number for the probe varies in the dispersive fiber resonator. We define the dephasing parameter \( \varphi \) resulting from the change of the wave number \( \Delta k \) as \( \varphi = \Delta k LN \) where \( N \) is the number of round trip. For the ring resonator, \( N = \gamma_0 / (n_g L / c) \). \( \gamma_0 \) is a cavity life time in the absence of Brillouin pump. \( \Delta k = n_g \Delta \omega / c \) in the dispersive medium can be easily derived from \( \frac{d\Delta k}{d\omega} = n_g / c \) (\( n_g \) is group index). We ignore additional dephasing resulting from the higher order dispersion, i.e. \( \frac{d^n n}{d\omega^n}, n > 1 \). Inserting \( \Delta k \) and \( N \) into \( \varphi \) and reminding that \( \varphi = 2\pi \) when \( \Delta \omega \) corresponds to the linewidth of the dispersive resonator, the relation between the WLC linewidth of \( \Delta \omega \) and \( \Delta \omega = 2\pi / \gamma_0 \) for normal ring resonator is \( \Delta \omega_0 n_g / n_g = \Delta \omega \).
C. WLC demonstration under Brillouin single absorption (gain) condition

In the WLC experiment, first, we employ Brillouin absorption spectrum.

Fig. 7.2 Schematics of experimental setup to observe WLC effect: LIA, lock-in amplifier; EOM, electro-optic modulator; FPC, fiber polarization coupler; PMC, polarization maintaining coupler; VC, variable coupler. The optical chopper combined with LIA will be used to capture the data presented in Fig. 7.7.

Fig. 7.2 displays the experimental set-up for WLC demonstration. We use the same fiber ring cavity made of Corning SMF-28e with a free spectral range (FSR) of 17.32MHz as used in the previous chapter. The cavity length was adjusted so that one of the cavity modes corresponds to Brillouin frequency ($\nu_B$). A fiber loop mirror [69] is used to stabilize a fast fluctuation of the laser diode (LD) [71]. 50:50 polarization maintaining coupler (PMC) divides the 90% LD output. One is amplified by Er-doped fiber amplifier (EDFA) to provide a Brillouin single pump. A probe is modulated by EOM at Brillouin frequency which was previously measured as 10.867GHz. The fundamental is suppressed by adjusting D.C. on EOM. The two sidebands are delivered to a Fabry-Perot filter.
(FP) spectrum filter. It is actively controlled by an A.C. servo and Lock-in-amplifier (LIA) to filter one of the sidebands. The upper sideband ($\nu_L + \nu_B$) is transmitted to interact Brillouin absorption for WLC demonstration. After FP filter is locked to $\nu_L + \nu_B$, the modulation frequency of the probe is slowly swept around $\nu_B$ as amount of $\Delta \nu = 3$MHz to attain a resonance peak. We use the lower sideband ($\nu_L - \nu_B$) to observe a narrowing peak due to normal dispersion in a single gain and compare to the broadened cavity response with the upper sideband probe ($\nu_L + \nu_B$).

Fig.7.3(a) and (b) displays experimental results and numerical simulations of the cavity resonance for different pump power. To obtain the probe transmission (black) without SBS inside the cavity, we modulate the probe at 7.0145GHz which is sufficiently separated from Brillouin shift frequency ($\nu_B = 10.867$GHz). As such, it propagates the cavity without interacting the Brillouin pump while the cavity is locked to the pump resonance. The linewidth of the ordinary cavity is measured as 0.24MHz. We change the pump power below Brillouin threshold to avoid mixing the probe with Brillouin lasing noise. As increasing pump power, we observe the broadening (narrowing) of the resonance peak with the interaction with Brillouin absorption (gain). The probe transmission is lower (higher) than the reference in the presence of Brillouin absorption (gain) than the reference transmission. We attribute such transmission variation to the induced (reduced) cavity internal loss by Brillouin absorption (gain). Fig.7.3(b) displays the simulations for the cavity response associated with the experimental results. In chapter VI, we defined G factor as $-1/2 g_0 I_p$ for absorption
(1/2g_2I_p for gain). Note that in single absorption (gain) configuration, G implies absorption (gain) amplitude at Brillouin resonance (v=v_B) and is proportional to the input pump intensity.

Fig. 7.3 (a) Experimental data of the cavity response for Brillouin absorption and gain with varying pump power (b) Simulation results indicating close agreement with the experimental results. The colors of the peaks corresponds to the same color of the peaks as indicated in Fig.7.3(a). G used in simulation and the calculated n_g for the give G are that G=0.0002, n_g =1.4538 for yellow, G=0.0003, n_g =1.4557 for cyan, G=0.0005, n_g =1.4595 for magenta, G=−0.0005, n_g =1.4405 for blue, G=−0.0015, n_g =1.4214 for green, G=−0.0018, n_g =1.4156 for red. The negative(positive) G represents absorption(gain). (c) Black is a reference resonance peak without SBS. Red represents the cavity response at WLC condition (n_g=0, G=0.0759). Green is simulation result where we assume that the cavity fiber is non-dispersive (i.e. n_g=n_r, n_r =1.45 for SMF-28e) but take into account the loss induced by Brillouin absorption in the calculation.
In simulation, we choose $\alpha = \sqrt{0.97}, t = \sqrt{0.945}$ and $G$'s to obtain the graphs which are consistent with the experimental data. The simulations confirm our interpretation of the transmission variation in the experiment. Following the transmission as a function of $\alpha$ presented in ref.[54], the transmission characteristics of the cavity without the pump is over-coupled ($\alpha > t$) so that the internally circulating field coupled into the output of the resonator $\sqrt{K_a}$ is excessive compared to the transmitted field $\sqrt{T_a}$. After applying Brillouin pump, the probe of $\nu_L + \nu_B$ undergoes $\alpha \exp(\alpha_B L)$ such that $\sqrt{K_a}$ is reduced to be equal to $\sqrt{T_a}$ at critical coupling point ($\alpha \exp(\alpha_B L) = t$). $\sqrt{T_a}$ completely interferes with $\sqrt{K_a}$ at 2mW pump ($|G| = 0.0005$) and thus the probe transmission vanishes at resonance. As the pump power becomes higher (4mW, $|G| = 0.0015$, and 6mW, $|G| = 0.0018$), $\sqrt{K_a}$ decreases again due to the augmented Brillouin absorption loss i.e. $\sqrt{T_a} > \sqrt{K_a}$. Note, $|b_1|^2$ corresponds to resultant power after the interference. As the more absorption is induced, it is observed to increase in under-coupled region ($\alpha \exp(\alpha_B L) < t$). When gain is introduced to the resonator, $\sqrt{K_a}$ is increased larger than in the absence of the pumps. As such, $|b_1|^2$ is raised after the inclusion of the Brillouin gain.

The gain amplitude at $\nu = \nu_B$ corresponds to $|G|$ for the absorption of the equal pump power. However, we also notice that in the numerical simulation different $|G|$'s are chosen to provide consistency with the experimental results for the given power values.
For example, $|G|=0.0005$ and $0.0015$ are for the gain and for the absorption at the pump of 2mW, respectively. The power of the resonating pump with the cavity depends on the stability of the cavity length lock to fulfill the resonance. Since the achievable gain (absorption) amplitude at $\nu=\nu_B$, $|G|$ in determined by such resonating power, the gain amplitude ($|G|=0.0015$) is high compared to the absorption ($|G|=0.0005$) due to the more stabilized lock. Fig.7.3 indicates that due to small group index change, the resonance peak broadening(narrowing) with the allowed pump power below Brillouin lasing threshold is small. Furthermore, the absorption causes the linewidth to broaden. Therefore, it is hard to conclude if the broadening results from negative dispersion or the induced absorption. The lasing occurs at the cavity mode close to $\nu_L-\nu_B$ within Brillouin gain profile. To observe the probe at $\nu_L+\nu_B$ embedded in the lasing background signal, we can possibly use the optical filter elements such as fiber bragg grating (FBG) and add it to port 3 of circulator 1 in Fig.7.2. As such, we can filter the lasing signal from the cavity resonance associated with Brillouin absorption probe ($\nu_L+\nu_B$). We can increase the pump power without restraint to attain $n_g=0$ for ideal WLC effect. Before proceeding to the experimental demonstration, we simulate the WLC response displayed in Fig.7.3(c). In the simulation, the WLC exhibits the finite linewidth due to the third order dephasing term $k_3$. Since $\sqrt{K_a_2}$ is reduced by the extra Brillouin absorption, $\sqrt{T_a_1}(T=0.945)$ contributes to the transmitted power $|b|^2$ associated with the resonance peak without interfering with $\sqrt{K_a_2}$. Therefore, $|b|^2$ approaches to complete
transmission \( |b_t|^2 = \sqrt{T}a_t = a_t \) at WLC condition. For comparison, we simulate \( |b_t|^2 \) (green) for the normal cavity \((n_g=n_f)\) with the loss corresponding to Brillouin absorption which is induced in the presence of WLC effect. Note that WLC response is broader compared to the normal cavity (black), however, the reduction in cavity buildup factor due to additional Brillouin loss makes WLC effect undistinguishable from the non-dispersive cavity with equal loss (green).

**D. WLC demonstration under Brillouin gain doublet condition**

Next, we used Brillouin dual gains to demonstrate WLC. The cavity length is controlled to resonate with bi-frequency pumps as presented in chapter VI.

![Fig.7.4 Schematics of experimental setup for WLC with dual gains](image)

Fig.7.4 displays the experimental setup. Light is modulated by EOM1 to provide the bi-frequency pumps with the fundamental eliminated. The modulation frequency is chosen to be 17.32MHz. EOM2 modulates the probe at \( v_B \) with the fundamental suppressed. The EOM2 output is inserted into the fiber ring cavity through an optical circulator.
without blocking the upper sideband ($\nu_L + \nu_B$). Due to strong Brillouin absorption at $\nu_L + \nu_B$ inside the cavity, we expect that the anti-stokes component ($\nu_L + \nu_B$) of the probe is depleted and negligibly affects on Brillouin interaction between the pumps and the stokes probe ($\nu_L - \nu_B$).

Fig.7.5 (a) Experimental data with varying the pump power, Gain separation is 34.64MHz equal to 2$\times$FSR, (b) Numerical simulations done with the parameters that $G=0.001(n_g=1.4499)$ for 2mW, $G=0.013(n_g=1.4338)$ for 8mW, $G=0.0014(n_g=1.4326)$ for 8.5mW, $G=0.015(n_g=1.4314)$ for 9.2mW and $G=0.0155(n_g=1.4499)$ for 9.6mW. The group indexes are calculated from $G$ values. The colors of the peaks represent the power values indicated in Fig.7.5(a). (c) Cavity modes(black bars), pump spectrums(red arrows) and gain lines(red peaks), a, b and c correspond to Brillouin shift frequency (10.867GHz). FSR is 17.32MHz.

Fig.7.5(a) and (b) illustrate the experimental results and numerical simulations. In the experiment, we monitored the pump resonance peak from port 3 of the variable
coupler (V.C.) and adjusted the coupling ratio of V.C. to be the critical coupling point \[ t = \alpha \] i.e. the transmission power is zero at resonance. To evaluate the cavity linewidth under no-gain condition, we measured the full width half maximum (FWHM) of the pump resonance. It was 0.36MHz. The critically coupled cavity becomes black out at the resonance due to the perfect destructive interference between \( \sqrt{7a_1} \) and \( \sqrt{K a_2} \). In Fig.7.5(a), the probe transmission peak confirms that the cavity in the presence of the 2mW pumps is near by the critical point. Its linewidth is close to 0.36MHz. As increasing pump power, however, the probe experiences more gains such that the complete destruction of \( |b_1|^2 \) doesn’t occur due to the excessive \( \sqrt{K a_2} \). The resonance peaks at resonance approaches to the D.C. level of \( \sqrt{7a_1} \). We increased the pump power until the lasing occurred. The simulations in Fig.7.5(b) confirm such a gain effect. To obtain agreement with the experimental data, we choose \( t = \sqrt{0.93}, \alpha = \sqrt{0.93} \) and \( G \) values. For the pump of 9.2 and 9.6mW, the transmission peaks exhibit the horizontal area concomitant with distortion around resonance. We attribute this effect to mode competition between the stokes probe and the residual anti-stokes wave. In the simulation, we raised \( d^3 \beta / d \omega^3 \) independently from \( d^3 \gamma / d \omega^3 \) to include the dephsing effect resulting from such a competition. Also, the experimental results show the broadened peaks compared to the simulations due to the depletion of the anti-stokes probe. Fig.7.5(c) illustrates the configurations of the cavity mode, dual gains and pump spectrums associated with the experiment.
Fig. 7.6 (a) Experimental data comparing the ring cavity response for gain doublets to the cases for a single gain with varying pump power. Black is the response without Brillouin interaction. The anti-stokes component ($\nu_{\text{L}}+\nu_{\text{B}}$) of the probe is blocked by the FP spectrum filter. (b) Numerical simulations showing the agreement with the experimental data. The colors correspond to those of the graphs in Fig. 7.6(a). Blue dot represents the case of 4mW pump. The parameters are $G=1.9\times10^{-4}$, $n_g=1.4518$ (1.5mW), $G=3.8\times10^{-4}$, $n_g=1.4536$ (2mW) for single gain and $G=0.5\times10^{-3}$, $n_g=1.4481$ (4mW) for dual gains, (c) Experimental data showing the cavity response with different pump power from (a). (d) Numerical simulations with parameters that $G=3.8\times10^{-4}$, $n_g=1.4536$ (2mW), $G=1.0\times10^{-3}$, $n_g=1.4595$ (4mW) for a single gain and $G=0.8\times10^{-2}$, $n_g=1.447$ (8mW), $G=1.8\times10^{-2}$, $n_g=1.4433$ (12mW) (e) Configurations of cavity modes (black bars), pump spectrums (red arrows) and gains (red peaks). a,b,c are Brillouin shift frequency (10.867GHz), For the comparison of linewidth, we moved the data to align the rising portions of the peaks in (a),(c)
Again, we use the setup presented in Fig.7.2 to block the upper sidebands $(\nu_L + \nu_B)$. In fig.7.6(a)–(d), we compare the transmission peaks under the gain doublet condition to the cases of the single gain. Also, the reference transmission without Brillouin interaction is displayed. The gain separation is 69.28MHz (=4FSR) indicated in Fig.7.6(e). The reference cavity linewidth was determined first by scanning the probe around 7.0145GHz. In the gain doublet system, Brillouin gain compensates for the internal cavity loss so as to narrow resonance peak. Also the linewidth is broadened by WLC effect under dual gains. Since there is no experimental diagnostics to separate them, we observe the combination of these two effects. Fig.7.6(a) indicates that for the case of 4mW bi-frequency pumps, WLC effect is balanced by the linewidth narrowing due to the gain. As a result, its linewidth is almost equal to the reference peak. As increasing the pump power, WLC effect is small compared to the linewidth narrowing induced by the gains. With the dual pumps of 8 as well as 12 mW, we observed the transmission peaks narrower than the reference. Note, however that in the cases of the single pump, there is additional linewidth narrowing caused by normal dispersion. Therefore, it is noticeable that the probe transmissions with the 4 and 8mW dual pump are seen to be more broadened than the cases of the 1.5 and 2mW single pump, respectively. In the experiment, the resonances (minimum transmission) were observed at the same frequency as the reference.

We revised the cavity length so that FSR obeys $\text{FSR}(N+1/2) = \nu_B$. We can convince that the gain peaks don’t support any cavity mode. By this choice, we can avoid allowing the Brillouin-shifted wave (stokes wave) to resonate simultaneously. As
such, Brillouin lasing threshold grows higher than the previous cases. The new FSR was measured 17.4016MHz. The gain separation was chosen to be $3 \times \text{FSR}(52.2\text{MHz})$. We increased the pump power to the level where Brillouin lasing occurs at modes closer to the gain peaks. However, we stopped increasing before the probe mode starts lasing.

Fig.7.7 (a) Probe transmission profiles showing a small WLC effect. It is separated by the optical chopper from the lasing signal at the modes close to the gain peaks. When the pump power reaches 18mW, the probe mode also starts lasing. (b) FSR(=17.4016MHz) is adjusted to be larger than the gain bandwidth ($\Gamma_B$). The gain peaks are shifted from the pump spectrums as amount of $\nu_B$ such that according to $\text{FSR}(N + 1/2) = \nu_B$, the peaks is located at the center of the two neighboring cavity modes. a, b, c are $\nu_B$ (10.867GHz)
Again, we used the setup displayed in Fig.7.2. The optical chopper is added to separate the probe from the lasing signal of the gain peaks. To evaluate the cavity linewidth without SBS process, we scanned the probe around 8.7008GHz which is sufficiently separated from $\nu_B (10.867\text{GHz})$.

Fig.7.7 illustrates the experimental data and the configurations of cavity modes, pump spectrum, dual gain peaks. The cavity response at 14mW becomes broad due to the small amount of WLC effect. From the simulations and experimental results so far, we found that the WLC effect under given experimental parameters is small due to the small group index change ($\Delta n_g = -0.01 \sim -0.03$, i.e. $n_g \approx n_f$).

**E. Investigation of experimental constraints in realizing WLC based on SBS in conventional optical fibers**

To create noticeable WLC effect, we investigate necessary parameters to induce $n_g=0.1, 0.725$ and 1.1. By the relationship \(\Delta \omega_c n_g / n_g = \Delta \omega \), we can anticipate that the WLC linewidth will broaden approximately a factor of 14.5 for $n_g=0.1$, 2 for $n_g=0.725$ and 1.32 for $n_g=1.1$ in comparison to the normal cavity. For the investigation, we choose FSR to yield \(\text{FSR}(N+1/2) = \nu_B\). The gain separation is chosen to correspond to FSR so that single longitudinal cavity mode is located at the center of the two gains. To observe WLC effect, we scan the probe frequency around such a mode. We inspect allowable FSR’s of WLC for the given $n_g$ to keep below Brillouin threshold at the probe scanning mode. First, let us consider gain that the probe experiences during one round trip in the
ring cavity so-called gain per pass. The cavity resonance frequency ($\nu_0$) in the middle of two gains is equal to $\nu_B$, due to the cavity design rule $\text{FSR}(N + 1/2) = \nu_B$ as well as the proper gain separation. In the double gains, Eq(7.5) indicates the probe at the resonance ($\nu_0 = \nu_B$) experiences the exponential Brillouin gain of $\alpha_{\text{Br}(\nu=\nu_B)} = 2G_i / (1 + 4 \Delta^2 / \Gamma_B^2)$. The probe is scanned around $\nu_B$ within the spectral region where negative dispersion occurs. To observe the linewidth broadening due to WLC effect, the scan range is required to be larger than FWHM of the reference cavity. We consider the cases that the cavity FSR is almost equal to the width measured at the bottom of the gain profile i.e. $\sim 2\Gamma_B$. Due to high Q of fiber cavities, FWHM is less than $\sim 2\Gamma_B$ a factor of 10~100. The minimum scan range is substantially less than $2\Gamma_B$ and thus most of such a scan region exhibits the constant Brillouin gain close to $\alpha_{\text{Br}(\nu=\nu_B)}$. Therefore, we can assume that the probe feels the constant gain $\alpha_{\text{Br}(\nu=\nu_B)}$ while scanning around $\nu_0$. Before proceeding to find gain per pass for the probe, let us discuss the effective cavity length ($L_{\text{eff}}$). In numerical simulation, we will introduce the significant internal cavity loss such that $L_{\text{eff}}$ should be taken into account. Note, $\alpha$ is associated with the loss characteristics in our case. As such, $L_{\text{eff}}$ in ref.[56, 64, 68, 72] is redefined as $L_{\text{eff}} = -L(1-\alpha)/(\ln \alpha)$ in terms of the cavity length. It leads to the exponential gain per pass for the probe $\alpha_{\text{Br}(\nu=\nu_B)}L_{\text{eff}}$. Using $L_{\text{eff}}$, the modified gain is changed to $\alpha \exp(\alpha_{\text{Br}}L_{\text{eff}})$. According to $a_2 = \alpha \exp(\alpha_{\text{Br}}L_{\text{eff}}) e^{i\theta} b_2$, $\alpha \exp(\alpha_{\text{Br}}L_{\text{eff}})$ represents the round trip gain in the presence of Brillouin pumps. Sufficient pump power can allow values of $\alpha \exp(\alpha_{\text{Br}}L_{\text{eff}}) \geq 1$. At
$\alpha \exp(\alpha_{\text{Br}} L_{\text{eff}}) = 1$, the induced Brillouin gain completely compensates for the internal cavity loss and thus the cavity reaches at the lasing threshold. As we are interested in observing WLC effect under Brillouin threshold, the cavity gain factor is $\alpha \exp(\alpha_{\text{Br}} L_{\text{eff}}) < 1$. It leads to the condition below Brillouin threshold at the probe scanning mode i.e. $\alpha_{\text{Br}(v=v_0)} L_{\text{eff}} < -\ln(\alpha)$. Define $\frac{d\beta}{d\omega} = 2G(d\gamma/d\omega)$ where $\gamma$ is the sum of two Lorentzians remaining after factoring $G = g_c I_p/2$ from $\beta$. Using $k_i = n_i/c + \frac{d\beta}{d\omega}|_{k_0} = n_g/c$ together with the new definition, we can express $G = (n_g - n_i)/(2c\gamma')$ where $\gamma' = \frac{d\gamma}{d\omega}|_{k_0}$ is negative. It is instructive to graphically compare $-\ln(\alpha)$ to $\alpha_{\text{Br}(v=v_0)} L_{\text{eff}}$ for $n_g=0.1, 0.725$, and $1.1$.

Fig. 7.8 For $n_g$’s, the curves represent $\alpha_{\text{Br}(v=v_0)} L_{\text{eff}}$ associated with different $\alpha$ values. The straight lines corresponds to $-\ln(\alpha)$. The operational FSR is FSR$>176$MHz for $n_g=0.1$, $\alpha=0.855$, FSR$>95$MHz for $n_g=0.725$, $\alpha=0.855$, FSR$>160$MHz for $n_g=0.725$, $\alpha=0.911$, FSR$>48$MHz for $n_g=1.1$, $\alpha=0.855$, FSR$>80$MHz for $n_g=1.1$, $\alpha=0.911$, and FSR$>180$MHz for $n_g=1.1$, $\alpha=0.955$. $\alpha=0.955$ and $\alpha=0.911$ for $n_g=0.1$, and $\alpha=0.955$ for $n_g=0.725$, there is no available operational FSR.
Fig. 7.8 illustrates the lasing threshold $-\ln(\alpha)$ and the exponential gain per pass $\alpha_{\text{Br}(v=\nu_0)}L_{\text{eff}}$ as function of FSR. For illustration, we consider different internal cavity loss so that $\alpha=0.955, 0.911, 0.855$. We choose $t=0.955 \, n_f=1.45$. Based on the previous measurement presented in chapter VI, we use $\Gamma_B=10\text{MHz}$. Remind that the gain separation (2$\Delta$) is equal to FSR. Due to the low threshold in high Q resonator ($\alpha=1$), $-\ln(\alpha)$ reduces as we withdraw the cavity loss i.e. $\alpha\to1$. To understand the behavior of $\alpha_{\text{Br}(v=\nu_0)}L_{\text{eff}}$, note that $G$ should be essentially increased to induce more group index change ($\Delta n_g$). The gain $\alpha_{\text{Br}(v=\nu_0)}$ seen by the probe increases proportionally to $G$ and thus small $n_g$ (large $\Delta n_g$) requires higher $\alpha_{\text{Br}(v=\nu_0)}L_{\text{eff}}$. Since the inclusion of the cavity loss reduces $L_{\text{eff}}$, $\alpha_{\text{Br}(v=\nu_0)}L_{\text{eff}}$ for the same $n_g$ increases as $\alpha\to1$. The cavity mode for the probe should stay below lasing threshold. As such, for WLC demonstration, we can employ the cavity FSR relevant to the condition $\alpha_{\text{Br}(v=\nu_0)}L<-\ln(\alpha)$, the so-called operational FSR. For example, the operational FSR is FSR$>176\text{MHz}$ for $n_g=0.1$, $\alpha=0.855$. Since the $n_g$ reduction entails additional $\alpha_{\text{Br}(v=\nu_0)}L_{\text{eff}}$, note that for $n_g=0.1$ with $\alpha=0.955, 0.911$ $\alpha_{\text{Br}(v=\nu_0)}L_{\text{eff}}$ is higher than the lasing threshold in all FSR’s of interest.

Fig. 7.9 illustrates $G$ for $n_g=0.1, 0.725, \text{and} 1.1$. To understand the behavior of $G$, we discuss about $n_g$ together with the negative dispersion slope as well as gain separation. Negative dispersion slope becomes large to reduce the group index. Fig. 7.9 suggests that either larger $G$ for constant gain separation or narrow gain separation for constant $G$ leads to the small $n_g$ associated with the steeper negative dispersion.
Fig. 7.9 Exponential Brillouin gain peak amplitudes for different $n_g$ values

Fig. 7.10 Pump power being required to obtain $n_g$. Note the power within the operational FSR mentioned in Fig. 7.8.
Fig. 7.10 displays the input pump power $I_j$ for the cavities with different internal loss. Using Brillouin coefficient $g_0 = 0.7682 \times 10^{-11}$ m/W [68], we first find the pump intensity per unit area $I_p$ inside cavity. Consider briefly the polarization property of the fiber. Taking the polarization into account, $G$ can be rewritten as $1/2g_0I_p\kappa$ where $\kappa = 1$ if the polarization is maintained or otherwise $0$. While the probe propagates inside the cavity, its polarization direction is properly adjusted to correspond to the pumps. Hence, the previous definition $G = 1/2g_0I_p$ (i.e. $\kappa = 1$) is suitable for our interesting cases. To demonstrate WLC, we control the cavity length to force Brillouin pumps to be resonant with the cavity. According to the pump build-up factor $\eta = (1 - |t|^2)/(1 - \alpha|t|^2)$, $I_j$ is related to $I_p$ by $I_j = I_pA_{\text{eff}}/\eta$. We choose $A_{\text{eff}} = 5 \times 10^{-11}$ m$^2$ for a silica single mode fiber [68]. Consider $I_j$ within the operational FSR. To achieve the given $n_g$ in the operational FSR, $I_j$ is essentially needed higher than the maximum power achievable from commercial EDFAs as well as the power limitation imposed on the fiber optic elements such as circulators and couplers. Even though we can experimentally increase $I_j$ such a high level, $G$ stops increasing due to Brillouin gain saturation. In the previous Brillouin characteristics measurement, the fiber used for our cavity showed it was saturated at $G=0.04$. Inserting $I_j$ into $G$, however, note $G = (1/2g_0)(I_j\eta/A_{\text{eff}})$. As such, the problem with the saturation is overcome by using fibers of high Brillouin coefficient and small effective area. Tellurite fiber ($g_0 = 1.6986 \times 10^{-10}$ m/W, $A_{\text{eff}} = 0.6967 \times 10^{-11}$ m$^2$) or $\text{As}_2\text{Se}_3$ chalcogenide fiber ($g_0 = 6.08 \times 10^{-9}$ m/W, $A_{\text{eff}} = 3.9 \times 10^{-11}$ m$^2$) has 20~800 times larger $g_0$ but also smaller $A_{\text{eff}}$ than conventional fibers. However, their transmission loss is major
drawback when the cavity is controlled to achieve the pump resonance. We built a 6m ring resonator which was composed of 4m tellurite fiber and 2m SMF28e. While scanning the cavity, the pumps were coupled into the cavity to obtain the resonance peak. Due to the loss, the coupled light $\sqrt{K_a}a_2$ was completely absorbed before traveling one round trip such that we observed the only transmitted field $\sqrt{A}a_1$. Tellurite fiber was reported to exhibit large loss at low input power [56].

**F. Alternative experimental set-up for future works**

Fig.7.11. Schematics of proposed setup for WLC demonstration

Alternatively, we propose a WLC illustrated in Fig.7.11. A fiber ring resonator is built by a coupler spliced with the high Brillouin coefficient fibers. Low loss optical circulator is used to insert Brillouin pumps into the ring [73, 74]. The pumps propagate without resonating with the cavity and thus the transmission loss problem is avoided due to no requirement of the cavity length control. However, we need more input power than
the resonantly enhanced pump cavity. The high power EDFA compensates it. Furthermore, high $g_0$ fibers allow us to drop the input power level which is required to achieve sufficient $\Delta n_g$ for large WLC effect.
VIII. CONCLUSION

Techniques to control dispersion in a medium have attracted much attention due to potential applications to devices such as ring laser gyroscopes, interferometric gravitational wave detectors, data buffers, phased array radars and quantum information processors. Of particular interest is an optical resonator containing a medium with an anomalous dispersion corresponding to fast-light, which behaves as a White Light Cavity (WLC). A WLC can be tailored to improve the sensitivity of sensing devices as well as to realize an optical data buffering system that overcomes the delay-bandwidth product of a conventional cavity.

This thesis describes techniques to tailor the dispersion for fast-light in intracavity media. We present first a demonstration of fast-light a photorefractive crystal. When placed inside a cavity, such a medium could be used to enhance the bandwidth of a gravitational wave detector. We then describe how a superluminal laser can be realized by adding anomalously dispersive medium inside a ring laser. We identify theoretical conditions under which the sensitivity of the resonance frequency to a change in the cavity length is enhanced by as much as seven orders of magnitude. This paves the way for realizing a fast-light enhanced ring laser gyroscope, for example. This is followed by the development of a novel data buffering system which employs two WLC systems in series. In this system, a data pulse can be delayed for essentially an arbitrary amount of time, without significant distortion. The delay time is independent of the data bandwidth, and is limited only by the attenuation experienced by the data pulse as it bounces between two high-reflectivity mirrors. Such a device would represent a significant
breakthrough in overcoming the delay-time bandwidth product limitation inherent in conventional data buffers.

We then describe our experimental effort to create a fiber-based WLC by using stimulated Brillouin scattering (SBS). Experimental results, in agreement with our theoretical model also presented here, show that the WLC effect is small under the conditions supported by current fiber optic technology. We conclude that future efforts to induce a large WLC effect would require fibers with high Brillouin coefficient and low transmission loss, as well as optical elements with very low insertion loss and high power damage thresholds.
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www.furukawa.co.jp/review/fr022/fr22_09.pdf

APPENDIX

A. Analysis of cavity output

In chapter IV, we used Fourier analysis to find the data pulse output of Fabry-Perot (FP) cavity. In this appendix, Laplace and Fourier analysis are presented to calculate the FP output. We choose the amplitude of an input data pulse in time domain as

\[ f(t) = 1 - e^{-\alpha t} - e^{\alpha(t-b)} \text{ (for } 0 < t < b) \]
\[ f(t) = -e^{-\alpha t} \text{ (for } t > b) \]  \hspace{1cm} (A.1)

\( \alpha \) and \( b \) are positive and real numbers. Laplace transform (LT) of the pulse amplitude function is written as

\[ L(f(t)) = F(s) = \int_0^\infty f(t)e^{-st} dt \]  \hspace{1cm} (A.2)

More explicitly, it is

\[ F(s) = \int_0^\infty (1 - e^{-\alpha t})e^{-st} dt - \int_0^{b}\int_0^{\alpha(t-b)}e^{-st} dt - \int_0^\infty e^{-\alpha t} dt - \int_0^b e^{-st} dt \]

Finally, we obtain

\[ F(s) = \frac{\alpha}{s(s + \alpha)} + \frac{1}{s - \alpha} (e^{-sb} - e^{-\alpha b}) - \frac{1}{s} e^{-sb} \]  \hspace{1cm} (A.3)

Next, we recall the transfer function of FP

\[ H(\omega) = \frac{Te^{-\frac{n\omega L}{c}}}{1 - Re^{-\frac{n\omega L}{c}}} \]  \hspace{1cm} (A.4)

where \( R \) and \( T \) is transmission and reflection coefficient for intensity, respectively, \( n \) is refractive index, \( L \) is cavity length. According to A.4, \( H \) in s-domain is given by
The inverse LT of the product of $F(s)$ and $H(s)$ becomes

$$L^{-1}(H(s)F(s)) = \frac{1}{2\pi j} \int_{γ-j\infty}^{γ+j\infty} \left[ \frac{\alpha \frac{Te^{−\frac{\beta s}{c}}}{s(s+\alpha)} + \frac{1}{s-\alpha}(e^{sb}−e^{−sb})\frac{Te^{−\frac{\beta s}{c}}}{1-\Re e^{−\frac{\beta s}{c}}} - \frac{1}{s}(e^{sb}−e^{−sb})\frac{Te^{−\frac{\beta s}{c}}}{1-\Re e^{−\frac{\beta s}{c}}} \right] e^{st} ds$$

Using partial fraction, Eq. A.6 becomes,

$$L^{-1}(H(s)F(s)) = \frac{1}{2\pi j} \int_{γ-j\infty}^{γ+j\infty} \left[ \frac{T\exp\left(\frac{t-nL}{c}\right)}{s(1-\Re e^{−\frac{2nL}{c}})} - \frac{1}{s-\alpha}(e^{−\frac{t-b}{c}}) - \frac{1}{s}(e^{−\frac{t-b}{c}}) \right] e^{st} ds$$

Using inverse LT as below

$$L^{-1}\left(\frac{e^{−\tau}}{s}\right) = \frac{1}{2\pi j} \int_{γ-j\infty}^{γ+j\infty} e^{s(t-\tau)} ds = u(t-\tau)$$

$$L^{-1}\left(\frac{e^{−\tau}}{s+\alpha}\right) = \frac{1}{2\pi j} \int_{γ-j\infty}^{γ+j\infty} e^{s(t-\tau)} ds = u(t-\tau)e^{−\alpha(t-\tau)}$$

where $u(t)$ is a step function i.e. $u(t)=0$ for $t<0$, $u(t)=1$ for $t>0$
Eq(A.8) gives the FP output pulse in time domain

\[
L^{-1}(H(s)F(s)) = f_{\text{out}}(t)
\]

\[
= \sum_{k=-\infty}^{\infty} \text{TR}^{k-1}u\left( t - \frac{n(2k-1)L}{c} \right) - \sum_{k=-\infty}^{\infty} \text{TR}^{k-1}u\left( t - \frac{n(2k-1)L}{c} \right) \exp\left[ -\alpha \left( t - \frac{n(2k-1)L}{c} \right) \right]
\]

\[
+ \sum_{k=-\infty}^{\infty} \text{TR}^{k-1}u\left( t - \frac{n(2k-1)L}{c} - b \right) \exp\left[ -\alpha \left( t - \frac{n(2k-1)L}{c} - b \right) \right] \tag{A.11}
\]

Next, let us consider Fourier analysis to obtain the FP output. Assuming that the carrier frequency of the envelop function \(f(t)\) is \(\omega_0\), Fourier transform (FT) of the input pulse is given by

\[
F(f(t)) = F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)e^{j\omega t}e^{-j\omega dt}
\]

\[
= \frac{1}{\sqrt{2\pi}} \int_{0}^{b} \left( 1 - e^{-\omega t} - e^{j(\omega-b) t} \right) e^{-j\alpha t}e^{-j\alpha dt} dt + \frac{1}{\sqrt{2\pi}} \int_{b}^{\infty} (-e^{-\omega t})e^{j\omega t}e^{-j\omega dt} \tag{A.12}
\]

\[
= \frac{1}{\sqrt{2\pi}} \left\{ -e^{-j(\omega-\omega_0)b} - \frac{e^{-j(\omega-\omega_0)b}}{\alpha - j(\omega - \omega_0)} + \frac{1}{\alpha - j(\omega - \omega_0)} - \frac{1}{\alpha + j(\omega - \omega_0)} \right\}
\]

Using Eq.(A.4) and (A.12), inverse FT of \(H(\omega)F(\omega)\) is

\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H(\omega)F(\omega)e^{j\omega t} d\omega
\]

\[
= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \left\{ \frac{e^{-j(\omega-\omega_0)b}}{\alpha - j(\omega - \omega_0)} - \frac{e^{-j(\omega-\omega_0)b}}{\alpha - j(\omega - \omega_0)} + \frac{1}{\alpha - j(\omega - \omega_0)} + \frac{1}{\alpha + j(\omega - \omega_0)} \right\} \frac{Te^{-\delta}}{1 - Re^{-\delta}e^{j\omega_0 t}} d\omega \tag{A.13}
\]

Define \(\delta = \omega - \omega_0\) and change the integration variable from \(\omega\) to \(\delta\), Eq(A.13) becomes
\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H(\omega)F(\omega)e^{j\omega t} d\omega
\]

\[
= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \left\{ -\frac{e^{-j\delta b}}{\alpha - j\delta} - \frac{e^{-j\delta b}}{\alpha + j\delta} + \frac{1}{j\delta} \right\} + \left\{ e^{-\alpha b} - \frac{1}{\alpha - j\delta} - \frac{1}{\alpha + j\delta} \right\} \cdot \frac{\text{Te}^{-j(n\omega_0 + \delta)t}}{c} e^{j(n\omega_0 + \delta)t} d\delta
\]

\[
(A.14)
\]

Again, express \(1 - \text{Re}^{-j2n(\omega_0 + \delta)/c}\) in the form of summation

\[
f_{\text{out}}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \left\{ -\frac{e^{-j\delta b}}{\alpha - j\delta} - \frac{e^{-j\delta b}}{\alpha + j\delta} + \frac{1}{j\delta} \right\} + \left\{ e^{-\alpha b} - \frac{1}{\alpha - j\delta} - \frac{1}{\alpha + j\delta} \right\} \cdot \text{Te}^{-j(n\omega_0 + \delta)t/c} e^{j(n\omega_0 + \delta)t} \sum_{N=1}^{\infty} R_{N-1} N^{1/2} \exp \left( -2j \frac{(N-1)n(\omega_0 + \delta)L}{c} \right) d\delta
\]

\[
(A.15)
\]

Before evaluating the integral value, let us review the definite integral \([1]\) in complex plan

\[
I = \oint_C f(z) e^{j\alpha z} dz \quad (A.16)
\]

where \(z\) is a complex variable and \(C\) is a closed contour. Fig. A.1 illustrates the possible integral path for different sign of \(a\). Here are the required conditions for \(f(z)\). First, if \(a>0\)(\(a<0\)), then \(f(z)\) is analytic in the upper(lower) half-plane except for number of singularities and \(\lim_{|z| \to \infty} f(z) = 0\), \(0 \leq \arg(z) \leq \pi\). The radius \(R_1\) and \(R_2\) for the contour \(C_1\) and \(C_2\), respectively are infinite. By residue theorem, Eq.(A.16) can be evaluated as

For \(a>0\), \(\int_{-\infty}^{\infty} f(x)e^{j\alpha x} dx + \lim_{R_1 \to \infty} \oint_{C_1} f(x)e^{j\alpha x} dx = 2\pi j \sum \text{residues in upper half plane} \quad (A.17)\)

For \(a<0\), \(\int_{-\infty}^{\infty} f(x)e^{j\alpha x} dx + \lim_{R_1 \to \infty} \oint_{C_2} f(x)e^{j\alpha x} dx = -2\pi j \sum \text{residues in lower half plane} \quad (A.18)\)

Right hand side (RHS) in Eq.(A.18) contains minus sign due to clockwise integration.
The conditions for \( f(z) \) allows us to use Jordan’s lemma[1]. As such, the second terms in left hand side of Eq(A. 17,18) becomes zero. It leads us to

For \( a>0 \),

\[
\int \int \int \int \int \int_{-\infty}^{\infty} f(x)e^{iax} \, dx = 2\pi j \sum \text{residues in upper half plane (A.19)}
\]

For \( a<0 \),

\[
\int \int \int \int \int_{-\infty}^{\infty} f(x)e^{iax} \, dx = -2\pi j \sum \text{residues in lower half plane (A.20)}
\]

Next, we consider the case that singularities of \( f(z) \) are placed on x-axis. Let us assume there is a singularity at \( z=0 \). We can estimate Eq.(A.16), following the contours indicated in Fig. A.2. For each case of \( a \), two different closed contour can be considered. Using the integral notation, For \( a>0 \), if excluding the singularity,

\[
\oint_{C} = \int_{-\infty}^{r} + \int_{c_{2}} + \int_{r} + \int_{c_{1}} \quad \text{. According to Jordan’s lemma,} \quad \int_{c_{1}} = 0 \quad \text{. It leads to}
\]

\[
\int_{-\infty}^{\infty} = -\int_{c_{2}} = j\pi \delta_{0} \quad \text{where} \quad \delta_{0} \quad \text{is the residue of} \quad f(z) \quad \text{at} \quad x=0 \quad \text{. Note the radius of the}
\]
The semicircular detour around the singularity is infinitesimal, i.e. \( \int_{-\infty}^{0} + \int_{0}^{\infty} = \int_{-\infty}^{\infty} \). If including the singularity, \( \oint_{C} \) is equal to \( 2\pi j \delta_0 \). It becomes \( \int_{-\infty}^{\infty} = 2\pi j \delta_0 - \int_{C} = j\pi \delta_0 \).

For \( a<0 \), \( \oint_{C} \) is equal to \( \int_{-\infty}^{\infty} + \int_{0}^{\infty} + \int_{C} = 0 \) where the singularity is outside the contour \( C \). It gives \( \int_{-\infty}^{\infty} = -j\pi \delta_0 \). Including it, \( \oint_{C} \) is equal to \( \int_{-\infty}^{\infty} + \int_{0}^{\infty} + \int_{C} = -2\pi j \delta_0 \). As a result, \( \int_{-\infty}^{\infty} = -j\pi \delta_0 \).

Hence, for the case that the singularity is placed on the x-axis, we can estimate

For \( a>0 \), \( \int_{-\infty}^{\infty} f(x)e^{j\pi x}dx = j\pi \delta_0 \) (A.21)

For \( a<0 \), \( \int_{-\infty}^{\infty} f(x)e^{j\pi x}dx = -j\pi \delta_0 \) (A.22)

Fig A.2 Possible integral paths showing that \( C_2' \) includes the singularity but \( C_2 \) excludes it for (a) \( a>0 \) (b) \( a<0 \)
With the Eqs(A.19–22), we are ready to evaluate the Fourier integral in Eq(A.15). We expand the factors in \{ \} of Eq(A.15) and evaluate term by term.

The first term \( f_{\text{out1}}(t) \) related to \( -e^{j\bar{b}/j\delta} \) is given by

\[
f_{\text{out1}}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{j}{\delta} e^{jin_{j} tr} \sum_{n=1}^{\infty} R^{N-1} \exp\left[ -j\frac{1}{c} \left( (2N-1)n\omega L \right) + j\left( t - b - \frac{2N-1}{c} nL \right) \right] d\delta \quad \text{(A.23)}
\]

There is singularity at \( \delta = 0 \). According to the range of \( t \), we can use one of the integral paths in Fig. A.2(b) and evaluate (A.23).

For \( t < (2N-1)nL/c \). Use the path in Fig.A.2(b)

\[
f_{\text{out1}}(t) = \frac{1}{2} e^{jin_{j} tr} \sum_{n=1}^{\infty} R^{N-1} \exp\left[ -j\frac{1}{c} \left( (2N-1)n\omega L \right) \right] \quad \text{(A.24)}
\]

For \( (2N-1)nL/c < t < b + (2N-1)nL/c \). Use the path in Fig.A.2(b)

\[
f_{\text{out1}}(t) = \frac{1}{2} e^{jin_{j} tr} \sum_{n=1}^{\infty} R^{N-1} \exp\left[ -j\frac{1}{c} \left( (2N-1)n\omega L \right) \right] \quad \text{(A.25)}
\]

For \( t > b + (2N-1)nL/c \). Use the path in Fig.A.2(a)

\[
f_{\text{out1}}(t) = -\frac{1}{2} e^{jin_{j} tr} \sum_{n=1}^{\infty} R^{N-1} \exp\left[ -j\frac{1}{c} \left( (2N-1)n\omega L \right) \right] \quad \text{(A.26)}
\]

The second term \( f_{\text{out2}}(t) \) related to \( -e^{-j\bar{b}/(\alpha - j\delta)} \) is given by

\[
f_{\text{out2}}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{-j}{\delta + \alpha j} e^{jin_{j} tr} \sum_{n=1}^{\infty} R^{N-1} \exp\left[ -j\frac{1}{c} \left( (2N-1)n\omega L \right) + j\left( t - b - \frac{2N-1}{c} nL \right) \right] d\delta \quad \text{(A.27)}
\]

There is singularity at \( \delta = -\alpha j \). According to the range of \( t \), we can use \( C_1 \) or \( C_2 \) in Fig. A.1 and evaluate (A.27).

For \( t < (2N-1)nL/c \). Use the contour \( C_2 \). According to Eq.(A.20)
\[ f_{\text{out}2}(t) = -e^{i\omega_j t} \sum_{N=1}^{\infty} R_{N-1} \exp \left[ -j \frac{(2N-1)\omega_0 L}{c} + \alpha \left( t - b - \frac{(2N-1)nL}{c} \right) \right] \] (A.28)

For \((2N-1)nL/c < t < b + (2N-1)nL/c\). Use \(C_2\) and Eq.(A.20)

\[ f_{\text{out}2}(t) = -e^{i\omega_j t} \sum_{N=1}^{\infty} R_{N-1} \exp \left[ -j \frac{(2N-1)\omega_0 L}{c} + \alpha \left( t - b - \frac{(2N-1)nL}{c} \right) \right] \] (A.29)

For \(t > b + (2N-1)nL/c\). Use \(C_2\) and Eq.(A.19). As such, there is no singularity inside \(C_2\)

\[ f_{\text{out}2}(t) = 0 \] (A.30)

The third term \(f_{\text{out}3}(t)\) related to \(1/j\delta\) is given by

\[ f_{\text{out}3}(t) = \frac{1}{2\pi} \int_{-\delta}^{\delta} e^{i\omega_j t} \prod_{N=1}^{\infty} R_{N-1} \left[ -j \frac{(2N-1)\omega_0 L}{c} + \alpha \left( t - b - \frac{(2N-1)nL}{c} \right) \right] d\delta \] (A.31)

There is singularity at \(\delta = 0\). We can follow the paths in Fig.A.2

For \(t < (2N-1)nL/c\). Use the path in Fig.A.2(b)

\[ f_{\text{out}3}(t) = -\frac{1}{2} e^{i\omega_j t} \prod_{N=1}^{\infty} R_{N-1} \exp \left[ -j \frac{(2N-1)\omega_0 L}{c} \right] \] (A.32)

For \(t > (2N-1)nL/c\). Use the path in Fig.A.2(a)

\[ f_{\text{out}3}(t) = \frac{1}{2} e^{i\omega_j t} \prod_{N=1}^{\infty} R_{N-1} \exp \left[ -j \frac{(2N-1)\omega_0 L}{c} \right] \] (A.33)

The fourth term \(f_{\text{out}4}(t)\) related to \(e^{-j\delta}/(\alpha - j\delta)\) is given by

\[ f_{\text{out}4}(t) = \frac{1}{2\pi} \int_{-\delta}^{\delta} \frac{je^{-\alpha b}}{\delta + \alpha j} e^{i\omega_j t} \prod_{N=1}^{\infty} R_{N-1} \left[ -j \frac{(2N-1)\omega_0 L}{c} + \alpha \left( t - b - \frac{(2N-1)nL}{c} \right) \right] d\delta \] (A.34)

There is singularity at \(\delta = -\alpha j\). We can use \(C_1\) or \(C_2\) in Fig. A.1 and evaluate (A.34)

For \(t < (2N-1)nL/c\). Use the contour \(C_2\). According to Eq.(A.20)
\begin{align*}
f_{\text{out}4}(t) &= e^{-\alpha t} e^{j\omega t} T \sum_{N=1}^{\infty} R^{-1} \exp \left[ -j \frac{(2N-1)n\omega L}{c} + \alpha \left( t - \frac{(2N-1)nL}{c} \right) \right] \tag{A.35}\end{align*}

For \( t > (2N-1)nL/c \). Use \( C_1 \). Due to the absence of the singularity in the contour

\( f_{\text{out}4}(t) = 0 \tag{A.36} \)

The fifth term \( f_{\text{out}5}(t) \) related to \(-1/(\alpha + j\delta)\) is given by

\begin{align*}
f_{\text{out}5}(t) &= \frac{1}{2\pi} \int \sum_{N=1}^{\infty} R^{-1} \exp \left[ -j \frac{(2N-1)n\omega L}{c} + j \left( t - \frac{(2N-1)nL}{c} \right) \delta \right] d\delta \tag{A.37}
\end{align*}

There is singularity at \( \delta = \alpha \). Using \( C_1 \) or \( C_2 \) in Fig. A.1 and evaluate (A.37)

For \( t < (2N-1)nL/c \). There is no singularity in \( C_2 \).

\( f_{\text{out}5}(t) = 0 \tag{A.38} \)

For \( t > (2N-1)nL/c \). Use \( C_1 \).

\begin{align*}
f_{\text{out}5}(t) &= -e^{j\omega t} T \sum_{N=1}^{\infty} R^{-1} \exp \left[ -j \frac{(2N-1)n\omega L}{c} - \alpha \left( t - \frac{(2N-1)nL}{c} \right) \right] \tag{A.39}\end{align*}

In order to verify that LT and FT provide same result, we obtain the output pulses from LT Eq.(A.11) as well as FT Eq.(A.24~26, 28~30, 32, 33, 35, 36, 38, 39). And plot the output pulses. For illustration, we choose the FP cavity parameters as free spectral range (FSR) 2.9GHz, Full width half maximum (FWHM) 1.46MHz. We choose \( \alpha = 1.46 \times 10^7 \), \( b = 7.7616 \times 10^{-7} \) so that the bandwidth of the pulse spectrum is a half of the cavity FWHM. Fig. A.3 illustrates the input pulse, the FP outputs produced by LT and FT. Note the pulse in Fig. A.3.(b) correspond to that in Fig. A.3.(c). We have shown that the LT and FT presented in Appendix A are the powerful technique to analyze arbitrary pulse propagation through FP.
Fig. A.3 (a) input pulse f(t), FP Output calculated by (b) LT (c) FT.

In next section, we present the matlab codes for simulations displayed in Fig. A.3

B. Matlab codes for Fig. A.3

1. Laplace analysis FPout.m

This code is used to produce the numerical simulation result in Fig. A.3 (b).

n=1; % refractive index for vacuum between two Fabry-perot (FP) mirrors.  
trans=sqrt(0.001); % Amplitude transmission coefficient for FP mirrors  
refl=sqrt(0.999); % Amplitude reflection coefficient for FP mirrors  
c=3e8; % Speed of light in free space  
Finesse=refl/(1-refl^2); % Finesse of FP  
w0=(2*pi*c)/(1550e-9); % Angular frequency for 1550 nm light  
m=3.3*10^4; % Mode number of 1550 nm light in FP  
L=2*pi*c*m/(n*w0) % FP cavity length  
FSR=c/(2*n*L) % Free spectral range  
FWHM_Cavity=FSR/Finesse % FWHM of FP cavity  
FWHM_Pulse=FWHM_Cavity*0.5 % Define bandwidth of input pulse spectrum  
t0=1/FWHM_Pulse % Define bandwidth of input pulse spectrum  
alpha=1/(beta*t0) % alpha in f(t)  
b=t0+2/alpha*log(2) % Time where the pulse intensity falls a half
N=t0/10^2 %time increment
t=[0:N:3*t0]; %create time domain

%Generate input pulse
f1=1-exp(-alpha*t);
for t_index=1:length(t)
    if t(t_index)>=0&t(t_index)<b
        f2(t_index)=exp(alpha*(t(t_index)-b))
    else
        f2(t_index)=1;
    end
end
f=f1-f2; %input pulse
M=1500; % number of round trip in the FP cavity
T=trans^2;
R=refl^2;
for t_index=1:length(t)
    for k=1:M
        if t(t_index)>=0&t(t_index)<((2*k-1)*n*L/c) % time range
            out(k,t_index)=0; %output for given time range
        elseif(t_index)>=((2*k-1)*n*L/c)&t(t_index)<=((2*k-1)*n*L/c)+b %time range
            out(k,t_index)=T*R^(k-1)-T*R^(k-1)*exp(-alpha*(t(t_index)-
            (2*k-1)*n*L/c))-T*R^(k-1)*exp(alpha*(t(t_index)-(2*k-1)*n*L/c))*exp(-
            alpha*b); %output for given time range
        else %time range
            out(k,t_index)=T*R^(k-1)-T*R^(k-1)*exp(-alpha*(t(t_index)-
            (2*k-1)*n*L/c)+T*R^(k-1)*exp(alpha*(t(t_index)-(2*k-1)*n*L/c))-b)-
            T*R^(k-1)*exp(alpha*(t(t_index)-(2*k-1)*n*L/c))*exp(-alpha*b)-T*R^(k-
            1); %output for given time range
        end
    end
    output(t_index)=sum(out(:,t_index)); %Amplitude of output pulse
end
int_output=output.^2; %Output intensity
int_input=f.^2; %Input intensity
figure(1)
plot(t,f1,'b',t,f2,'r') %plot element functions for input pulse
grid on
figure(2)
plot(t,int_input,'b') %plot input pulse
grid on
figure(3)
plot(t,int_output,'b') %plot output pulse
grid on

2. Fourier_anlaysis_FPout.m

This code is used to produce the numerical simulation result in Fig. A.3.(c)

n=1; %refractive index for vacuum between two Fabry-perot(FP) mirrors.
trans=sqrt(0.001); %Amplitude transmission coefficient for FP mirrors
refl=sqrt(0.999); %Amplitude reflection coefficient for FP mirrors
C=3e8; %Speed of light in free space
Finesse=refl/(1-refl^2); %Finesse of FP
w0=(2*pi*c)/(1550e-9); %Angular frequency for 1550nm light
m=3.3*10^4; %Mode number of 1550nm light in FP
L = 2*pi*c*m/(n*w0) % FP cavity length
FSR = c/(2*n*L) % Free spectral range
FWHM_Cavity = FSR/Finesse % FWHM of FP cavity
FWHM_Pulse = FWHM_Cavity*0.5 % Define bandwidth of input pulse spectrum
t0 = 1/FWHM_Pulse % time width of the input pulse
beta = 0.1;
alpha = 1/(beta*t0) % alpha in f(t)
b = t0*2/alpha*log(2) % time where the pulse intensity falls a half
N = t0/10^2 % time increment
t = [0:N:3*t0]; % create time domain

% Generate input pulse
f1 = 1 - exp(-alpha*t);
for t_index = 1:length(t)
    if t(t_index) >= 0 & t(t_index) < b
        f2(t_index) = exp(alpha*(t(t_index)-b))
    else
        f2(t_index) = 1;
    end
end
f = f1 - f2; % input pulse
M = 1500; % number of round trip in the FP cavity

% output pulse
T = trans^2;
R = refl^2;
for t_index = 1:length(t)
    for k = 1:M
        if t(t_index) >= 0 & t(t_index) < ((2*k-1)*n*L/c)
            first(k,t_index) = 1/2*exp(j*w0*t(t_index))*T*R^(k-1)*exp(-j*(2*k-1)*n*w0*L/c)*f_out
            second(k,t_index) = -exp(j*w0*t(t_index))*T*R^(k-1)*exp(-j*(2*k-1)*n*w0*L/c+alpha*(t(t_index)-b-(2*k-1)*n*L/c))*f_out
            third(k,t_index) = 1/2*exp(j*w0*t(t_index))*T*R^(k-1)*exp(-j*(2*k-1)*n*w0*L/c)*f_out
            forth(k,t_index) = 0;
        elseif t(t_index) >= ((2*k-1)*n*L/c) & t(t_index) <= ((2*k-1)*n*L/c+b)
            first(k,t_index) = 1/2*exp(j*w0*t(t_index))*T*R^(k-1)*exp(-j*(2*k-1)*n*w0*L/c)*f_out
            second(k,t_index) = -exp(j*w0*t(t_index))*T*R^(k-1)*exp(-j*(2*k-1)*n*w0*L/c+alpha*(t(t_index)-b-(2*k-1)*n*L/c))*f_out
            third(k,t_index) = 1/2*exp(j*w0*t(t_index))*T*R^(k-1)*exp(-j*(2*k-1)*n*w0*L/c)*f_out
            forth(k,t_index) = 0;
        else % time range
            first(k,t_index) = -1/2*exp(j*w0*t(t_index))*T*R^(k-1)*exp(-j*(2*k-1)*n*w0*L/c)*f_out
            second(k,t_index) = 0;
            third(k,t_index) = 1/2*exp(j*w0*t(t_index))*T*R^(k-1)*exp(-j*(2*k-1)*n*w0*L/c)*f_out
            forth(k,t_index) = 0;
        end
        fifth(k,t_index) = exp(j*w0*t(t_index))*T*R^(k-1)*exp(-j*(2*k-1)*n*w0*L/c-alpha*(t(t_index)-(2*k-1)*n*L/c)*f_out
    end
end

else % time range
    first(k,t_index) = -1/2*exp(j*w0*t(t_index))*T*R^(k-1)*exp(-j*(2*k-1)*n*w0*L/c)*f_out
    second(k,t_index) = 0;
    third(k,t_index) = 1/2*exp(j*w0*t(t_index))*T*R^(k-1)*exp(-j*(2*k-1)*n*w0*L/c)*f_out
    forth(k,t_index) = 0;
end

end
fifth(k,t_index)=\exp(j*w_0*t(t_index))*T*R^{(k-1)}*\exp(-j*(2*k-1)*n*w_0*L/c-alpha*(t(t_index)-(2*k-1)*n*L/c)); \% f_{out}

total(k,t_index)=first(k,t_index)+second(k,t_index)+third(k,t_index)-forth(k,t_index)-fifth(k,t_index);

output(t_index)=sum(total(:,t_index)); \% Amplitude of output pulse

int_output=abs(output).^2; \% Output pulse intensity

figure(1)
plot(t,f1,'b',t,f2,'r') \% element functions for input pulse
grid on

figure(2)
plot(t,int_input,'b') \% Plot input pulse
grid on

figure(3)
plot(t,int_output,'b') \% plot output pulse
grid on
C. Photo images of the experimental setup

Photo images of the experimental setup are shown in this section with brief captions. Detailed information can be found in chapter VI and VII.

Fig. C-1 1550nm Laser diode, current driver circuit and temperature controller circuit

Fig. C-2 10Gb/s and 2.5Gb/s electro-optic modulators
Fig. C-3 (a) Black foam box mounted on vibration absorber to shield fiber ring cavity from noise (b) Fiber ring cavity inside the box

Fig. C-4 (a) Fiber loop mirror (b) Fiber polarization controller
Fig. C-5 (a) Optical chopper wheel (b) Fabry-Perot spectrum analyzer

Fig. C-6 Er-doped fiber amplifier (EDFA), Maximum power output is 250mW
Fig. C-7 (a) A.C. servo (b) Electronics; Lock-in-amplifiers (LIA), Radio-frequency wave Generator, High Voltage Amplifier, and D.C. supplier
D. Copies of publications and preprints
Demonstration of a simple technique for determining the $M/#$ of a holographic substrate by use of a single exposure

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We propose and demonstrate a simple technique for determining the $M/#$ parameter of a holographic recording material. In this method, divergent object and reference beams are used to produce a spatially varying index modulation. One can analyze the resultant diffraction pattern to find $M/#$ by using only a single grating; existing techniques require many gratings. © 2004 Optical Society of America

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The dynamic range of a holographic medium is an important parameter for determining storage density and diffraction efficiency of holographic memory systems and holographic beam combiners.\textsuperscript{1–4} In these applications, many holographic gratings are multiplexed in the medium at the same spatial location. $M/#$ is a parameter that defines the dynamic range of the holographic medium; it is essentially $\pi/2$ times the ratio of the maximum achievable index modulation and the index modulation that corresponds to a unity diffraction efficiency grating. The existing techniques\textsuperscript{4–8} for measuring $M/#$ require one to write many holograms on the material. Here we discuss a technique to determine $M/#$ for a holographic recording material that is potentially simpler, and we present simulated and experimental results for a photopolymer-based holographic recording medium.

Typically, illumination of a holographic substrate with a spatially periodic, sinusoidal intensity pattern produces a periodic index modulation $n(x) = n_0 + n' \cos(Kx)$, where $n_0$ is the spatially averaged index of refraction of the medium, $n'$ is the index modulation depth, and $K$ represents the wave number of the grating. When a laser beam of wavelength $\lambda$ illuminates this grating at the Bragg angle, diffraction efficiency $\eta$ is given by\textsuperscript{9}

$$\eta = \frac{I_d}{I_0} = \sin^2\left(\frac{\pi n' \alpha d}{\lambda}\right),$$

where $I_0$ is the input intensity, $I_d$ is the diffracted intensity, $d$ is the thickness of the substrate, and $\alpha$ is the obliquity factor determined by the orientation of the grating. A characteristic scale for the index modulation is $n_e = \lambda/(2\alpha d)$, so $\eta$ becomes

$$\eta = \sin^2\left(\frac{\pi}{2} \frac{n'}{n_e}\right), \quad \eta = 1, \quad n' = n_e.$$ \hspace{1cm} (2)

In many situations the modulation depth\textsuperscript{4} can be modeled as

$$n' = n_m \left[1 - \exp\left(-\frac{t}{\bar{t}}\right)\right],$$

$$\bar{t} = \frac{\lambda}{2 \pi n_m}, \quad t = \text{exposure time}, \quad \bar{t} = \text{time constant that depends on the material sensitivity and the intensity of the writing laser beams, and } n_m \text{ is the maximum index modulation. A convenient way to quantify the value of } n_m \text{ is through the use of } M/# \text{, which can be defined as } M/# = (\pi n_m)/(2n_e).$$

By using only a single grating; existing techniques require many gratings. For notational convenience we define a scaled version of this expression:

$$Q = n_m/n_e,$$

such that $M/# = (\pi/2)Q$. When $Q$ is an integer, it represents essentially the maximum number of orthogonal, unit diffraction efficiency gratings that can ideally be written in a given spatial location.

Consider a situation in which $N$ equalized diffraction efficiency gratings are multiplexed on a single substrate by use of the Bragg (angle or wavelength) orthogonality condition. For $N > Q$ the diffraction efficiency for each grating can be approximated by $\eta = (\pi^2/4)Q/N^2$\textsuperscript{4,5} More generally, if the diffraction efficiencies of the gratings are not identical, it is possible to define and measure $M/#$ of the material from the relation\textsuperscript{4,5}

$$Q \approx \sum_{i=1}^{N} \sqrt{\eta_i}, \quad \eta_i \ll 1.$$ \hspace{1cm} (4)

Although one can measure $M/#$ by using one exposure in certain cases,\textsuperscript{5,7} in general to measure $M/#$ by this approach may require one to write many holograms. As an alternative method, one can also use the fact that the diffraction efficiency of a single grating in the small index-modulation limit is a quadratic function (to first order) of the exposure time, described by

$$\eta(t) \approx Q^2\left(\frac{t^2}{\bar{t}^2}\right)\left(\frac{\pi^2}{4}\right),$$

which follows directly from Eqs. (2), (3a), and (3b). $M/#$ can thus be determined from the curve that defines the diffraction efficiency of a single hologram as a function of exposure time. This method also requires recording many successive holograms with different exposures on the holographic substrate.\textsuperscript{4}

In this Letter we offer a potentially simpler approach to determining $M/#$ from a single recording on the holographic medium. To illustrate this method we first combine Eqs. (2) and (3a) to express the diffraction efficiency as a function of time:
\[
\eta(t) = \sin^2\left[\frac{\pi}{2} Q \left(1 - \exp\left(-\frac{t}{\tau}\right)\right)\right].
\]  
(6a)

Now, according to the generalized optical pumping model,\(^4\) the saturation rate \(\tau^{-1}\) depends linearly on the intensity of the radiation for writing the grating: \(\tau^{-1} = \beta I\), where \(\beta\) is the sensitivity of the medium and \(I\) is the amplitude of the intensity modulation, defined as

\[
I = I[1 + \cos(K_G x)],
\]  
(6b)

where \(K_G\) is the grating vector. For typical values of \(I\) used, the value of \(\beta\) can be assumed to be a constant. If the value of \(I\) depends on position \(r\) as well, we can write

\[
\eta(t, r) = \sin^2\left(\frac{\pi}{2} Q \left[1 - \exp[-\beta I(r)t]\right]\right). 
\]  
(7)

Specifically, let us consider a situation when two equal-intensity, coherent Gaussian beams write a grating in a holographic medium. The intensity distribution will be

\[
I(r) = 2I_0 \exp(-2r^2/\omega_0^2) [1 + \cos(K_G \cdot r)],
\]

\(K_G = K_1 - K_2\),

(8)

where \(K_1\) and \(K_2\) are the propagation wave vectors, \(I_0\) is the intensity at the center of each beam, and \(\omega_0\) is the Gaussian beam radius of each writing beam. Comparing Eq. (8) with Eq. (6b), we find that \(I = 2I_0 \exp(-2r^2/\omega_0^2)\). When this expression is used in Eq. (7), the resultant diffraction efficiency is given by

\[
\eta(t, r) = \sin^2\left(\frac{\pi}{2} Q \left[1 - \exp[-f(r)t/\tau]\right]\right),
\]

\(0 \leq f(r) \leq 1\),

(9)

where \(f(r) = \exp(-2r^2/\omega_0^2)\) and \(\tau = 1/2\beta I_0\). Across the spatial profile of the writing beams, the value of \(f(r)\) varies from 1 in the center for \(r = 0\) to a value of 0 for \(r \gg \omega_0\).

Now, if \(t/\tau = 5\), for example, then, at \(r = 0\), \(\exp[-f(r)t/\tau] \) approaches zero. However, for \(r \gg \omega_0\), \(f(r) \ll 1/5\) and \(1 - \exp[-f(r)t/\tau]\) approaches unity. This argument holds for larger values of \(t/\tau\) as well. Thus, for \(t/\tau \geq 5\), the quantity \(1 - \exp[-f(r)t/\tau]\) varies monotonically from one to zero. Therefore the total number of circular fringes is of the order of \(Q/2\) for \(t/\tau \geq 5\). Accordingly, we note that for the proper exposure time one can be sure to observe the full number of fringes. To be more precise, let us express \(Q\) as follows:

\[
Q = 2m + n + \alpha, \quad \alpha < 1, \quad n = 0, 1.
\]  
(10)

In this notation \(\alpha\) is the fractional part of \(Q\) and \(n\) determines whether \(Q\) is odd or even. Consider first the case \(n = 0\) and \(\alpha = 0\). In this case, for \(t/\tau \geq 5\) the number of full circular fringes equals \(m\) with a null at the center. Consider next the situation in which \(n = 1\) and \(\alpha = 0\). In this case the number of full circular fringes will still be \(m\), but there will be a peak at the center. Finally, for \(\alpha \neq 0\) the efficiency at the center will have a dip if \(n = 1\) and a peak if \(n = 0\). The actual value of the efficiency at the center reveals the value of \(\alpha\).

We studied the phenomenon by using simulations. Figure 1 shows the result for dependence of diffraction efficiency on exposure time for an even-\(Q\) value material with a plane-wave readout beam. Normalized diffraction efficiency is plotted versus radial distance.

Fig. 1. Result of simulation showing the evolution of the diffracted pattern as a function of holographic exposure for an even-\(Q\) \([m = 5, n = 0, \alpha = 0]\) in Eq. (10) value material with a plane-wave readout beam. Normalized diffraction efficiency is plotted versus radial distance.

Fig. 2. Result of simulation for the diffraction pattern for fractional \(Q\) with a plane-wave readout beam \([m = 5, n = 0, \alpha = 0.2]\) in Eq. (10). Normalized diffraction efficiency is plotted versus radial distance.

Fig. 3. Hologram writing and readout geometry.
As one reaches the optimum limit for holographic exposure, the number of fringes that are visible in the diffracted beam reaches a maximum. Exposure time $T$ is labeled in the top right corner of each graph. All images are diffracted beams except for image 1, which is the transmitted beam for $T = 26$ s.

The expected dark center for an even-$Q$ material. Figure 2 shows the simulation result for diffraction for a material with an odd-$Q$ value plus a fractional part. As expected, there is an intensity peak at the center of the diffraction pattern and a dip that is due to the fractional part. The value at the center yields the value of fractional $\alpha$ as 0.2, resulting in a $Q$ value of 11.2, which corresponds to an $M/\#$ value of 17.584.

To show the principle of operation experimentally, we used a dye-doped polymer material called Memplex.\textsuperscript{10} This material has a $Q$ value of 6, as claimed by the manufacturer. Figure 3 shows the combined setup for hologram writing and readout. Writing was done with a frequency-doubled Nd:YAG laser ($\lambda = 532$ nm), and readout was performed with a He–Ne laser operating at 632.8 nm. This material required baking after holographic exposure. During this experiment, the exposure times were gradually increased. After exposure, the material was baked until the number of observable interference fringes reached maximum. Figure 4 shows the results for a series of exposures for the holographic substrate. It shows that, as we reach the optimum limit for holographic exposure, the number of interference fringes visible in the diffracted beam reaches a maximum (three in this case). Thus the $Q$ for our material is $\sim 6$. This value of $Q$ yields an $M/\#$ of 9.42 for the material.

We have proposed and demonstrated a simple approach to determining the parameter $M/\#$ for any holographic recording material. This easy-to-use technique will be attractive for holographic data storage when \textit{a priori} knowledge about the storage material is valuable in determining the storage density and the recording schedule for the holograms.

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References

In recent years there has been a keen interest in producing high-power lasers by using the method of beam combination. For some applications, such as a Doppler laser radar, it is necessary to ensure that the combined output is spectrally narrow. This requirement can be met by use of a coherent beam combiner (CBC).\textsuperscript{1–6} For example, to circumvent the damage threshold as well as the saturation constraints of optical amplifiers, one can first split a master oscillator into $N$ copies, each of which is then amplified without affecting their mutual coherences. The amplified beams are then combined by the CBC. In principle, an $N \times 1$ CBC system with amplification can be implemented with a tree of conventional beam splitters, as shown in Fig. 1A.\textsuperscript{4} At every node of the tree there is a 50/50 beam splitter. The same tree of beam splitters operating in reverse combines the beams. Maximum output is ensured by phase locking, which can be implemented with electro-optic modulators with feedback, for example. However, a much more robust system that requires fewer optical components can be constructed with coherent holographic beam combiners (CHBCs), as shown in Fig. 1B. In addition, a CHBC can be used as a high-precision surface sensor, as discussed below. A CHBC is a holographic structure with $N$ superimposed common-Bragg-angle gratings that one can prepare by recording the holograms sequentially, with the reference wave incident at a fixed angle and the object wave incident at a different angle for each of the $N$ exposures.\textsuperscript{1,7}

In this Letter we demonstrate a CHBC that uses volumetric multiplexing of gratings in a thick polymeric substrate. Our experimental results compare well with the theoretical model based on the coupled-wave theory of multiwave mixing in phase gratings.\textsuperscript{8–12} We assume that the gratings are recorded in a lossless dielectric material by plane waves propagating in the $x$–$z$ plane and polarized in the $y$ direction. We restrict our analysis to the Bragg-matching condition; the coupled-wave equations are\textsuperscript{5}$C_R \partial R / \partial z = -j \Sigma k_m S_m$ and $C_S \partial S / \partial z = -j k_m R$. Here $R$ and $S_m$ are the amplitudes of the reference and the $m$th diffracted waves, respectively. We define the obliquity factors $C_R$ for the reference and $C_S$ for the $m$th diffracted waves as $C_R = \rho / \beta$ and $C_S = \sigma_m / \beta$, where $\rho$ and $\sigma_m$ are the corresponding wave vectors and $\beta$ is the propagation constant. The coupling constant is $k_m = \pi n_m / \lambda$, where $n_m$ is the amplitude of spatial modulation of the refractive index. The $m$th grating is characterized by grating strength $v_m = \pi n_m d / (\lambda (C_R C_S)^{1/2})$, where $d$ is the thickness of the material. We define the diffraction efficiency of the $m$th grating by $\eta_m = (C_S / C_R) S_m (d) S_m^* (d)$.

In the beam-splitter mode, input wave $R$ illuminates $N$ superimposed gratings at the common Bragg angle and couples into diffracted waves $S_1 \ldots S_N$. The waves produced by the CHBC have equal and maximum diffraction efficiencies when the $N$ grating

![Fig. 1. A, $N \times 1$ CBC implemented with a tree of conventional 50/50 beam splitters. B, $N \times 1$ CHBC ($\triangleright$, amplifier; $\otimes$, phase lock).](image-url)
strengths satisfy the condition $\left(\sum_{m} k_m^2\right)^{1/2} = \pi/2$. Therefore, the optimal grating strengths are $v_1 = v_2 = \ldots = v_N = \pi/2, N$. From time-reversal symmetry of Maxwell’s equations, it follows that a beam combiner must show maximum diffraction efficiency for the same grating strengths that would yield maximum diffraction efficiencies for the beam-splitter mode. The boundary conditions for the gratings in a beamsplitter mode at $z=0$ can be written as $R(z)=1$ and $S_1(z)=S_2(z)=\ldots=S_N(z)=0$. The solutions of the differential equations at $z=d$ are $R(d)=\cos(\sqrt{N}\psi)$ and $S_m(d)=-j(1/\sqrt{N}) \sin(\sqrt{N}\psi)$.

In the beam-combiner mode the waves $S_1 \ldots S_N$ illuminate the superimposed gratings, each at the corresponding Bragg angle, thus producing a combined diffracted wave $R$. In the presence of a linear phase delay between the $N$ input waves, the boundary conditions at $z=0$ are $R(0)=0$, $S_1(0)=1$, $S_2(0)=\exp(j\psi)$, $S_3(0)=\exp[j(2\psi)]$, and $S_4(0)=\exp[j(3\psi)] \ldots S_N(0) = \exp[j(N-1)\psi]$, where $\psi$ is the phase delay. Defining $\gamma = \Sigma (jIC_{S_m})k_m^2/C_{\psi_S}$, we obtain the solution at $z=d$ to be given by $R(d) = -j[I(C_{S_m})/(C_{\psi_S})]^{1/2}(\Sigma k_m \exp[j(m-1)\psi]) \sin[(\gamma//IC_{S_m})^{1/2}d]$. The intensity of the output wave of a beam combiner is $I = R(d)R^*(d)$.

Our holograms were written and read with a frequency-doubled cw Nd:YAG laser operating at 532 nm. We recorded six angle-multiplexed holograms in the photopolymer-based Memplex thick holographic material developed by Laser Photonics Technology, Inc.\(^{13}\) The incident angle of reference wave $R$ was held constant during every exposure. Beams $S_1 \ldots S_6$ were recorded at fixed angular intervals. During the readout, reference beam $R$ illuminated the holograms at the common Bragg angle, and beams $S_1 \ldots S_6$ were reconstructed simultaneously. The numerical simulation results for beam profiles are presented in Fig. 2a. Figures 2b and 2c show the experimental transmitted ($T$) and diffracted beam profiles. We estimated the value of $\nu=0.23\pi$ by fitting the numerical simulation curves to the experimentally observed beam profiles. This value is only slightly larger than $\nu=\pi/2, \nu=0.204\pi$, at which the maximum diffraction efficiency is achieved.

The optical setup for demonstrating a six-beam combiner is presented in Fig. 3. The input beam from the laser illuminates the hologram in the direction of reference $R$. The gratings act as a beam splitter, producing six diffracted waves (indicated by solid lines in the figure). The six waves are collimated by a lens and reflected by a tilted mirror. The lens is placed a focal length away from both the CHBC and the mirror to create a 4$f$ imaging system. The mirror is rotated with a piezo-electric element by a small angle to vary the phase delay. The angle is small enough that the reflected beams remain Bragg matched. Phase delay $\phi$ between the adjacent waves is a constant for a given angle of the mirror. The six reflected waves illuminate the hologram in the beam-combiner mode. The combined beam is partially reflected by a 60% reflecting beam splitter and monitored by a photodetector.

Figure 4 (top) shows the numerical simulations of the equations above for the output intensity of a six-beam combiner with unit intensity input beams as a function of phase delay $\phi$ for three values of $\nu$, 0.204$\pi$, 0.123$\pi$, and 0.08$\pi$. The intensity profile obtained by solution of the coupled-wave equations resembles the familiar multiple-beam interference pattern. We estimate the finesse ($F$) of the CHBCs by dividing the peak-to-peak angular bandwidth [called the free angular range (FAR)] by the half-peak intensity angular bandwidth ($\Delta \phi_{1/2}$). Whereas the maximum intensity varies as a function of $\nu$, all three cases have the same finesse value, $F=6$. Note that this value is the same as that of $N$, the number of beams combined, as is to be expected because the finesse of any resonator is directly related to the number of beams that contribute to the peak of the interference curve. Figure 4 (bottom) shows the intensity of the partially reflected combined beam measured by the photodetector. We estimate the finesse of this six-beam combiner to be 5.7, which is very close to the theoretical value of 6.

We denote by $I_0$ the sum of the intensities of diffracted beams $S_1 \ldots S_6$ measured just after the CHBC when it is operating as a beam splitter. As $\nu=0.23\pi$ for our CHBC, the peak of the output intensity of the
As we described above, a CHBC may be useful in producing high-power laser beams. In addition, a CHBC may find applications as a high-precision surface sensor. For instance, if we replace the rotating mirror with some unknown specularly reflecting surface in the experimental setup in Fig. 3, we can use the CHBC to perform a multiple-beam interferometric study of the unknown surface. Sharp fringes can be obtained with multiple-beam interferometry, thus providing a much higher resolution than that achievable with simple two-beam interferometry. For example, one can detect unknown surface displacement $D$ by observing a shift of the fringes with an accuracy limited by $(\text{SNR})\Delta D = (\lambda F) / \sigma$, where $\sigma$ is the signal-to-noise ratio. Previously reported holographic multiple-beam interferometry techniques relied on the nonlinear properties of the holographic material, thus generating the beams as multiple diffraction orders.\textsuperscript{14–17} Using a CHBC made with volume gratings allows for interferometry with a much higher number of beams ($\sim 1000$) and, therefore, higher precision in measurements.\textsuperscript{18} In addition, it is possible to make a CHBC by angle multiplexing beams in horizontal and vertical directions and thus to obtain two-dimensional information about the vibrating surface.

To summarize, we have demonstrated a CHBC for six beams at 532 nm that uses volumetric multiplexing of gratings in a thick polymeric substrate. Our experimental results compare well with the theoretical model based on the coupled-wave theory of multimode mixing in a passive medium.

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References

In this paper, we present the coupled-wave analysis for multiple thick holographic gratings for optical beam combining. Starting with the basic model of incoherently superimposed multiple gratings used to form the beam-combiner, we derive equations that describe wave propagation in a holographic substrate containing angular multiplexed gratings with either sinusoidal refractive index or absorption modulation including the off-Bragg matched condition. Numerical results are presented for multiple thick holographic gratings in three different geometries: the beam-splitter mode, the beam-combiner mode, and the cross-coupled mode. The effect of non-equal beam intensities and phased angle variation on the efficiency of beam combiner have been investigated. Experiment is performed to create an efficient six beam splitter-combiner using a thick photopolymer based holographic material. The experimental measurement for efficiency in beam combining compares well with the predicted value from our theoretical model.

**Keywords:**

1. INTRODUCTION

Various industrial and technological fields require cost effective and efficient ways to create high power lasers. An efficient incoherent beam-combiner could be very useful to generate high power laser beams from multiple low power sources. One useful method to combine beam is a holographic beam combiner with incoherently superimposed index gratings recorded in a holographic media. This is a very simple, cost-effective and efficient method. The only constraint of a holographic beam combiner is the diffraction efficiency of multiple gratings recorded on the holographic material, and the cross-talk among the gratings.

In a previous paper, we demonstrated a coherent holographic beam combiner (CHBC). We recorded six superimposed holographic gratings with a reference wave at a fixed incident angle and six objective waves at different incident angles. In the beam splitter mode, the beam profiles for transmitted and diffracted waves were monitored and were compared with numerical simulations to determine the strengths of the 6 gratings. In the reverse mode, we demonstrated combination of six beams into a single beam, by controlling the relative phases between the six beams. The quality of the CHBC for the six beams was evaluated by comparing the experimental data with numerical simulations.

In this paper, we extend the previous study which was mainly focused on the beam combiner mode, and present a complete theoretical analysis along with simulation results for a holographic beam combiner which could act as a beam splitter and a cross-coupler in alternate geometries. We have used the coupled-wave theory to describe an angle-multiplexed multiple-grating holographic medium as an efficient optical element for beam combining, splitting and cross-coupling. The grating modulation strength for optimum performance in each mode of its operation is determined by numerically solving coupled-wave equations for a holographic substrate with \( N \)-multiplexed gratings. The analysis is restricted to thick index gratings in a lossless medium under Bragg matched condition. Our numerical results show that 100% conversion is achievable for each of these modes. Effect of non-equal beam intensities and phase angle variation between adjacent waves...
on beam combining have been studied using numerical examples. A thick photopolymer based holographic material is used in our experiment to create an efficient beam combiner using angle multiplexing. The grating strength is measured from the experimentally observed beam profiles using a single exposure technique recently proposed by us for the M# measurement. Grating modulation close to the maximum diffraction efficiency condition has been achieved experimentally for six multiplexed gratings on a holographic substrate to demonstrate an efficient beam combining. In what follows, we review the coupled-wave theory and present an analysis for multi-wave mixing in a thick holographic medium in the context of beam combining, splitting and cross-coupling.

Diffraction efficiency of a thick hologram and the amplitudes of signal and reference waves can be obtained from the coupled wave theory for thick hologram gratings. Thick holographic gratings cause energy exchange between the incoming reference wave and the outgoing signal wave at or near the Bragg angle. The analysis of the Bragg-matched diffraction for two sets of incoherently superimposed gratings using coupled wave theory was later developed by Case. He determined the efficiencies of the diffracted waves for an index-modulated lossless holographic material and experimentally demonstrated two different modes of operation such as the beam splitter and combiner. Magnusson considered the first and higher diffracted orders in gratings with arbitrary shapes and all possible incident angles including the Bragg angle. Furthermore, equations for efficiencies of higher order diffracted beams, which can be significant in holographic gratings with large refractive index modulations, were obtained by using the coupled wave analysis. A different approach to analyze diffraction from two incoherently superimposed gratings was investigated by Zhao et al. They described a volume-index grating vector and examined the Bragg matching and pseudo phase matching cases. Kobolla also investigated a beam splitter with multiple gratings recorded sequentially by one reference wave and object waves in a lossless holographic material and under the Bragg-matched condition.

We consider a general number of gratings recorded by one reference wave and multiple object waves. The incident angle of the reference wave is unchanged, while the object waves with a varying incident angle for each sequential exposure interfere with the reference wave. We then calculated the diffraction efficiency for the cases (i) an incoming wave in the direction of the reference wave, (ii) several input waves with equal intensities propagating in the direction of the object waves. Diffraction efficiency for the incoming waves with non-equal intensities is also studied. In our experiment, six gratings are recorded by angular multiplexing which is considered as a six beam combiner. In order to predict the efficiency of the beam combiner, grating strength is evaluated by comparing the experimental results to our simulation in the beam splitter mode. The six beam combiner is demonstrated using a simple set-up with a plano-convex lens and a mirror.

2. COUPLED-WAVE ANALYSIS FOR MULTIPLE THICK HOLOGRAPHIC GRATINGS

We derive equations that describe wave propagation in angular multiplexed grating structure with a sinusoidal refractive index (or absorption) modulation including the off-Bragg matched condition. We consider multiple incoherently superimposed gratings for our designs of beam combiner, beams splitter and beam coupler. Figure 1 shows the basic diagram of the thick grating model used in our analysis.

For simplicity, we assume that the gratings are recorded in an optical material, which is infinite in extent along the x and y direction, and all the waves are propagating in the x-z plane, and polarized in the y direction. The grating is defined by the grating vector \( \mathbf{k}_g \) and its length is \( k_g = 2\pi/\Lambda_g \), where \( \Lambda_g \) is the period of the nth multiplexed grating.

When the wave is polarized perpendicular to the plane of incidence, wave propagation in presence of multiple gratings can be described in terms of the scalar wave equation as

\[
\nabla^2 E(x, z) + k^2 E(x, z) = 0
\]

where \( E(x, z) \) is the complex amplitude distribution, assumed to be independent of \( y \). Material modulations constituting the holographic gratings can be expressed by

Fig. 1. Schematic illustration of the reconstruction of object waves from the superimposed multiple holographic gratings illuminated by a reference wave.
either the spatial variations of the relative dielectric constant $\varepsilon$, or the conductivity $\sigma$. If $\varepsilon_0$ and $\sigma_0$ are the amplitudes of the spatial modulations of the $h$th grating, the modulated quantities of $\varepsilon$ and $\sigma$ can be written as

$$
\varepsilon = \varepsilon_0 + \sum_{n=1}^{N} \varepsilon_n \cos(\mathbf{K}_h \cdot \mathbf{X})
$$

$$
\sigma = \sigma_0 + \sum_{n=1}^{N} \sigma_n \cos(\mathbf{K}_h \cdot \mathbf{X})
$$

(2)

where $\varepsilon_0$ is the average dielectric constant, $\sigma_0$ the average conductivity and $\mathbf{X} = (x, y, z)$. This results in a spatially modulated propagation constant $k$

$$
k^2 = \frac{\omega^2}{c^2} \varepsilon - j\omega\mu\sigma
$$

(3)

where $c$ is the light velocity in free space and $\mu$ is the permeability of the material. Inserting Eq. (2) into Eq. (3), we obtain

$$
k^2 = \beta^2 - 2j\alpha\beta + \sum_{n=1}^{N} 2\beta\kappa_n (e^{-j\kappa_n x} + e^{j\kappa_n x})
$$

(4)

where

$$
\beta = \frac{\omega}{c} \sqrt{\varepsilon_0}, \quad \alpha = \frac{\mu \sigma_0}{2\sqrt{\varepsilon_0}}
$$

(5)

and the coupling constant $\kappa_n$ between the reference wave $S_0$ and the $h$th diffracted wave $S_h$ from the $h$th grating is

$$
\kappa_n = \frac{\pi n_n}{\lambda} - j \frac{\alpha_n}{2}, \quad n_n = \frac{e_n}{2\sqrt{\varepsilon_n}}; \quad \alpha_n = \frac{\mu \sigma_n}{2\sqrt{\varepsilon_n}}
$$

(6)

where $n_n$ and $\alpha_n$ are the spatial modulation amplitudes of the refractive index and the absorption constant. The diffracted waves and the reference wave have the propagation vectors $\rho_0$ and $\rho_n$ respectively that satisfy the following phase matching relation

$$
\rho_n = \rho_0 - \mathbf{K}_h
$$

(7)

The components of $\rho_n$ and $\mathbf{K}_h$ are given by

$$
\rho_n = \begin{bmatrix} \sin \theta \\ 0 \\ \cos \theta \end{bmatrix}, \quad \mathbf{K}_h = \begin{bmatrix} \sin \phi_h \\ 0 \\ \cos \phi_h \end{bmatrix}
$$

(8)

At the Bragg angle, $\theta_{\text{Bragg}}$, of the $h$th grating

$$
\beta^2 - \rho_b^2 = 0
$$

(9)

Combining Eqs. (7) and (8), and using the Eq. (9), one obtains

$$
\cos(\phi_h - \theta_{\text{Bragg}}) = K_h / 2\beta
$$

(10)

We describe the waves in the medium (Fig. 1) with complex amplitudes $S_0$ and $S_h$ that vary along $z$. The total electric field incident on the gratings can be written as

$$
E = S_0(z)e^{-j\omega z} + \sum_{h=1}^{N} S_h(z)e^{-j\omega z}
$$

(11)

By using Eqs. (4) and (11) in (1), we obtain

$$
\begin{align*}
\left( S_0 - 2jS_0\rho_0 + (\beta^2 - \rho_b^2)S_0 - 2j\alpha\beta S_0 + 2\beta \sum_{n=1}^{N} \kappa_n S_n \right) e^{-j\omega z} \\
+ \sum_{n=1}^{N} \left( S_n - 2jS_n\rho_0 + (\beta^2 - \rho_b^2)S_n \right) e^{-j\omega z} = 0
\end{align*}
$$

(12)

where the primes indicate spatial derivatives of the field amplitudes in the $z$ direction. The propagation vector $\rho_0$ of the reference wave $S_0$ is equal to the wave propagation vector in the absence of the gratings, i.e., $\beta^2 - \rho_b^2 = 0$. We assume that the energy exchange between the diffracted orders is slowly varying. Consequently, the second order differential terms can be neglected. One can define the obliquity factors $C_0$, $C_h$ and dephasing factor $\theta_h$ for the $h$th grating as

$$
C_h = \frac{\rho_{0h}}{\beta}, \quad C_h = \frac{\rho_{nh}}{\beta}, \quad \theta_h = \frac{\beta^2 - \rho_b^2}{2\beta}
$$

(13)

where $\rho_{0h}$ and $\rho_{nh}$ are the $z$-components of $\rho_0$ and $\rho_n$ respectively. By substituting Eq. (13) in Eq. (12), one can obtain the following equations for the amplitudes of diffracted and transmitted waves corresponding to $N$ superimposed gratings

$$
\begin{align*}
C_0 S_0 + \alpha S_0 &= -j \sum_{h=1}^{N} \kappa_h S_h \\
C_h S_h + (\alpha + j \theta_h) S_h &= -j \kappa_h S_0
\end{align*}
$$

(14)

These coupled-wave equations are solved numerically for multiple thick gratings to describe three different modes of operation as beam-splitter, beam-combiner, and cross-coupler.

3. SIMULATION RESULTS FOR MULTIPLE BRAGG GRATINGS IN THREE DIFFERENT MODES

In the common-Bragg-angle structure with $N$ incoherently superimposed gratings, the reference wave is common to every exposure so that the $N$ gratings have the common Bragg angle ($\theta_{\text{Bragg}} = \theta_{\text{Bragg}} = \cdots = \theta_{\text{Bragg}} = \theta_{\text{Bragg}}$) and the readout wave has the same wavelength as that used for recording. There are three different modes in which the common-Bragg-angle structure can be read out:”

(i) The beam-splitter mode, where the structure is illuminated with the reference wave at the common Bragg angle. In this case, a single input wave reconstructs $N$ object waves.

(ii) The beam-combiner mode is the time-reversal case of the first one. In this case, the $N$ object waves illuminate the structure at the respective Bragg angles and thus, reconstruct the reference wave.

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(iii) The cross-coupled mode, where the structure is illuminated with waves propagating in the direction of randomly picked object waves. For this mode, the input waves are initially diffracted into the direction of the reference wave. Successively, the reconstructed reference wave diffracts again into the directions of other object waves before leaving the grating structure.

We have investigated the efficiencies of the output waves for all the above three cases. It is assumed that a holographic material has \( N \) lossless dielectric gratings \( (\alpha = 0) \) and the Bragg-matched condition \( (\vartheta_B = 0) \) is satisfied. Using these assumptions, Eq. (14) can be written as

\[
C_n^* S_n = -j \sum_{m=1}^{N} C_m S_m \quad C_0 S_0 = -j \kappa_0 S_0
\]

The efficiency of the output wave \( S_n \) is defined as:

\[
\eta_n = \frac{|C_n|^2 S_n^* S_n^*}
\]

Equations (7) and (8) give the \( z \) component of the propagation vector of the object wave for the \( h \)th grating. Inserting it in Eq. (13), the oblique factors \( C_n \) and \( C_0 \), and the dephasing measure \( \vartheta_n \) for the common Bragg angle structure in the Bragg matched condition can be obtained as

\[
C_0 = \frac{\cos \theta_{Bragg}}{\beta}, \quad C_n = \frac{\cos \vartheta_{Bragg} - K_n \cos \vartheta_B}{\beta}, \quad \vartheta_n = 0
\]

where \( \theta_{Bragg} \) is the common-Bragg-angle which is same as the incident angle of the reference wave.

The grating strength for a single hologram can be defined as

\[
\nu = \frac{\pi n_r d}{\lambda \sqrt{C_r C_0}}
\]

where \( d \) is the thickness of a holographic material, \( C_r \) and \( C_0 \) are the obliquity factors of reference and object wave respectively, and \( \lambda \) is wavelength of light. The grating strength \( \nu_n \) for the \( h \)th grating is given by

\[
\nu_n = \frac{\pi n_r d}{\lambda \sqrt{C_r C_0}}
\]

We consider six recorded gratings in the medium and the grating strengths for all these gratings are considered equal \((\nu_1 = \nu_2 = \nu_3 = \nu_4 = \nu_5 = \nu = 0)\). The diffraction efficiencies of the waves at \( z = d \) for the three different configurations such as beam combiner, splitter and cross-coupler modes are plotted as a function of the equalized grating strength normalized by \( \nu_0 = \pi/2 \) in Figures 2, 3, 4, and 5. During the holographic recording, beam \( S_0 \) is used as a reference wave and the others \( S_1 \sim S_6 \) as objective waves. The common-Bragg-angle is \( \pi/6 \) and the first grating formed by the interfering beams \( S_0 \) and \( S_1 \) is non-slanted \((\vartheta_1 = \pi/2)\). The gratings are recorded in a sequential order by interfering beams \( S_1 \) to \( S_6 \) with reference wave \( S_0 \). The angle difference between two object waves \( S_j \) and \( S_k \), is constant i.e., the six gratings are recorded using equal angular spacing. For the splitter mode, the boundary condition at \( z = 0 \) is

\[
S_j(0) = S_j(0) = S_i(0) = S_j(0) = S_i(0) = S_j(0) = S_i(0) = S_j(0) = 1
\]

where \( S_j(z) \) is the amplitude of the reference wave and \( S_j(z) \) are the amplitudes of the six object waves. The beam splitter divides the incident wave \( S_j \) into the six waves with equalized maximum diffraction efficiencies when the equalized grating strength \( \nu \) is 0.204 \( \pi \) as shown Figure 2. The simulation result shows that
Fig. 4. Relative efficiencies $\eta/\eta_0$ for the cross-coupling plotted as a function of normalized grating strength. The input wave incident in the direction of the beam $S_1$ is illuminating the 6 multiplexed gratings. $S_0$ may predict the efficiency of beam combiner for the case while non-equal amplitudes on the efficiency of beam combiner so that we investigate the influence of non-equal input beam intensities on the efficiency of beam combiner so that we may predict the efficiency of the beam combiner for the real case. Efficiency of the output waves are plotted in Figures 6(a, b and c) for the case while non-equal amplitude waves are incident on the beam combiner. As shown, the same grating strength as when a beam splitter reaches the maximum diffraction efficiency. Figure 3 shows all input waves from $S_1$ to $S_6$ are fully combined and consequently the outgoing wave $S_0$ has the maximum efficiency when the individual grating strength is $\frac{\pi}{2}$. For the cross-coupled mode, we considered two cases. In the first case, when an input wave is incoming along the direction of beam $S_3$, the efficiencies of outgoing waves are analyzed. The boundary conditions are

$$S_1(0) = S_1(0) = S_0(0) = S_0(0)$$
$$= S_0(0) = S_0(0) = 0$$

Since the beam-splitter acts as a beam-combiner in reverse, the beam combiner must show maximum efficiency at the output waves of the 1:6 beam splitter from $S_1$ to $S_6$ reach maximum diffraction efficiencies when all grating strengths are equalized to $\frac{\pi}{2}$. The result can be extended to the case of a beam splitter with $N$ partial waves with equalized maximum diffraction efficiency, when sum of the $N$ grating strengths satisfy the equation

$$\left(\sum_{i=1}^{N} p_i^2\right)^{1/2} = \frac{\pi}{2}.$$

For the beam combiner operation, the boundary condition is given by

$$S_1(0) = S_1(0) = S_1(0) = S_1(0)$$
$$= S_0(0) = 1, \quad S_0(0) = 0$$

(21)

The efficiencies of outgoing waves are shown in Figure 5. At the grating strength $0.4\pi$, five input waves produce a diffracted wave in the direction of beam $S_0$ and this diffracted wave is completely diffracted again into the direction of beam $S_1$ before leaving a holographic material. The outgoing wave in the direction of beam $S_0$ disappears and the efficiency of the output wave in the direction of beam $S_0$ shows maximum value. At the grating strength $0.2\pi$, the five input waves are diffracted into the direction of beam $S_0$ and the reconstructed beam $S_0$ is partially diffracted again into the direction of beam $S_1$. However, the reconstructed beam is mostly outgoing, and efficiency of the outgoing wave in the direction of beam $S_0$ reaches maximum.

During the experiment, it is difficult to perfectly equalize the intensities of all the input waves. Therefore, we investigate the influence of non-equal input beam intensities on the efficiency of beam combiner so that we may predict the efficiency of the beam combiner for the real case. Efficiency of the output waves are plotted in Figures 6(a, b and c) for the case while non-equal amplitude waves are incident on the beam combiner. As shown,
a small amplitude variation of the input intensities causes a small loss in output intensity, resulting in output less than the sum of the square of amplitude for all the inputs. In Figure 6(a), for the case that the amplitude difference between adjacent inputs is 16% of the strongest input amplitude, the output intensity of $S_0$ is about 76% of the sum of the intensities. Thus in the worst case when the input amplitude difference is large, the output of $S_0$ shows only a 24% drop in efficiency. Figures 6(b) and (c) show the output is 90% and 98% of the sum of input intensity when the amplitude difference is 10% and 5% respectively. Therefore, the output loss caused by non-equal input waves is concluded to be very small if the intensities of input waves are adjusted within 5%.

Relative efficiency can be obtained by calculating square of the input amplitudes when normalized grating strength $\nu/\pi/2$ is set to a zero. However, there exists a difference in value between the relative efficiency obtained from the simple calculation and from numerical simulation shown in graphs above. Such discrepancy results from the ratio of the obliquity factors, $(C_h/C_0$ in Eq. (16) which is considered in the numerical simulation, but not in the simple calculation.

4. Determination of Equalized Grating Strength by Analyzing the Output Waves in the Beam Splitter Mode

In this section, we estimate equalized grating strength by analyzing the output waves when an input wave propagates along the direction of reference wave. The reference is diffracted by 6 gratings recorded using angular multiplexing. The images of the diffracted waves are compared to the graphs of numerical simulation which provides the intensities of output waves for an arbitrary equalized grating strength as a function of the beam radius.

The input wave $S_0$ illuminates $N$ superimposed holograms with equalized gratings at the common Bragg angle and is diffracted into waves $S_1 \sim S_N$. The output waves $S_1 \sim S_N$ propagate along the direction of each object wave. The boundary conditions for Eq. (12) at $z = 0$ can be written as

$$S_0(0) = 1, \quad S_1(0) = S_2(0) = \cdots = S_N(0) = 0 \quad (24)$$

We are interested in the amplitudes of output waves at $z = d$. The solutions of the differential equations at $z = d$ are written as

$$S_0(d) = \cos \left( \sqrt{\sum_{h=1}^{N} \nu_h^2} \right) \quad (25)$$

$$S_h(d) = -j \frac{\nu_h}{\sqrt{\sum_{k=1}^{N} \nu_k^2}} \sin \left( \sqrt{\sum_{k=1}^{N} \nu_k^2} \right)$$

where $\nu_h$ is the grating strength for the $h$th hologram.

Consider that the holograms are recorded using a Gaussian beam. The index modulation amplitude is then proportional to the intensity of the recording beam. The grating strengths of all recorded holograms are equal and
will show the Gaussian profile along the beam radius. The grating strengths for each hologram can be expressed as
\[ v_h = \pi n_h d / \sqrt{\lambda C_0 C_h} = A \exp\left(-\frac{2r^2}{\omega_0^2}\right), \]
where \( C_0 \) and \( C_h \) are the obliquity factors of \( S_0 \) and \( S_h \) respectively, \( n_h \) are the index modulation amplitudes, \( A \) is an equalized grating strength value at the center of the recording beam, and \( \omega_0 \) is the Gaussian beam radius of the recording beam.

Readout is performed at the same wavelength as that used for hologram recording. Combining Eqs. (25) and (26), \( S_0 \) and \( S_h \) can be written as
\[ S_0(d) = \cos\left(\sqrt{N}A \exp\left(-\frac{2r^2}{\omega_0^2}\right)\right) \]
\[ S_h(d) = -j \frac{1}{\sqrt{N}} \sin\left(\sqrt{N}A \exp\left(-\frac{2r^2}{\omega_0^2}\right)\right) \]

Readout beam also shows the Gaussian profiles radially. Multiplying the Eq. (27) by a Gaussian envelope, the amplitude behavior of outgoing waves at \( r \) in the radial direction can be expressed by
\[ S_0(r) = \cos\left(\sqrt{N}A \exp\left(-\frac{2r^2}{\omega_0^2}\right)\right) \exp\left(-\frac{r^2}{\omega_1^2}\right) \]
\[ S_h(r) = -j \frac{1}{\sqrt{N}} \sin\left(\sqrt{N}A \exp\left(-\frac{2r^2}{\omega_0^2}\right)\right) \exp\left(-\frac{r^2}{\omega_1^2}\right) \]

where \( \omega_1 \) is the Gaussian radius of the reading beam. From Eq. (28), it is noticed that when \( N \) superimposed holograms are considered as a single hologram, an effective accumulated grating strength for \( N \) holograms at the center of beam is \( \sqrt{N} \) times the grating strength of an individual grating denoted as \( A \). One can insert Eq. (28) into Eq. (16) and obtain the efficiency of the output wave as a function of \( r \)
\[ \eta_h = \frac{1}{C_0} \left| S_h(r) S_h^* (r) \right|, \quad (h = 0 \sim N) \]

where \( S_h^* (r) \) represents the conjugate of \( S_h(r) \).

Figure 7 shows the combined experimental setup for the hologram writing and readout. The holograms are recorded using an Nd-YAG laser (\( \lambda = 532 \) nm) and the read-out is performed with the same laser used in recording. We recorded 4 holograms in the 1st sample and 6 holograms in the 2nd sample with different exposure schedule using angular multiplexing. In this set-up, the incident angle of the reference wave \( S_0 \) is fixed during every exposure. Therefore, a common Bragg angle can be established. During the readout, a beam illuminates the holograms at the common Bragg angle and the beams \( S_1 \sim S_N \) are reconstructed simultaneously under the Bragg matched condition. Angle displacement between adjacent object waves is constant. The holographic material with 4 holograms was baked for 3 hours and that with 6 holograms for 2 hours. We compared observed experimental images with simulation results from optional \( A \) values and evaluated the optimized \( A \) value which makes efficiency profiles identical to the experimentally observed images of output waves.

Figure 7. Recording set-up for angular multiplexing using the Nd-YAG laser. The read-out is performed using another Nd-YAG laser.

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Table I. Experimentally observed transmitted and diffracted patterns for 4 superimposed gratings. The profiles show the relative efficiency $\eta_1/\eta_0$ as a function of the beam radius. The unit of beam radius along the $x$-axis is in “$\mu$m” and the peak efficiency $\eta_0$ is normalized to unity.

<table>
<thead>
<tr>
<th>$A = 1.08\pi$</th>
<th>Exposure time (sec)</th>
<th>Images</th>
<th>Profiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_0$</td>
<td></td>
<td><img src="image1" alt="Image" /></td>
<td><img src="graph1" alt="Graph" /></td>
</tr>
<tr>
<td>$S_1$</td>
<td>13</td>
<td><img src="image2" alt="Image" /></td>
<td><img src="graph2" alt="Graph" /></td>
</tr>
<tr>
<td>$S_2$</td>
<td>13</td>
<td><img src="image3" alt="Image" /></td>
<td><img src="graph3" alt="Graph" /></td>
</tr>
<tr>
<td>$S_3$</td>
<td>17</td>
<td><img src="image4" alt="Image" /></td>
<td><img src="graph4" alt="Graph" /></td>
</tr>
<tr>
<td>$S_4$</td>
<td>21</td>
<td><img src="image5" alt="Image" /></td>
<td><img src="graph5" alt="Graph" /></td>
</tr>
</tbody>
</table>

Tables I, II show the transmitted and diffracted images of 4 and 6 superimposed holograms, and the exposure schedules used. In Table I, the diffraction image of $S_2$ shows low efficiency as compared with that of $S_1$, $S_3$, $S_4$ and a fringe in the bright region is observed inside the ring possibly because of the mode hopping of the laser during the 2nd exposure. By comparing the number of rings observed in the images with the profiles, one can also determine the equalized grating strength of 4 holograms to be $1.08\pi$. In Table II it is also determined that the equalized grating strength of the 6 holograms reached a value of $0.23\pi$. It is higher than $0.204\pi \approx \frac{\pi}{2\sqrt{6}}$, the value at which the simulated beam splitter was shown to have the maximum diffraction efficiency. The sample adequately functions as a high efficiency beam splitter. If used in reverse, it can also function as a high efficiency beam combiner based on our discussions before.

Table III shows the intensities of the transmitted wave, $S_0$, and diffracted waves $S_1$ to $S_6$ measured using the 2nd sample shown in Table II. The output intensity data in Table III indicates that the input beam is diffracted into 6 beams with the almost equalized intensity. As mentioned earlier, the equalized grating value is measured to be $0.23\pi$ which deviates slightly from the optimum value.
Table II. Experimentally observed transmitted and diffracted patterns for 6 superimposed gratings.

<table>
<thead>
<tr>
<th>A = 0.23π</th>
<th>Exposure time (sec)</th>
<th>Images</th>
<th>Profiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₀</td>
<td></td>
<td><img src="image1" alt="Image" /></td>
<td><img src="profile1" alt="Profile" /></td>
</tr>
<tr>
<td>S₁</td>
<td>4</td>
<td><img src="image2" alt="Image" /></td>
<td><img src="profile2" alt="Profile" /></td>
</tr>
<tr>
<td>S₂</td>
<td>3.5</td>
<td><img src="image3" alt="Image" /></td>
<td><img src="profile3" alt="Profile" /></td>
</tr>
<tr>
<td>S₃</td>
<td>3.5</td>
<td><img src="image4" alt="Image" /></td>
<td><img src="profile4" alt="Profile" /></td>
</tr>
<tr>
<td>S₄</td>
<td>3.2</td>
<td><img src="image5" alt="Image" /></td>
<td><img src="profile5" alt="Profile" /></td>
</tr>
<tr>
<td>S₅</td>
<td>3.2</td>
<td><img src="image6" alt="Image" /></td>
<td><img src="profile6" alt="Profile" /></td>
</tr>
<tr>
<td>S₆</td>
<td>3.2</td>
<td><img src="image7" alt="Image" /></td>
<td><img src="profile7" alt="Profile" /></td>
</tr>
</tbody>
</table>
Table III. Intensities of the transmitted and diffracted waves measured in experiment using the 2nd sample shown in Table II.

<table>
<thead>
<tr>
<th>S0</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.53</td>
<td>0.32</td>
<td>0.5</td>
<td>0.44</td>
<td>0.34</td>
</tr>
</tbody>
</table>

0.204π. However it clearly demonstrates a near maximum diffraction efficiency 6 beam splitter.

5. INFLUENCE OF PHASE ANGLE VARIATION BETWEEN THE INPUT WAVES ON THE INTENSITY OF AN OUTPUT WAVE

An output of a six beam combiner is investigated using a numerical simulation based on the assumption that at the entrance of a holographic material, phase difference exists between each input wave. We also imposed the additional condition that the phase angle difference between adjacent input waves is constant. Therefore, the boundary conditions at \( z = 0 \) are

\[
\begin{align*}
S_0(0) = 0, \quad S_1(0) = 1, \quad S_2(0) = e^{j\phi}, \quad S_3(0) = e^{2j\phi}, \\
S_4(0) = e^{3j\phi}, \quad \ldots \quad S_6(0) = e^{6j(\pi - 1)\phi}
\end{align*}
\]  

(30)

where \( \phi \) is the phase difference between adjacent waves. We obtain the solution of the amplitude of \( S_0 \) at \( z = d \)

\[
S_0(d) = -\frac{1}{2} \sqrt{\frac{\prod_{k=1}^{N} C_k}{C_0^{2N}}} \left[ \sum_{l=1}^{N} s_l e^{j(0-1)\phi} \right] \\
\times \left[ \exp \left( j \frac{\gamma}{\prod_{k=1}^{N} C_k} d \right) - \exp \left( -j \frac{\gamma}{\prod_{k=1}^{N} C_k} d \right) \right]
\]  

(31)

with \( \gamma = \sum_{k=1}^{N} \left[ \frac{\prod_{i=1}^{N} C_k}{C_0^2} \right] \). The intensity of the output wave, \( I \) can be written as

\[
I = S_0(d)S_0^*(d)
\]  

(32)

In our analysis for the intensity profiles of output wave, it is assumed that the grating strengths for \( N \) holograms are equalized to the value at which beam combiner shows maximum diffraction efficiency. This assumption yields the equalized grating strength for \( N \) superimposed holograms as \( \nu_l = \nu_2 = \nu_3 = \ldots = \nu_{N-1} = \nu_N = \nu = \pi/2\sqrt{N} \). The beam intensities from Eq. (32) are plotted as a function of phase difference \( \phi \) between adjacent input waves in Figures 8(a), (b), (c) for three cases considering 3, 6, and 9 incoming waves and the grating strengths are equalized to the value of \( \pi/2\sqrt{N} \). In the three cases, in-phase input waves are completely combined and the intensities reach maximum value which is equal to the sum of the intensity of individual input waves. Figures 8(a), (b), (c) correspond to the cases where the number of input waves is 3, 6, and 9, respectively.

![Fig. 8](image-url)
(b), (c) show the finesse that corresponds to the number of input waves and the half-width shrinks as the number of input waves increases. The finesse of 6 and 9 beam combiners are two and three times higher than that of a 3 beam combiner respectively.

Figure 9(a) shows the output intensity of 6 beam combiner is plotted as a function of grating strength $\nu$ for the case when all the input waves are in phase. Figure 9(b) shows the output intensity as a function of the phase angle between the adjacent waves for three different values of $\nu$ that correspond to output intensities 2, 4 and 6, as shown in Figure 9(a). The height of peak decreases as the grating strength reduces. The height of the solid line plot in Figure 9(a) is equal to the maximum intensity value. The finesse of the three peaks have the same value of 6, regardless of the grating strength.

6. EXPERIMENTAL DEMONSTRATION OF SIX-BEAM COMBINER VARYING PHASE ANGLE BETWEEN ADJACENT WAVES

Using our simulation, we first studied the intensity profile of the output wave $S_0$ of a six beam combiner. Figure 10 shows the result for the case when grating strength is set equal to that for holograms in the 2nd sample, 0.23$\pi$, discussed earlier. Figure 10 shows that the maximum peak height from the experiment with the 2nd sample can be expected to be 5.8.

Experimentally, we combined the six waves using the 2nd sample. Figure 11 shows the experimental set-up using a plano-convex lens that is used to demonstrate the beam combiner. The input wave from Nd-YAG laser illuminates the holograms in the direction of reference $S_0$ which is split into 6 waves. The waves represented by solid lines are generated in the beam splitter mode. The flat surface of the lens faces towards the hologram which acts like a point source with six split waves. Both surfaces of the lens contribute to refractions to minimize the spherical aberration. The six waves emerge parallel on the other side of the lens and are reflected by the mirror. Since the angle between the adjacent object waves is set constant via holographic recording, the phase delay between the adjacent diffracted waves $\phi$ is constant for a given angle of the mirror. The six reflected waves illuminate the holograms in the beam combiner mode. Combined beam is partially reflected by the beam splitter placed after the hologram. The partially reflected beam is monitored by photo detector while adjusting the mirror angle to continuously vary the phase angle difference.

The sum of the intensities of individual beams (defined as $6I_0$) is measured using an amplified photodetector whose output reads 760 mV after the splitter in Figure 11. $I_0$ stands for the average intensity of each individual beam.

Fig. 9. (a) Simulation results showing the intensity of the output wave as a function of grating strength $\nu$; Six input waves have unit intensity and are considered to be in-phase i.e., $\phi$ is set to zero in Eq. (31). (b) Simulation result showing the intensity of the output wave as a function of phase angle between the adjacent waves plotted for three values of $\nu$. The solid line corresponds to $\nu = 0.6492$, the dashed line for $\nu = 0.3850$, and the dotted line for $\nu = 0.2500$ in Eq. (31).

Fig. 10. The intensity of $S_0$ is plotted as a function of the phase angle difference between the adjacent waves for $\nu = 0.23\pi$. 
At this point, the undeflected beam was measured as 20 mV. For a $\nu$ value 0.23$\pi$, it is estimated from our theory that the maximum height of peak will correspond to a voltage 734 mV. Figure 12 shows the intensity of the partially reflected combined beam measured by the photodetector. It is noted from the 3rd and 4th peak on the left that the half width covers 0.7 fractional divisions in one rectangular box and the distance between the peaks is 4 fractional divisions. This measurement yields the finesse of six beam combiner as 5.7 which is approximately equal to the value that we estimated theoretically. The height of the highest peak is measured as 265 mV at the detector shown Figure 11. If we take into account the beam splitter reflectivity to be 61.54% and the reflection at the surface of the hologram, the intensity of the combined beam should correspond to 475.6 mV at the detector. The experimental output is still lower than the theoretically expected output of 734 mV which is attributed to residual imperfections in the experimental process and the inherent plane wave approximations in the coupled-wave theory.

Degradation of the efficiency of a holographic beam combiner can also be caused by either a cross coupled mode resulting from the double diffraction of the reconstructed wave $S_0$. In this case, input waves are not completely diffracted into the direction of reference wave $S_0$ as the reconstructed wave $S_0$ is partially diffracted again into the directions of the six object waves ($S_1 \sim S_6$) before leaving the holographic gratings. Also, if the input wave illuminates a location away from the center of the hologram, the grating strength ($0.23\pi$ at the center of the hologram) decreases along the radius of the hologram showing the Gaussian distribution. Additionally, the error in angle displacement during holographic recording can also cause phase angle difference variation between the adjacent beams. Such difference variation can result in phase angle mismatch for the input beams that can neither be in-phase simultaneously nor being fully combined.

7. CONCLUSION

In summary, we presented numerical results using the coupled-wave theory that revealed the grating modulation strength for efficient beam combining, and predicted the effect of non-equal beam intensities and phase angle variation on the performance of a beam combiner. We also demonstrated an experimental realization of a beam combiner.
combiner using angle multiplexed holographic gratings on a thick polymeric substrate. Experimentally measured efficiency in beam combining agrees with our theoretical model.

References and Notes

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Fast-light in a photorefractive crystal for gravitational wave detection

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Abstract: We demonstrate superluminal light propagation using two frequency multiplexed pump beams to produce a gain doublet in a photorefractive crystal of Ce:BaTiO\textsubscript{3}. The two gain lines are obtained by two-wave mixing between a probe field and two individual pump fields. The angular frequencies of the pumps are symmetrically tuned from the frequency of the probe. The frequency difference between the pumps corresponds to the separation of the two gain lines; as it increases, the crystal gradually converts from normal dispersion without detuning to an anomalously dispersive medium. The time advance is measured as 0.28 sec for a pulse propagating through a medium with a 2Hz gain separation, compared to the same pulse propagating through empty space. We also demonstrate directly anomalous dispersion profile using a modified experimental configuration. Finally, we discuss how anomalous dispersion produced this way in a faster photorefractive crystal (such as SPS: Sn\textsubscript{2}P\textsubscript{2}S\textsubscript{6}) could be employed to enhance the sensitivity-bandwidth product of a LIGO type gravitational wave detector augmented by a White Light Cavity.

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References and links

In this paper, we show that the two wave mixing between pump and probe pulses in a photorefractive crystal creates a double gain profile similar to that of the bi-frequency pumped Raman gain in rubidium we have used previously, with the corresponding anomalous dispersion. We demonstrate superluminal propagation of pulses in such a medium. The anomalous dispersion produced in this way can also be employed to realize a WLC. The experiment reported here used a green laser at 532 nm, with a photorefractive crystal that has anomalously produced in this way can also be employed to realize a WLC. The Raman gain in rubidium we have used previously, with the corresponding anomalous dispersion. We demonstrate superluminal propagation of pulses in such a medium. The wavelength of the Rb-transition, however, is not suitable for many of the applications listed above. For example, we have previously demonstrated one approach that uses a dispersive vapor medium within the cavity.

Specifically, the anomalous dispersion was produced by a rubidium vapor with bi-frequency pumped Raman gain. A WLC operating at the wavelength of the Rb-transition, however, is not suitable for many of the applications listed above. For example, in order to apply the WLC concept for enhancing the bandwidth-product of a LIGO-type gravitational wave detector, optical data buffering with a delay far exceeding the limit imposed by the delay-bandwidth product of a conventional cavity, a zero-area Sagnac ring laser gravitational wave detector with augmented strain sensitivity, and a super-sensitive ring laser gyroscope.

These applications are all based on so-called the White Light Cavity (WLC). A WLC is a cavity which resonates over a broader range of frequencies than ordinary empty cavities of equal length and finesse without a reduction in the cavity lifetime. As such, it can circumvent the tradeoff between the resonance bandwidth and the field build-up factor that ordinary cavities entail. A WLC also has the property that if the cavity length is moved away from the equal length and finesse without a reduction in the cavity lifetime, the frequency offset needed to restore the resonance is much larger than that for a conventional cavity, thus making it a more sensitive displacement and rotation sensor than an empty cavity can be.

A variety of approaches have been proposed and studied for realizing a white light cavity experimentally. [8-10]. For example we have previously demonstrated one approach that uses a dispersive vapor medium within the cavity. [3] Specifically, the anomalous dispersion was produced by a rubidium vapor with bi-frequency pumped Raman gain. A WLC operating at the wavelength of the Rb-transition, however, is not suitable for many of the applications listed above. For example, in order to apply the WLC concept for enhancing the bandwidth-sensitivity product of a LIGO-like gravitational wave detector, it is necessary to realize a WLC that operates at 1064 nm.

In this paper, we show that the two wave mixing between pump and probe pulses in a photorefractive crystal creates a double gain profile similar to that of the bi-frequency pumped Raman gain in rubidium we have used previously, with the corresponding anomalous dispersion. We demonstrate superluminal propagation of pulses in such a medium. The anomalous dispersion produced in this way can also be employed to realize a WLC. The experiment reported here used a green laser at 532 nm, with a photorefractive crystal that has a relatively slow (~seconds) response time. However, the technique demonstrated here is generic enough so that it can be employed with other photorefractive media as well.
example, a crystal of SPS [11,12], is sensitive at 1064 nm, and has a much faster response time, on the order of a millisecond, which suffices because LIGO does not seek to detect gravitational waves more than a few kHz in frequency. There is also reason to believe that photorefractive gain in quantum wells at 1064 nm [13] or an alternative approach involving gain depletion in quantum dot amplifiers [14] may prove suitable. The goal is to create a WLC which operates at 1064 nm for enhancing the sensitivity bandwidth product of LIGO-type gravitational wave detectors. The intent of this paper is to demonstrate the feasibility of the photorefractive approach.

2. Gain Doublet in photorefractive regime

The gain doublet and corresponding anomalous dispersion are the product of non-degenerate two-wave mixing and angular multiplexing in a photorefractive crystal. Here, we summarize briefly the physical model used to study this process. A space charge field is generated by the interference of two strong pump beams with a weak probe, so that the refraction index is modulated by the electro-optic effect. The probe is coupled with the pump beams due to these refractive index gratings [15]. Assuming that the intensity of each pump is much higher than that of the probe, in undepleted pumps approximation the phase and intensity coupling coefficients can be written as functions of the angular frequencies of the pumps and the probe [16]:

\[
\begin{align*}
\Gamma_{in} & = \sum_{j=1,2} \frac{\Gamma_{0j}}{2} \left[ \frac{d}{1 + (\omega_s - \omega_{pj})^2 \tau_j^2} \right] \\
\Gamma_{ph} & = \sum_{j=1,2} \frac{\Gamma_{0j}}{2} \left[ \frac{d(\omega_s - \omega_{pj}) \tau_j}{1 + (\omega_s - \omega_{pj})^2 \tau_j^2} \right]
\end{align*}
\]

where \(\Gamma_{in}\) and \(\Gamma_{ph}\) are the intensity and phase coupling coefficients respectively. \(\Gamma_{0j}\) depends on the incident angle between the \(j\)th pump and the probe [17]. \(\tau_j\) is the rise time of the space charge field induced by the pumps, and \(d\) is the effective interaction length. \(\omega_s\) and \(\omega_{pj}\) are the angular frequencies of the probe and the \(j\)th pump respectively. The intensity gain and the phase shift of the probe are determined by these coefficients. For our experiment, the 1st and 2nd pump beams are up-shifted and down-shifted respectively by \(\Delta \omega\) from the source frequency of \(\omega_0\), so that \(\omega_{p1} = \omega_0 + \Delta \omega\), \(\omega_{p2} = \omega_0 - \Delta \omega\). This creates two gain lines separated by \(2\Delta \omega\), and a region of anomalous dispersion occurs between them. The probe is pulsed, with a Gaussian spectrum centered around \(\omega_0\).

The group velocity of the Gaussian pulse can be expressed as

\[v_g = C/\left(1 + C \times \partial \Gamma_{ph} / \partial \omega\right)\] [18], where \(C\) is the speed of light in vacuum and \(n\) is the mean index. If the carrier frequency of the probe pulse is placed in the middle of the anomalous dispersion region and the pulse bandwidth is smaller than the anomalous dispersion bandwidth, the group velocity in the photorefractive medium can become larger than \(C\), or may even become negative. It is instructive to view these parameters graphically.

We consider the probe pulse to be of the form \(\exp\left(-t^2/t_0^2\right)\). We choose the pump intensities to be equal so that \(\Gamma_{01} = \Gamma_{02} = \Gamma_0\) and \(\tau_1 = \tau_2 = \tau_M\). For illustration, we consider \(\Gamma_0 d = 6\), \(\tau_M = 1.1\) sec, and \(t_0 = 0.6\) sec. Figure 1(a) shows the normalized intensity and phase coupling coefficients (\(\Gamma_{in}\), \(\Gamma_{ph}\)), and the Fourier Transform (S) of the probe as functions of \(\omega - \omega_0\), for \(\Delta \omega = 0\). Here the two gains overlap exactly and behave like a single gain line. The dispersion around the probe carrier frequency in this case is normal, and the \(v_g\)
of the probe becomes smaller than $C$. The width of the gain here is close to $\tau^{-1}_M$. In Figs 1(b)-1(d), we plot the same parameters for increasing separation between the pump frequencies. For $\Delta\omega=1\text{Hz}$ (Fig. 1(c)), the separation is comparable to the gain width, so that two gain peaks are clearly distinguishable. This leads to anomalous dispersion between the peaks, as shown. In Fig. 1(d), the separation becomes sufficiently wide so that the anomalous region covers the whole bandwidth of the Gaussian spectrum of the pulse.

![Fig. 1](image)

**Fig. 1.** Numerical, normalized plots of $S(\omega)$, the Fourier transform of the input pulse (solid line), $\Gamma_{in}$ (dashed line), and $\Gamma_{ph}$ (dotted line). The input pulse is assumed to be of the form $\exp(-t^2/t_0^2)$. For these plots, we have used for $t_0=0.6\text{sec}$, $\Gamma_{in}=6\text{sec}$ and $\tau_1=\tau_2=\tau_m=1.1\text{sec}$. The four sets are for four different gain separations, $2\Delta\omega=(a) 0\text{Hz}, (b) 1\text{Hz}, (c) 2\text{Hz}, (d) 4\text{Hz}$

### 3. Pulse propagation in the gain doublet system of photorefractive materials

For the case of the Gaussian input pulse of the form $\exp\left(-t^2/t_0^2\right)$, the output pulse coupled in the frequency domain can be expressed as [11,16]:

$$S(d, \omega_s) = S(0, \omega_s) \exp(\Gamma_{m1} + \Gamma_{m2}) \exp\left(i(\Gamma_{ph1} + \Gamma_{ph2})\right)$$

(2)

Here $d$ is the propagation distance of the probe inside a photorefractive crystal. $S(0, \omega_s)$ is the Fourier transform of the input pulse at the entrance of the material. Hence, one can obtain the total output signal intensity in the time domain by squaring the inverse Fourier transform of $S(d, \omega_s)$. Figures 2(a)-2(d) show the numerical simulation of the normalized output signal intensity for different gain line separations corresponding to each case in Figs. 1(a)-1(d). In Figs. 2(a) the output is clearly delayed compared to the pulse propagating in free space, as
Figure 2(b) shows the resultant output from propagation under the two gain peaks configuration associated with Fig. 1(b). To understand the behavior of the probe in this case, note brief that the gain in Fig. 1(b) is peaked at \(\omega_0 \pm \Delta \omega\) with a deep valley in the middle. As such, the components of the probe spectrum at \(\omega_0 \pm \Delta \omega\) get amplified disproportionately, leading to a two peaked spectrum. This leads to the beat note at 1Hz in Fig. 2(b). Furthermore, each of these peaks experiences normal dispersion, which leads to slowing of the probe, also evident in Fig. 2(b). In the case of Fig. 1(c), the gain separation is large enough so that the spectrum of the probe is almost completely within the region where the dispersion is anomalous (i.e. negative). This leads to advancement of the probe pulse, as can be seen in Fig. 1(c). The advance in this case is rather small due to the moderate steepness in the anomalous dispersion, and is evident only near the peak. Note also that a residual beat note is present at 2Hz, as expected. As the gain separation increases further (Fig. 1(d)), the slope of the anomalous dispersion become smaller, thus reducing the pulse advancement, as seen in Fig. 2(d). It is evident from Figs. 1(c) and 1(d) that \(\hat{\partial}^2 \Gamma_{\text{ph}} / \partial \omega^2 \neq 0\) for \(\omega \neq \omega_0\). The resulting group velocity dispersion [18, 19] causes pulse compression in this case, as can be seen clearly in Figs. 2(c) and 2(d).

4. Experimental pulse propagation set-up and result

We carried out an experiment that corresponds closely to the simulation. The experimental set-up is illustrated schematically in Fig. 3. A collimated 532nm doubled Nd-YAG laser was split into a probe and two pump beams. The acousto-optic modulators (AOMs) were driven by frequency synthesizers (PTS’s). The IQ modulator (JCIQ-176M, Minicircuits) and the AOM generated the Gaussian probe pulse with a carrier frequency of \(f_L+110\text{MHz}+4\text{Hz}\) corresponding to \(\omega_0/2\pi\), where \(f_L\) is the laser frequency. The Gaussian pulse with a temporal width of \(t_0=0.6\text{sec}\) was generated by a DAQ-Card (DAQCard−6036E, National instrument).
with a repetition period of 20sec for \( \Delta \omega/2\pi = 0 \) and 0.5Hz and, 10sec for \( \Delta \omega/2\pi = 1 \) and 2Hz. The first and second pumps (P1 and P2) were shifted by \( \pm \Delta \omega/2\pi \) from \( f_L + 110 \text{MHz} + 4 \text{Hz} \), respectively, thereby producing two gain peaks with a separation of \( 2 \Delta \omega/2\pi \). The incident angle of the probe is approximately 90° from the C-axis of the Ce:BaTiO\(_3\) crystal used for this experiment. P1 and P2 were angular multiplexed at angles of 40° and 60° with respect to the probe. The probe was coupled to the two pumps over an interaction length of 0.5cm inside the crystal. The polarization direction of all the beams and the C-axis were parallel to the optical table. A part of the probe was split-off to provide a reference pulse. This probe was monitored simultaneously with the pulse that traveled through the crystal, and was used to determine the degree of pulse delay/advancement and compression.

Fig. 3. Schematic diagram of the experimental set-up; AOM : Acousto-optic modulator, B.S: Beam Splitter, B.C : Beam Collimator, H.P: Half-wave plate, M: Mirror, PTS : Frequency synthesizer, Pump1: \( f_L + 110 \text{MHz} + 4 \text{Hz} + \Delta \omega \), Pump2: \( f_L + 110 \text{MHz} + 4 \text{Hz} - \Delta \omega \), Probe : Gaussian pulse with the carrier frequency of \( f_L + 110 \text{MHz} + 4 \text{Hz} \)

Figures 4(a)-4(d) show the normalized reference and output signals as \( \Delta \omega \) increases. For \( \Delta \omega = 0 \) in Fig. 4(a), the two gains coincide to form a single gain which results in the time delay (1.3sec) of the Gaussian input pulse. As \( \Delta \omega/2\pi \) increases to 0.5Hz, parts of the probe spectrum shifted from \( \omega_0 \) by \( \pm 0.5 \text{Hz} \) (the maximum gain and slowing region) become the primary frequency components which are amplified and delayed, thereby resulting in the beat frequency of 1Hz, corresponding to \( 2 \Delta \omega/2\pi \), as shown in Fig. 4(b). In Fig. 4(c), since the gain doublet is sufficiently separated so that the spectrum of the probe exists entirely within the anomalous dispersion region, the output is advanced by 0.28sec compared to the reference. As \( \Delta \omega/2\pi \) increases up to 2Hz, the anomalous dispersion becomes very small. Hence, it is observed that the output has virtually no advancement, as can be seen in Fig. 4(d)
5. Experimental dispersion measurement set-up and result

In these pulse propagation experiments, the dispersion was not measured directly, but rather inferred from the gain conditions and the group velocity. To measure the dispersion directly, we used a different experimental configuration, as shown in Fig 5, top. Briefly, the output of the frequency-double Nd:YAG laser at 532 nm was split in two parts. One part was frequency shifted by an AOM, operating at 40 MHz. The other part remained unshifted. The unshifted beam was split further into two parts: one denoted as pump 1 and the other as signal 1. Similarly, the shifted beam was split further into two parts: pump 2 and signal 2. Pump 2 (signal 2) was combined with pump 1 (signal 1) with a beamsplitter. The combined beam containing both pumps was applied to the crystal directly. The combined beam containing both signals was reflected by a mirror mounted on a PZT (Piezoelectric Transducer) before being applied to the crystal, with a relative angle of about fifteen degrees with respect to the combined-pumps beam.
A voltage varying quadratically with time was applied to the PZT. This moved the mirror with a velocity that varied linearly with time. As such, each of the signal beams reflecting off this mirror experienced a Doppler effect induced frequency shift which also varied linearly with time. Thus, the effect of the PZT scan was to tune the frequency of signal 1 (signal 2) around the frequency of pump 1 (pump 2). The profile of the scanning voltage was chosen to be so that the frequency difference between the signal and the pump varied from -0.5 Hz to 0.5 Hz.

Consider first the experiment when signal 2 was blocked, but pump 1, pump 2 and signal 1 were on. The dispersion experienced by signal 1 due to the presence of pump 1 was measured using the following approach. A 1 kHz modulation was added to the scanning voltage applied to the PZT, using the local oscillator from a lock-in amplifier. This produced sidebands which interfered with a part of the pump diffracted into the direction of the probe due to the induced grating, producing a beat note. The phase of this beat signal depends upon the phase of the probe beam, and thus upon the index of refraction it experiences while passing through the crystal. The beat note was demodulated with a reference signal, producing an output proportional to the phase of the probe beam. Scanning the frequency of the probe (via the quadratic voltage applied to the PZT) while recording this signal allowed us to map out the index of refraction of the material as a function of frequency.

The dispersion observed using this method is shown in the left trace in the bottom part of Fig. 5. As expected, the dispersion is normal at the center, and anomalous on the sides, corresponding to the gain of signal 1. Even though pump 2 is present in the system, the dispersion as seen by signal 1 here is due primarily to the effect of pump 1. The experiment was then repeated with signal 1 blocked, but pump 1, pump 2 and signal 2 were on. The resulting dispersion, due to the gain of signal 2 caused primarily by pump 2, is shown in the right trace in the bottom part of Fig. 5. The two traces, taken together, and separated by 40 MHz, can be interpreted to be the dispersion seen by a single signal beam when its frequency is scanned over the whole range. If pump 1 and pump 2 were closer together (e.g., with a frequency separation of 2 Hz), then the right edge of the left trace would merge into the left edge of the right trace, producing a linear but anomalous dispersion for a signal beam with its frequency in-between those of pump 1 and pump 2. Due to technical constraints, we were not
able to carry out this particular experiment with the frequencies of the two pumps so close to each other. In the experiment depicted in Fig. 3, the two pumps were angularly multiplexed. In contrast, in the experiment depicted in Fig. 5, the two pumps were superimposed on each other spatially. The latter approach may be more convenient for some applications.

As mentioned earlier, the anomalous dispersion demonstrated using photorefractive crystals in these two experiments implies that it should be possible to realize a White Light Cavity using such a medium. For application to gravitational wave detection, the wavelength of interest is 1064 nm, since it is used by the current embodiment of the LIGO interferometer, and high quality mirrors and detectors suited for LIGO have been developed at this wavelength. A crystal of SPS (Sn$_2$P$_2$S$_6$) or a quantum well material which is sensitive at this wavelength could be employed for this purpose. Furthermore, these materials have a much faster response time [11,12,13], which in turn means that the bandwidth of the negative dispersion regime can extend over a few kHz, which is optimal for detection of gravitational waves [5,7]

6. Conclusion

We have demonstrated the realization of negative dispersion and superluminal light propagation using dual-frequency pumped photorefractive crystals. The results show that that such a medium is a promising candidate for the design of a White light Cavity (WLC). We have previously demonstrated a WLC with the transmission bandwidth of approximately 10 MHz, using the anomalous dispersion between the Raman gain peaks of $^{85}$Rb atomic vapor. The desired bandwidth for LIGO applications is only a few kHz. The bandwidth of the crystal used in this experiment was on the order of 1Hz, which is too narrow for such an application. However, the bandwidth can be increased up to a few KHz by using a fast response material such as an SPS crystal or quantum wells.

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Superluminal Lasers: Dependence of Lasing Frequency on Group Index in an Active Cavity Above Threshold

H.N. Yum, M. Salit, K. Salit, M. S. Shahriar

Abstract:

We model the effect of anomalous dispersion in a laser cavity on the lasing frequency. We explore the application of this effect to improvement of Ring Laser Gyrosopes. We find that the sensitivity of the gyroscope depends on the variation of the steady-state index of refraction of the incavity medium (or media) as a function of lasing frequency, and that the factor by which the sensitivity is increased by the presence of anomalous dispersion is given by the group index of the materials.

Ring Laser Gyrosopes (RLGs) have been used in inertial navigation systems since the 1970s for everything from aerospace and military guidance systems to oil prospection. The basic principle is simple: the length of an optical cavity is perturbed under rotation by the Sagnac effect, so that the resonant frequency of the light with the cavity changes. The degree of this shift depends, as has been shown on the dispersion of the medium inside the cavity. In previous White Light Cavity (WLC) demonstration, it was realized by placing a fast-light medium inside a conventional cavity. The WLC becomes highly sensitive to mirror displacement and rotation. However, the concomitant broadening of the linewidth, by essentially the same factor, means that there is virtually no net improvement in the smallest measurable displacement or rotation. This constraint was eliminated if an active cavity is used, where the rotation (or any non-reciprocal change in the cavity length) is measured by monitoring the beat note between the counter-propagating lasing modes.

In this paper, we develop the theoretical model for WLC behavior of such an active resonator: a superluminal ring laser. Since the laser operates under the condition where the gain per pass balances the loss per pass, the conventional model for WLC does not apply. However, we show that if the gain profile is flat over the region of interest, with a dip in the middle, the lasing mode centered at the dip behaves like WLC, with its frequency becoming highly sensitive to mirror displacement or rotation. The enhancement factor can be as high as $10^7$, comparable to what is achievable in a passive cavity. We also present a physical interpretation of this behavior in terms of an effective Kramers-Kroenig relation, and show that the process works with both homogeneously or inhomogeneously broadened gain media.

First, we consider the light in a cavity which contains a dispersive medium. The resonant frequency can be described by:

$$\nu + \Phi = \Omega - \frac{1}{2} \nu \chi'(E, \nu)$$

(3.1)
where $\chi'$ is a real part of the susceptibility $\phi$ is a round-trip phase shift. $\nu$ is the resonant frequency of a medium-filled cavity, and $\Omega$ is that of the cavity in the absence of the medium so-called an empty cavity. $\Omega$ is equal to $2\pi cL/m$. The parameter $m$ is an integer and $c$ is the speed of light in free space. $L$ is the cavity length. Note $\chi'$ is itself frequency and intensity dependent. Let us assume that the resonance frequencies $\Omega$ and $\nu$ are both initially equal to $\nu_0$ when the cavity length is $L_0$. Let us further assume that the gain (or absorption) profile of the medium in the cavity is symmetric about $\nu_0$. For the sake of comparison, we define the normalized parameters, $\Delta = (\Omega - \nu_0)/\Gamma$, $\delta = (\nu - \nu_0)/\Gamma$ where $\Gamma$ is the linewidth of the absorption or gain line in radian. The derivatives $d\Delta/dL$ and $d\delta/dL$ represent the resonant frequency shifts induced by the perturbation of $L$ associated with the empty and the medium cavity, respectively. One can consider the ratio, $R = [d\delta/dL]/[d\Delta/dL]$ to determine if the amount of the frequency shift is enhanced ($R>1$) or diminished ($R<1$) by the dispersive intracavity medium. In steady state ($\phi = 0$), after dividing Eq(3.1) by $\Gamma$ and using $\nu\Gamma = \delta + \nu_0/\Gamma$, the differentiation of the resultant with respect to $L$ leads us to

$$\frac{d\delta}{dL} + \frac{1}{2} \frac{d\chi'}{dL} \nu + \frac{1}{2} \chi' \left( \frac{d\delta}{dL} \right)^2 = \frac{d}{dL} \left( \frac{\Omega - \nu_0}{\Gamma} \right) \quad (3.2)$$

By substituting $(d\chi'/d\delta)(d\delta/dL)$ for $d\chi'/dL$, the ratio $R$ can be expressed as

$$R = \frac{1}{1 + \frac{1}{2} \chi' + \frac{1}{2} \frac{d\chi'}{d\delta} \nu} \quad (3.3)$$

By applying $d\delta = 1/\Gamma \nu$ and $n_0 = \sqrt{1 + \chi'} = 1 + \chi'/2$, one can easily prove that the denominator in Eq.(3.3) is the group index, $n_g = 1 + \chi'/2 + [\nu/2 d\chi'/d\nu]$. For normal dispersion ($n_g > 1$) in active cavities, $R = n_g$ becomes less than one so that resonance frequency change induced by the length variation $dL$ is reduced. For $n_g < 1$, i.e. in the cavity filled with anomalous dispersion medium, the frequency shift is amplified to be equal to $1/n_g$ times the amount of the shift for the empty cavity. Eq.(3.3) can be applied to any intracavity medium. As such, if one can calculate the dispersion $d\chi'/d\nu$ of given medium, then the resonance frequency shift of the medium cavity can be predicted compared to that of the empty cavity. $\nu_0$ and $\Gamma$ are the medium property and thereby being independent from the length change $dL$. Eq. (3.3) can be modified as $(d\nu/dL)/(d\Omega/dL) = 1/n_g$. Recent study$^6$ showed $\Delta \nu = \Delta \nu_{ec}/n_g$ where the resonant frequency shift of a passive cavity $\Delta \nu$, an empty cavity $\Delta \nu_{ec}$. Now, note that the modified equation associated with an active cavity is consistent with the previous results. To interpret this result for the case of the active cavity, however, we must take into account the fact that the gain medium can be saturated. Indeed, the field inside a laser cavity increases until it saturates the gain medium such that the resulting gain exactly balances the loss within the cavity. This is the steady-state lasing condition. Field
amplitude and phase in the laser cavity are described by a set of self consistency equations. As such, the steady-steady gain condition can be expressed as:

\[
\dot{E} = -\frac{1}{2} \frac{\nu}{Q} E - \frac{1}{2} \nu E \chi''(E, \nu) = 0 \quad (3.4)
\]

where \( E \) is the laser field amplitude, \( \chi'' \) is an imaginary part of the susceptibility. \( Q \) represents loss characteristics of the cavity. It leads to

\[
\chi''(E, \nu) = -\frac{1}{Q} \quad \text{(for } E, \nu \neq 0) \quad (3.5)
\]

\( \chi'' \) implies the gain (or loss) characteristics and \( \chi' \) is relevant to refractive index. Note that \( \chi'' \) and \( \chi' \) depends on \( E \) as well as its frequency \( \nu \).

To understand the relationship between the gain saturation and the self-consistency equations, let us consider homogeneously broadened gain medium. Fig.3.1 illustrates the saturated gain corresponds to the cavity loss at steady state. We assume that a lasing mode frequency \( \nu \) exists within the gain profile but is apart from the gain center. The saturated gain \( G(\nu) \) and the non-saturated gain \( G_0(\nu) \) are expressed as \( G = \Delta N g(\nu) \) and \( G_0 = \Delta N_0 g(\nu) \), respectively where \( \Delta N \) is the population inversion in the presence of the lasing field, \( \Delta N_0 \) is the initial population difference without field and \( g(\nu) \) is a normalized Lorentzians lineshape function. Note that \( G \) and \( G_0 \) have the same

\[
\text{Fig. 1: Saturation of homogeneous gain profile to satisfy self-consistency equations at lasing frequency. Cold cavity represents a cavity before the gain reach to steady state. When the laser cavity reaches at steady state, the lasing beam experiences gain and index represented by red an blue points, respectively.}
\]

lineshape function. As such, the shape of profiles remains unchanged. The saturation effect by the lasing field is to deplete the population of the excited state i.e. reduction of \( \Delta N \). The magnitude of the homogeneous gain profile reduces but until the gain at the frequency of the lasing field becomes equal to the loss in the cavity. The question then becomes, what effect does this gain saturation have on the dispersion associated with the gain profile? One might at first think that the fact the steady-state gain is the same at every lasing frequency would mean that there could be no variation in the steady-state
index of refraction as a function of frequency. This is not the case, however. In a homogenously broadened medium, the saturation effect is simply to decrease the magnitude of $\chi'$ over the whole profile. Note that the frequencies for which $\chi'$ is plotted here are not in general resonant in the cavity and cannot be sustained in steady state. Only the single point at $\omega = \nu$, the lasing frequency, provides us information about the phase shift that would be seen by a resonating beam. However, if we draw this graph for a range of different choices of $\nu$, corresponding to different choices of cavity length, we will see that each choice determines a different index value at steady state. Thus, the index does indeed depend on lasing frequency. This variation is the primary reason that the resonant frequency of a laser cavity differs from that of a passive cavity of equal length, usually being closer to the center of the gain line, a phenomenon called mode-pulling.

Next, consider an inhomogeneously broadened gain medium. If the medium is inhomogeneously broadened, we may regard such inhomogeneous gain profile as the sum of many narrower, homogeneously broadened gain profiles individually centered at a different frequency. The saturation does not reduce the gain over the whole profile, but a localized frequency range around the lasing frequency. As such, the gain saturation occurs at the frequencies supported by the lasing mode. The lasing beam so-called hole burning laser bleaches a narrow band “hole” into the inhomogeneously broadened gain. This result is referred to as spectral hole burning. Next, let us imagine that a gain profile has four spectral holes in it separated approximately by the free spectral range (FSR) of the cavity. We assume that the inhomogeneous gain bandwidth is sufficiently broad so that it can sustain 4 cavity modes. The net gain can then be modeled as sum of a broad Lorentzian (or more accurately, a Voigt profile) and four narrow, inverted Lorentzians. The associated $\chi'$ at any frequency is then determined by the sum of the phase shift due to such a broad Lorentzian gain and that due to each of the narrow Lorentzian “absorption” features, as illustrated in Fig. 3.2.

Fig. 1: Saturation of inhomogeneous gain profile to satisfy self-consistency equations at lasing frequency or frequencies, and index profile corresponding to the saturated gain profile.

The phase associated with a hole is zero at the center of that hole, but there are two sources of the phase contributions. The first is the phase from the dispersion within the
absorption bandwidth of the hole. The second is phase from other burned holes at other frequencies. They may cause the phase to be different from that which would be associated with the broad gain alone. At any given frequency, in other words, the phase depends on the shape of the entire profile. Let us assume that the inhomogeneous gain support a single lasing mode. As such there is only one spectral hole. The dispersion at the lasing frequency will be unaffected by the saturation. As a result, the lasing beam experiences the single phase associated with the inhomogeneous gain. Again, the index of refraction depends on lasing frequency. This brief review is intended to make clear the distinction between the dispersion of a medium in free space, and the dependence on frequency of the steady-state refraction index in a laser cavity. Though related, they are not same. The group index to which we refer here is related to the velocity of groups composed entirely of lasing frequencies. Superluminal pulses of this type have in fact been observed in inhomogeneously broadened media.

We have established that the resonant frequency shift is induced by the change of the laser cavity length. It is inversely proportional to the group index of the material. Furthermore, we have shown that the dispersion inside a laser in steady state is not zero, though it does depend on the saturating field. We now ask how to achieve the dispersion profile we desire. Anomalous dispersion is required to maximize the dependence of lasing frequency on cavity length. The Kramers-Kronig relations tell us that anomalous dispersion is usually associated with an absorption feature, or equivalently, a reduction in gain. In this section, let us consider the case that the cavity contains a medium with a narrow absorption as well as a medium with a saturable and infinitely broad gain. This configuration creates the absorption dip in the broad gain, resulting in anomalous dispersion within the absorption bandwidth. We then express the net susceptibility of the media as a sum:

\[
\chi' = -\frac{N_i h\Omega}{\varepsilon_0 E^2} \left( \frac{2\Omega_i (\nu - \nu_0)}{2\Omega_i^2 + \Gamma_i^2 + 4(\nu - \nu_0)^2} \right) \quad (3.6a)
\]

\[
\chi'' = -\frac{N_e h\Omega}{\varepsilon_0 E^2} \left( \frac{\Omega_{ie} \Gamma_i}{2\Omega_i^2 + \Gamma_i^2} \right) + \frac{N_i h\Omega}{\varepsilon_0 E^2} \left( \frac{\Omega_i \Gamma_i}{2\Omega_i^2 + \Gamma_i^2 + 4(\nu - \nu_0)^2} \right) \quad (3.6b)
\]

Here \( \varepsilon_0 \) is the permittivity of free space. \( E \) is the field amplitude. \( h \) is plank’s constant. \( N_e \) and \( N_i \) represent the number of atoms per unit volume for the broadband gain medium and for the inserted narrow absorptive medium, respectively. We use the subscript “e” for the “envelope” gain profile and “i” for the narrower absorption profile, in order to remember that the absorption is “inside” the envelope of the gain. We assume \( \Gamma_i >> \Gamma_e \). \( \Gamma_e \) is the gain decay rate. \( \Gamma_i \) denotes the absorption linewidth. Note, \( \Gamma_e \) corresponds to \( \Gamma \) in Eq.(3.3). \( \nu_0 \) is the center frequency of the absorption line. The field with Rabi frequency \( \Omega_{ie} \) couples the background gain medium. The absorption medium is driven by the field with Rabi frequency \( \Omega_i \). \( \Omega_i \) and \( \Omega_{ie} \) are equal to \( \varphi_i E/h \) and \( \varphi_i E/h \), respectively, where \( \varphi_i \) and \( \varphi_e \) are the dipole moments associated with the media. We now define \( \xi \) and \( \xi_e \) such that \( \Omega_i^2 = \Gamma_i^2 E^2 \xi \) and \( \Omega_{ie}^2 = \Gamma_e^2 E^2 \xi_e \).
The field intensity inside the medium is given by \( I_{i, e} = \frac{E^2}{2\eta_{i, e}} \), where \( \eta_{i, e} \) is impedance. \( I_{i, e} \) can be rewritten as \( I_{i, e} = \Omega_{i, e}^2 \frac{h^2}{2\eta_{i, e}} \eta_{i, e}^2 \) by using the Rabi frequencies \( \Omega_{i, re} = \varphi_{i, e} E/h \). According to \( \Omega_{i, re}^2 = \Gamma_{i, e}^2 \) for the saturation, the saturation intensity is given by \( I_{\text{sat}, e} = \Gamma_{i, e}^2 \frac{h^2}{2\eta_{i, e}} \eta_{i, e}^2 \). As such, \( I_{i, e} / I_{\text{sat}, e} \) can be written as \( \Omega_{i, re}^2 / \Gamma_{i, e}^2 \).

Also, from \( I_{i, e} = \frac{E^2}{2\eta_{i, e}} \), we can derive \( I_{\text{sat}, e} = \frac{E^2}{2\eta_{i, e}} I_{\text{sat}, e} \). Using \( \Omega_{i, re} / \Gamma_{i, e} = \frac{E^2}{2\eta_{i, e}} I_{\text{sat}, e} \), one finds \( \xi_{i, e} = \frac{1}{2\eta_{i, e}} I_{\text{sat}, e} \). To understand physical characteristics of \( \xi_{i, e} \), note that according to \( I_{\text{sat}, e} = \Gamma_{i, e}^2 \frac{h^2}{2\eta_{i, e}} \eta_{i, e}^2 \), \( \eta_{i, e} I_{\text{sat}, e} \) consists of the medium-dependent parameters such as absorption bandwidth and dipole moments in the absence of the lasing frequency-dependent parameters. Therefore, one recognizes that \( \xi_{i, e} = \frac{1}{2\eta_{i, e}} I_{\text{sat}, e} \) is independent of the lasing frequency. For the sake of simplicity, we introduce \( G = N_e h\xi_{i, e} \Gamma_e / \varepsilon_0 \), \( H = N_i h\xi_{i, e} \Gamma_i / \varepsilon_0 \), \( \delta = (\nu - \nu_0) / \Gamma_i \) and \( B = 2E^2 \). Eq(3.6) is modified as

\[
\chi' = \frac{-2\delta H}{B^2 + 1 + 4\delta^2} \quad (3.7a)
\]

\[
\chi'' = -G \left( \frac{1}{B^2 + 1} \right) + H \left( \frac{1}{B^2 + 1 + 4\delta^2} \right) \quad (3.7b)
\]

We are now at the moment to evaluate the ratio \( R \) defined in Eq. 3.3. Before differentiating this \( \chi' \) with respect to \( \delta \), we should note that the steady-state field intensity inside the cavity is the frequency dependent function, i.e. \( B(\delta) \), so that the derivative of \( \chi' \) in Eq(3.7a) can be expressed as \( \delta \chi' / \delta \delta = \partial \chi' / \partial \delta + (\partial \chi' / \partial B) (\partial B / \partial \delta) \). \( \partial \chi' / \partial \delta, \partial \chi' / \partial B \) are assessed by partially differentiating Eq.(3.7a) with respect to \( \delta \) and \( B \), respectively. To calculate \( dB / \partial \delta \), we must find \( B \) as a function of \( \delta \). Let us recall the field amplitude equation in the laser cavity. In a steady state, it gives \( \chi'' = -1/Q \). Inserting \(-1/Q\) into \( \chi'' \) of Eq(3.7b) and modifying it, we find a quadratic equation for \( B \),

\[
\xi_e \xi_i B^2 + [\xi_e + \xi_i] (1 + 4\delta^2) - QG\xi_e + QH\xi_i B + 1 + 4\delta^2 - QG(1 + 4\delta^2) + QH = 0 \quad (3.8)
\]

Keeping in mind that the intensity is always a positive number, we can choose the solution for “\( B \)” which is positive over the lasing bandwidth. In Fig. 3.3. we can now plot the real part of the steady-state susceptibility as a function of frequency. For illustration, we consider \( Q = 3 \times 10^7 \), \( \Gamma_i = 10 \text{MHz} \) and \( \lambda_0 = 780 \text{nm} \), \( H = 5 \), \( G = 10 \) and \( \xi_i / \xi_e = 1.1333333938 \) (\( \lambda_0 = 2\pi c / \nu_0 \) where \( c \) is speed of light in vacuum)
Fig. 3. Real part of the steady-state susceptibility as a function of lasing frequency.
We are now able to find the partial derivatives $\partial \chi'/\partial \delta$, $\partial \chi'/\partial B$ from Eq(3.7a) and $B$ as function of $\delta$ from Eq(3.8).

The remaining procedure to calculate $R = [d\delta/dL]/[d\Delta/dL]$ is to express $\chi'$ and $d\chi'/d\delta$ in terms of $\delta$. $d\chi'/d\delta$ is obtained by differentiating $B$ with respect to $\delta$ and then inserting $\partial \chi'/\partial \delta$, $\partial \chi'/\partial B$, $dB/d\delta$ into $\partial \chi'/\partial \delta + (\partial \chi'/\partial B)(dB/d\delta)$. Substituting $B$ from Eq(3.8) into $\chi'$ in Eq(3.7a), one can find $\chi'$ as function $\delta$, as well. Inserting $\chi'(\delta)$, $d\chi'/d\delta$ into Eq(3.3), we obtain the ratio $R$ for the narrow absorption medium in the environment of the infinite broadband gain as a function of $\delta$. This $R$ is a measure of the degree to which the sensitivity of a laser cavity with these dispersive media is enhanced over that of an empty cavity. We note, however, the equation for $R$ is frequency dependent. This essentially tells us that once the lasing frequencies are outside of the anomalous dispersion regime, there is no enhancement of their sensitivity to length changes. We find, with the numbers above, the profile of $R$ gives a peak value of $11 \times 10^6$ and reduces to zero as leaving such a dispersion regime.

We find furthermore that the peak value of $R$, i.e. $R_p$ varies with the ratio $\xi_s/\xi_e$. Fig. 3.4 displays the normalized $R_{p,\text{nor}}$ by its maximum value for each case of the different H. For illustration, we use the same parameters as in Fig.3. We consider more H's corresponding the absorption dip. $R_{p,\text{nor}}$ changes very rapidly with $\xi_s/\xi_e$. Note, $\xi_s/\xi_e$ corresponds to $\Gamma_n^2 \varphi_s^2/\Gamma_e^2 \varphi_i^2$. Therefore, in order to significantly enhance the frequency shift compared to the empty cavity, one must carefully choose the media. Its $\varphi_s^2/\varphi_i^2$ can yield the appropriate $\xi_s/\xi_e$ for maximum $R_p$. Fig. 3.4 suggests that the optimal values of $\xi_s/\xi_e$ for maximum of $R_p$ varies with the depth of the absorption dip.
Fig. 4: Maxima of the ratio for different absorption depth. (red: H=2, green: 5, blue: H=8, black: H=10) as the function of $\xi / \xi_e$. The graphs are normalized by maximum for each case.

The variation of the absorption, “H,” also has the effect of shifting the frequency of the maximum sensitivity and altering the bandwidth of that sensitivity. Fig. 3.5 displays R for different absorption dip. For illustration, we adjust $\xi / \xi_e$ such that maximum R’s for H=2, 5, 8, 10 become equal to $1.1 \times 10^7$. The graph indicates that the peak of R is shifted from $\delta=0$, and the bandwidth of R broadens as increasing H from 2 to 10. We have carried out similar simulations for a wide variety of gain and absorption media, varying the bandwidth and depth of the gain and the “hole” in the gain, for homogeneous and inhomogeneously broadened media. The results in all cases support those we have seen in this presented example.

Fig. 5. R as function of $\delta$. $\xi / \xi_e$ is equal to 0.4533333530 for H=2 (red), 1.1333333938 for H=5 (green), 1.8133334374 for H=8 (blue), and 2.2666668013 for H=10 (black)

In conclusion, these results imply that we may achieve the same sensitivity in lasers, whether based on homogeneously or inhomogeneously broadened gain media,
which we have seen in passive cavities. Of course, an increased sensitivity is worthless if the noise is also increased. We will not go into detail here about the affects of the fast light medium on laser frequency noise. These details can be found in ref.16. However, we do wish to make one important point. The primary source of frequency noise for most lasers is mirror jitter, and we might expect that the fast light medium would increase the sensitivity of lasers to such jitter, thus, increasing the noise by the same factor as the sensitivity. However, laser gyroscopes do not involve direct frequency measurements, but rather measurements of the beat note between counter propagating modes, as illustrated in Ref. Error! Bookmark not defined. Mirror jitter, thermal effects, and other macroscopic noise sources affect both beams, and this common-mode noise cancels out, when one measures the beat frequency. The only noise on such a measurement is quantum noise, and Ref.16 shows that this is unaffected by the fast light material. Ultimately the details of real, physical systems will determine how well the approach works for gyroscope applications. These theoretical results, however, are sufficiently promising to justify significant further experimental investigation.
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Distortion free pulse delay system via a tunable bandwidth white light cavity

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Recently, a tunable bandwidth white light cavity (WLC) was demonstrated by using anomalously dispersive intracavity medium to adjust a cavity linewidth without the loss of a cavity buildup [1]. In this paper, we make use of such alterable bandwidth and maintainable buildup to design a distortion-free delay system for a data pulse. The system consists of physically identical two WLCs. Since the dynamic range of our system is associated with the linewidth of the WLCs, the system bandwidth is broadly tunable by controlling the parameters of the WLCs such as the first order and the third order terms in Taylor expansion of the dispersive index. Delay time is determined being independent of the system bandwidth such that our system offers a breakthrough to overcome a delay-time-bandwidth product problem in optical communication.

Due to significant applications to optical buffers, optical memories, the group index ($n_g$) has been controlled through manipulation of atomic resonances [2,3] or light interactions in non-linear materials such as Stimulated Brillouin scattering (SBS) in optical fibers [4–7], two-wave mixing in photorefractive crystal [8,9]. So far, in slow-light demonstration ($n_g > 1$), a light pulse was delayed less than or several times the pulse duration [2–4, 6–8]. SBS generated by Pulsed Brillouin pump in fibers [10] was used to store the optical pulse with the constraint on the storage time that should be less than the acoustic lifetime of nanoseconds in optical fibers. In stored-light method based on electromagnetically induced transparency (EIT), a photon wave packet is transferred to atomic coherence in collective atomic system so that the storage time corresponds to the coherence time of hyperfine transitions [11]. In low temperature environment, 20\,µs pulse was recorded as the coherence of the ground hyperfine states in Pr:Y$_2$SiO$_5$ for time scales of several seconds [12].

A material that is anomalously dispersive over a limited frequency range is known as a fast-light medium because it can support group velocities greater than the free-space speed of light. Optical cavities which contain such a fast light medium have a broadened linewidth without a reduction of the cavity buildup so-called White Light Cavity (WLC) [13]. It has been implemented in a cavity filled with rubidium vapor where Raman gain lines were produced around the cavity resonance to achieve the fast-light condition ($n_g < 1$). The pump intensity and the gain separation were controlled to adjust a negative dispersion slope so that the linewidth of the WLC was tunable. In the absence of the pumps, the WLC was converted to an ordinary cavity.

Utilizing such features of the WLC, in this paper we propose the details of a data pulse delay system. Due to the tunable bandwidth of the WLC, a data pulse is successfully delayed for sufficiently longer time than the pulse duration without distortion. Furthermore, the delay time is adjustable by switching the pumps to control WLC effect. Unlike the previous works [12], this system can be possibly realized by simpler experimental configurations in room temperature.

Before presenting a distortion free pulse delay system, we briefly review a theoretical model to explain the aspects of a WLC. Consider a tunable-bandwidth WLC. Fig.1 displays the WLC where two partial reflectors form into a typical Fabry-Perot cavity. For simplicity, the cavity is completely filled with a dispersive medium. In order to calculate the output of the WLC with a Gaussian data pulse, first, remind the transfer function analysis of a standard cavity [13] and
adapt the complex amplitude transfer function to the WLC. A monochromatic optical input wave, $E_\text{in} = E_0 e^{j\omega t}$, to a cavity with the intracavity medium can be related to an output field as

$$E_\text{out} = E_0 e^{j\omega t} \times \frac{e^{-jkL}}{1 - r^2 e^{-2jkL}} \quad (1)$$

where $E_0$ is the field amplitude, $L$ is the cavity length, $t$ and $r$ are the field amplitude transmission and the reflection coefficient, respectively. (For intensity, $R=r^2$, $T=t^2$ and $R+T=1$) and $\omega$ is an angular frequency of the optical field. $k$ is a wave number and is expressed as $k = \omega n/c$. Here, $n$ is the refractive index of the medium inside the cavity, and $c$ is speed of light in vacuum. For an empty cavity, $n$ is simply equal to the index of vacuum($=1$). For our case that the cavity is filled with the dispersive medium, $n$ is a function of $\omega$. Therefore, the wave number in the WLC can be modified as $cnk = \omega n$. $n(\omega)$ is expressed by a Taylor expansion around $\omega_0$ as $n(\omega) = n_0 + \frac{d}{d\omega}n|_{\omega=\omega_0} \cdot \omega_0 + \frac{d^2}{d\omega^2}n|_{\omega=\omega_0} \cdot \omega_0^2$. More specifically, it is equal to the center frequency of a gain doublet so that the medium obtains a negative dispersion asymmetrically around $\omega_0$ [1]. In the previous WLC experiment, the bi-frequency pumps provided two gain lines. The negative dispersion was created in the intermediate region between the two gains. $\omega_0$ was in the middle of the gain doublet so that the dispersion was asymmetric around $\omega_0$. Due to the asymmetrical dispersion, we can consider $n_2 = \frac{1}{2}d^2n/d\omega^2|_{\omega=\omega_0}$ as a null value and thus eliminate $n_2$ in the Taylor expansion of $n(\omega)$. A cavity response for the tunable-bandwidth WLC is obtained by inserting the Taylor expansion of $n(\omega)$ into $k$ of Eq(1). If the cavity represents a general system, then one can regard the cavity response as a transfer function of the system. Therefore, the transfer function of the WLC can be expressed as $H_{\text{WLC}}(\omega) = \frac{e^{-jkL}}{1 - r^2 e^{-2jkL}}$. 

Next, we extend a monochromatic input to an arbitrary pulse. Consider the Fourier transform of the pulse, $S(\omega)$. By convolution theorem, the output of a WLC is the inverse Fourier transform of the product of $S(\omega)$ and $H_{\text{WLC}}(\omega)$ and thus one can obtain the output intensity $|S_{\text{WLC}}(t)|^2$ where

$$S_{\text{WLC}}(t) = \int S(\omega)H_{\text{WLC}}(\omega)e^{j\omega t}d\omega.$$  

We choose the input pulse to be the form of $S(t) = \exp(-t^2/t_0^2)$, whose Fourier transform is written as $S(\omega) = t_0/\sqrt{2} \exp\left\{((\omega - \omega_0 - \xi)t_0)^2/4\right\}$. The carrier frequency of the pulse is upshifted as much as $\xi$ from the empty cavity resonance($\omega_0$). Likewise, for a reference pulse which propagates free space as much as the cavity length $L$, the resultant pulse after traveling is $S_{\text{free}}(t) = 1/\sqrt{2} \int S(\omega)H_{\text{free}}(\omega)e^{j\omega t}d\omega$ where a transfer function $H_{\text{free}}(\omega)$ is the phase change $\exp(-j\omega L/c)$ due to the propagation. Let us next consider the group velocity of the pulse in the WLC. The phase($\phi$) for an individual frequency wave after the propagation distance of $L$ can be written as
\[ \phi = \omega t - \frac{c n_{\text{eff}} L}{c} \quad (2) \]

where \( n_{\text{eff}} \) is the effective refractive index induced by the WLC. For simplicity, assuming that the pulse propagates the WLC without distortion, all frequency components are added in phase \[14\] at the exit of the WLC thereby \( d\phi/d\omega = 0 \). We can define the group velocity as \( v_g = L/t \). After differentiating Eq(2) with respect \( \omega \), \( L/t(=v_g) \) can be expressed as \( c/[n_{\text{eff}} + \omega(dn_{\text{eff}}/d\omega)] \). From \( v_g = c/[n_{\text{eff}} + \omega(dn_{\text{eff}}/d\omega)] \), we obtain the group index as \( n_g = n_{\text{eff}} + \omega(dn_{\text{eff}}/d\omega) \). Next consider \( n_g \) in terms of the phase angle of WLC.H.

If the complex transfer function \( H_{\text{WLC}} \) is written as \( |H_{\text{WLC}}| \exp(j\angle H_{\text{WLC}}) \) in terms of the amplitude \( |H_{\text{WLC}}| \) and the phase angle \( \angle H_{\text{WLC}} \), the output of the WLC in frequency domain can be written as \( S_{\text{WLC}}(\omega) = |H_{\text{WLC}}| \exp(j\angle H_{\text{WLC}})S(\omega) \). One can note that \( \angle H_{\text{WLC}}(\omega) \) is the phase resulting from the propagation inside the WLC thereby being equal to the second term in Eq(2).

In the treatment of pulse distortion, it is convenient to consider the group index dispersion i.e. group velocity dispersion. The degree of pulse stretch (or compression) in time domain after propagation through a medium of length \( L \) is given by \( \Delta T = (L/c)(dn_g/d\omega)\Delta \omega \) where \( \Delta \omega \) is that pulse bandwidth \[14\]. By the insertion of Eq(3), we obtain \( \Delta T = -(d^2\angle H_{\text{WLC}}/d\omega^2)\Delta \omega \) and thus confirm that the pulse maintains its original shape after propagation on the condition that its spectrum belongs to the spectral region where \( d^2\angle H_{\text{WLC}}/d\omega^2 = 0 \).

Fig.2~3 display the transfer functions \( (|H_{\text{WLC}}|^2, |H_{\text{EC}}|^2) \), the phases \( (\angle H_{\text{WLC}}, \angle H_{\text{EC}}) \), the frequency spectrum of the input pulse \( |S|^2 \) and the output pulses \( (|S_{\text{WLC}}|^2, |S_{\text{ref}}|^2) \). For illustration, we choose the parameters as cavity length \( L = 5 \text{cm} \), Finesse=999\((R=0.999)\), and Full Width Half Maximum (FWHM)=2.9MHz. Assuming \( n_3=0 \), \( n_1 \) is adjusted to satisfy the ideal WLC condition \[1\] where the linewidth of a WLC is infinite and the group index \( n_g \) is 0 for the cavity completely filled with a medium. Next, we discard the assumption and consider the case that the dephasing away from \( \omega_0 \) due to \( n_3 \) results in a cavity response drop, i.e. a finite WLC linewidth. For the Gaussian input pulse, the width is \( \Delta \nu_{\text{pulse}} = 29 \text{MHz} \) \((t_0=34\text{ns})\) and the carrier frequency is shifted from \( \omega_0 \) by \( \xi = 1.5 \times \Delta \nu_{\text{pulse}} \). Therefore, the frequency spectrum of the pulse is sufficiently separated from the spectral region of an empty cavity resonance. Later, we will discuss about the necessity for the pulse to be shifted in our delay system.

Fig.2(a) indicates that for a WLC associated with \( n_3 = 5.223 \times 10^{-35} \), the pulse spectrum \( (S(\omega)) \) belongs to the spectral region where the constant cavity response \( (|H_{\text{WLC}}|^2=1) \) ends and subsequently begins to reduce. Since the amplitude of the transfer function decreases with frequency variation, we can observe that the intensity of output pulse (red) is reduced in Fig.2(c).

Fig.2(b) illustrates \( d\angle H_{\text{WLC}}/d\omega < 0 \) in the pulse spectrum and thus \( n_g > 1 \). Due to \( v_g < c \), the pulse slows down compared to the reference which propagates free space as much as \( L \). With the consideration of \( d^2\angle H_{\text{WLC}}/d\omega^2 \), the pulse spectrum partially exists in the spectral region of \( d^2\angle H_{\text{WLC}}/d\omega^2 \neq 0 \) and thus one can observe the pulse compression in Fig.2(c).

Fig.3(a) suggests that the WLC associated with \( n_3 = 1.723 \times 10^{-36} \) resonates over broader spectral
range than the WLC of $n_3=5.223\times10^{-35}$. As indicated in Fig.3(b), therefore, the input pulse spectrum mostly includes in the spectral region of WLC where $\frac{d^2 H_{\text{WLC}}}{d\omega} = 0$ and $\frac{d^2 H_{\text{WLC}}}{d\omega^2} = 0$, minor frequency components are in $\frac{d^2 H_{\text{WLC}}}{d\omega^2} \neq 0$. Hence, in the insets of Fig.3(c) it is observed that $S_{\text{WLC}}(t)$ exhibits no time delay or advancement and but the minor pulse compression.

In order to design a pulse delay system, we utilize the characteristics of the WLC in Fig. 3 and choose the shifted Gaussian pulse $S(\omega) = t_0/\sqrt{2} \exp\left[\left((\omega - \omega_0 - \xi)t_0\right)^2/4\right]$ as an input pulse. Fig. 4 displays that the proposed system consists of two WLCs. The intermediate region between the WLCs is surrounded by the partial reflector (PR) on right hand side(RHS) of the left WLC(LWLC) and the PR on left hand side(LHS) of the right WLC(RWLC).

In the previous WLC demonstration, the interaction of bi-frequency pumps with the intracavity medium created a negative dispersion. With the pumps off, the dispersion vanished and the WLC was converted to an ordinary cavity. In the system of Fig.4 one can turn off the pumps to eliminate WLC effect on LWLC(or RWLC). Without WLC effect, the WLCs’ transfer functions are equal to that of an empty cavity as illustrated in Fig.3(a). The frequency separation of the two pumps and the intensity of the pumps are parameters which determine the slope of the dispersion. One can control the parameters to change $n_1$, $n_3$ and thus to manipulate the linewidth of LWLC(or RWLC). We use such WLC characteristics in the operation of the delay system. Here, consider an appropriate operation scheme to delay the pulse without distortion. Imagine a Gaussian data pulse whose bandwidth is broader than the linewidth of RWLC without WLC effect and the carrier frequency is shifted by $1.5\times\Delta\nu_{\text{pulse}}$ from the cavity resonance ($\omega_0$). Before the pulse enters into RWLC, one activates WLC effect. As illustrated in $S_\text{cavity}(t)$ & $S_\text{free}(t)$ of Fig.3(c), the pulse appears at the exit of LWLC without delay and any distortion compared to a reference pulse which propagates in free space. We assume that the intermediate zone between LWLC and RWLC is long enough to spatially confine the data pulse. Afterward, the pulse propagates in the middle of the two WLCs and reaches at the LHS PR of RWLC. Remind that RWLC in the absence of WLC effect converts to an empty cavity. The pulse is sufficiently shifted from the resonance frequency of $\omega_0$ so that the frequency spectrum of the pulse exists out of the FWHM($\Delta\nu_{\text{FWHM}}=2.9\text{MHz}$) of the empty cavity as illustrated in Fig.3(a). In such physical circumstances, therefore, RWLC is a simple reflector with the reflectivity of R and thus the pulse is reflected with minor loss due to the transmission of the PR. Likewise, before the reflected pulse returns to the RHS PR of LWLC, we deactivate WLC effect so that the pulse is reflected. It will travel multiple round trips between the WLCs until we activate WLC effect on RWLC. As soon as RWLC is activated, due to the wide linewidth of RWLC, the pulse passes through RWLC without frequency component loss and thus being free from distortion. Hence, the signal will be delayed as much as time elapse inside the intermediate zone, compared to the reference. Next, let us find a transfer function for the entire system which is composed of LWLC, RWLC and the region in the middle of the two WLCs. First, consider a transfer function in the intermediate zone. As illustrated in Fig.5, a monochromatic wave begins to propagate from RHS PR of LWLC and arrives at LHS PR of RWLC after N round trips. The wave before passing through the LHS PR is written as $E_0 e^{j\omega t} R^N e^{-j\Delta\nu_{\text{FWHM}}2N+1L_2/c}$ where $L_2$ is distance between the two PRs. Since $E_0 e^{j\omega t}$ represents the input wave, one can write the transfer function as

$$H_i(\omega) = R^N e^{-j\Delta\nu_{\text{FWHM}}2N+1L_2/c} \quad (4)$$
Since $H_{\text{WLC}}(\omega)$ describes the response of the WLCs, Eq(4) together with $H_{\text{WLC}}(\omega)$ leads us to a transfer function $H_{\text{total}}(\omega)$ of the pulse delay system in Fig. 4. $H_{\text{total}}(\omega)$ can be expressed as

$$H_{\text{total}}(\omega) = H_L(\omega)H_R(\omega)$$

where $H_R(\omega)$ and $H_L(\omega)$ are the transfer functions of RWLC and LWLC, respectively. Remember that the WLCs are identical thereby $H_R(\omega) = H_L(\omega) = H_{\text{WLC}}(\omega)$. Again, by convolution theorem, the output of the delay system can be written as

$$S_{\text{system}}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H_{\text{total}}(\omega)S(\omega)\exp(i\omega t)\,d\omega \quad (5)$$

Likewise, the reference pulse propagates as much as the length of the system and thus $S_{\text{free}}(\omega)$ is

$$S_{\text{free}}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S(\omega)H_{\text{free}}(\omega)\exp(i\omega t)\,d\omega$$

where

$$H_{\text{free}}(\omega) = \exp(-\omega^2(2L + L_2)/c)$$

Fig.6 graphically illustrates $S_{\text{free}}(t)$ and $S_{\text{system}}(t)$. Remind that we choose the input pulse to be same as the Gaussian pulse in Fig.3(a) and use the WLC in Fig.3(b) for RWLC and LWLC. For illustration, we consider $L_2=2557m$. The numerical simulation suggests that for one round trip($N=1$), the data pulse is delayed as much as $2L_2/c = 1.7 \times 10^{-5}$ sec. For one hundred round trips($N=100$), the delay time is observed as $200L_2/c = 1.7 \times 10^{-3}$ sec with an intensity attenuation due to the transmission of the PRs. It is equal to approximately 5000 times input pulse duration ($= 1.7 \times 10^{-3}/t_0 = 5000$). The bandwidth of a delay system can be defined as the maximum frequency spectrum width of a data pulse that the system can delay without distortion. One can note that $\Delta \nu_{\text{FWHM}}$ of WLC is an important parameter to determine our system bandwidth and it increases up to 340MHz in the presence of the WLC effect as shown in Fig.3(a). The data pulse of $\Delta \nu_{\text{pulse}}=29$MHz is successfully loaded to the intermediate zone through LWLC. The carrier frequency of the pulse is deviated from the empty cavity resonance as much as $1.5 \times \Delta \nu_{\text{pulse}} = 33.5$MHz. Since $\Delta \nu_{\text{FWHM}}$ of the WLCs in the absence of WLC effect is 2.9MHz, the pulse is non-resonant with the WLCs. Hence, loading to the intermediate zone and subsequently turning off LWLC, the pulse can be spatially confined inside the zone to delay until we activate RWLC. Since one can either downshift or upshift an input data pulse from the empty cavity resonance, it is noted from $H_{\text{WLC}}(\omega)$ in Fig.3(a) that the dynamic range of our system corresponds to two spectral regions divided by the resonance peak of $\Delta \nu_{\text{FWHM}} = 2.9$MHz. Therefore, our system bandwidth $\Delta \nu_{\text{system}}$ is approximately equal to $(340-2.9)/2=168$MHz. Due to the tunable WLC, it is easily expandable and thus one can obtain wider bandwidth than the current system. Another remarkable feature is that the delay time is independent parameter of $\Delta \nu_{\text{system}}$ and simply corresponds to time elapse inside the region between LWLC and RWLC. Therefore, our system provides a solution to the issue related to delay time-bandwidth product.

In conclusion, we have proposed that a WLC can be used to design an optical pulse delay system. The bandwidth $\Delta \nu(1/t_0=29$MHz) of a Gaussian data pulse ($= \exp(-t^2/t_0^2)$) is broader than the FWHM(2.9MHz) of the cavities in the system. However, the pulse was successfully loaded to the system without distortion by WLC effect. It was delayed being compared to a freespace propagating reference much greater than the input pulse duration. Our system bandwidth was determined to be equal to the half of the WLC linewidth thereby being tunable. Since the system bandwidth can be easily expanded broader than the frequency spectrum of the data pulse, an input data pulse can be arbitrarily chosen. Furthermore, the system delay time is
adjustable by switching WLC effect. The delay time can be augmented as much as required regardless of $\Delta \nu_{\text{system}}$, and thus our system is not constraint to the time bandwidth product. This represents the improvement over standard optical delay devices and is important step toward solving the delay time-bandwidth problem in such devices.
Fig. 1 Schematic of a tunable-bandwidth WLC; Two partial reflectors enclose the intracavity medium and form into the cavity with reflectivity $R$ and length $L$. 

Fig. 2 (a) Transfer functions for empty cavity (blue) and for WLC (red), and the Fourier Transform of Gaussian input (green) (b) Phase of $H(\omega)$ for empty cavity (blue) and for WLC (red). (c) $|S_{\text{free}}|^2$ (blue) and $|S_{\text{WLC}}|^2$ (red). The parameters of the intracavity medium are $n_1 = -8.223 \times 10^{-16}/\text{rad}$, $n_3 = 5.223 \times 10^{-35}/\text{rad}^3$

Fig. 3 For the medium with $n_1 = -8.223 \times 10^{-16}/\text{rad}$, $n_3 = 1.723 \times 10^{-36}/\text{rad}^3$ (a) Transfer functions for empty cavity (blue) and for WLC (red), and the Fourier Transform of Gaussian input (green) (b) Phase of $H(\omega)$ for empty cavity (blue) and for WLC (red). (c) $|S_{\text{free}}|^2$ (blue) and $|S_{\text{WLC}}|^2$ (red).
Fig. 4 Diagram of the proposed pulse delay system. Two identical WLC is separated by $L_2$.

Fig. 5 Monochromatic wave travels $N$ round trips between two reflectors.

Fig. 6 A series of pulses in time domain. At $t=0$, the reference and the data pulse are launched at the entrance of the RWLC. Blue is the reference pulse ($S_{\text{free}}(t)$). It propagates the optical path of $2L+L_2$ in free space and the center of the pulse appears at $t = (2L + L_2)/c \approx 8.55 \times 10^{-6}$ second. The data pulses is observed at $t = (2L + 3 \times L_2)/c \approx 2.56 \times 10^{-5}$ second for one round trip ($N=1$) and $t = (2L + 201 \times L_2)/c \approx 1.71 \times 10^{-3}$ second for $N=100$. 
References


Pulse delay via tunable white light cavity based on fiber optics

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Previously, we proposed a data buffering system free from a delay-time-bandwidth product problem in optical communication [1]. Since it is hard to build its optical configuration in free space, however, in this paper we design a delay system based on fiber ring resonators. The fiber-optic system retains the characteristics of the former buffer system where the delay time was adjustable independently from its bandwidth. In addition, we expand the achievable linewidth of a fiber optic White Light Cavity (WLC) and thus improve the dynamic range of the system up to several times Brillouin frequency of a fiber in use.

Slow- and fast-light demonstration in optical fibers has been attracted due to applicability to current optical devices such as to optical buffers, optical delay lines and fast memory access which are necessary for fiber optic communication system[2~5]. To delay a data pulse, slow light via the interaction of the pulse with a single Brillouin pump was a fundamental method. However, pulse delay time accomplished by such previous techniques was less than the pulse duration which is short delay from the view of practical points. A data pulse was stored by the writing pulse (Pulsed Brillouin pump) as an acoustic excitation and retrieved by the read-out pulse converting such excitation to the original data pulse [6]. The storage time was several times data pulse duration. Since the date was stored in the form of the decaying acoustic disturbance, however, the retrieved pulse was decayed and thus the storage time in this scheme was limited by the acoustic lifetime. Paradoxically, fast-light technique, which is based on negative dispersion, was proposed to build a data buffering system [1]. In a so called White light cavity (WLC), a cavity contains fast-light medium whose negative dispersion compensates for wavelength change. It resonates broader spectral range compared to an empty cavity of equal length and finesse without loss of cavity build-up [7]. The buffering system was composed of two WLCs in the version of Fabry-Perot (FP) and the intermediate delay zone. In theoretical analysis, a data pulse was delayed several thousands times pulse duration with minor attenuation. However, the intermediate zone was required to be sufficiently long for such a long delay time so that it is hard to be constructed in free space optics.

Coupling characteristics between a ring resonator and a waveguide has been an important research area in optical communication [8~12]. Their physical aspects such as power transfer and phase are consistent with that of a fiber ring resonator [13~16]. Its coupling coefficient is represented by the transmission in FP while transmission coefficient corresponds to the reflection. In fast-light demonstration based on Stimulus Brillouin Scattering (SBS), negative dispersion was produced by bi-frequency Brillouin pumps in optical fibers. Therefore, we can expect that the combination of the fiber resonator with the fast-light technique possibly provides a solution to created WLC effect in fiber resonator.

For a more practical application to existing telecommunication, in this paper we design a fiber white light cavity and employ it to build a fiber-based data buffering system. Another noticeable feature of the system is that the broadband negative dispersion (GHz) is obtained by using two broadened gains and thus we can extend the dynamic range to multiple times Brillouin frequency ($\nu_B$: 8~12GHz [17, 18]).

First, let us review an optical fiber resonator [13, 14] in Fig.1(a). We are interested in the field amplitude transfer characteristics $b_2/a_1$ of this configuration. On the condition that a 2 by 2 coupler is internally lossless, the relationship between the complex amplitudes $a_i$ and $b_i$.
can be described by the matrix for a 2 by 2 coupler

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} =
\begin{bmatrix}
  \sqrt{1-k} & j\sqrt{k} \\
  j\sqrt{k} & \sqrt{1-k}
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix}
\] (1)

where \( k \) is the intensity coupling coefficient. In addition, the transmission within the resonator can be described as \( a_2 = \alpha e^{\theta} b_2 \) in terms of the internal transmission factor \( \alpha \) and the round trip phase shift \( \theta \). For \( \alpha = 1 \), the coupled field \( a_2 \) travels one round trip without attenuation. \( \theta \) is written as \(-\omega n(\omega)L/c\) where \( L \) is the circumference of the ring resonator. \( n(\omega) \) is the refractive index of the fiber. In order to explain dispersion induced by SBS, it is expressed by a Taylor expansion about the cavity resonance frequency \( \omega_0 \). \( \omega_0 \) is close to the stokes frequency \( \omega - \nu_B \) which downshifted from Brillouin pump \( \omega \) as much as the Brillouin frequency \( \omega - \nu_B \) of the fiber. Using Eq.(1) together with \( a_2 = \alpha e^{\theta} b_2 \), we can derive the ratio of the complex transmission amplitude to the input,

\[
b_2 = \frac{\sqrt{1-k} - \alpha e^{\theta}}{1 - \alpha \sqrt{1-k} e^{\theta}} \] (2)

Likewise, the input and the circulating field are related by

\[
a_2 = \frac{\alpha \sqrt{k} e^{\theta}/\alpha}{1 - \alpha \sqrt{1-k} e^{\theta}} \] (3)

Next, we consider a fiber resonator coupled to a fiber. Fig. 1(b) displays that the input port of a second coupler is connected to that of the first coupler to construct a semicircle and the closed loop is formed by likewise connecting the output ports. It can be regarded as the microresonator coupled to a second waveguide in Ref.[8] and will be used as a building block for our fiber-based data buffering system. In the presence of the second coupler, we define the intensity coupling coefficients \( k_1 \) and \( k_2 \) (the subscripts ‘1’ and ‘2’ represent ‘first’ and ‘second’ couplers, respectively) and modify the internal transmission from \( \alpha \) to \( \alpha \sqrt{1-k_2} \).

Substituting \( k_1 \) and \( \alpha \sqrt{1-k_2} \) into \( k \) and \( \alpha \) in Eq(3) respectively, the ratio of the circulating amplitude to the input in the new configuration is written as

\[
a_2 = \frac{i\alpha \sqrt{k_1} e^{\theta}/\alpha}{1 - \alpha \sqrt{1-k_2} e^{\theta}} \] (4)

While the transmitted amplitude is related to \( a_1 \),

\[
b_1 = \frac{\sqrt{1-k_1} - \alpha \sqrt{1-k_2} e^{\theta}}{1 - \alpha \sqrt{1-k_1} e^{\theta}} \] (5)

Likewise, we also change from \( a_2 = \alpha e^{\theta} b_2 \) to \( a_2 = \alpha \sqrt{1-k_2} e^{\theta} b_2 \). With the consideration of the power transfer from \( b_2 \) to \( a_1 \) by the second coupler, we write \( a_1 \),

\[a_1 = \sqrt{\alpha_k} \sqrt{k_2} e^{\theta} b_2 \] (6)

Inserting \( a_2 = \alpha \sqrt{1-k_2} e^{\theta} b_2 \) into Eq.(4) and substituting the resulting \( b_2 \) from such insertion into Eq.(6), one can obtain \( a_2/a_1 \),

\[
\frac{a_2}{a_1} = \frac{-\sqrt{\alpha \sqrt{k_1} \sqrt{k_2}} e^{\theta}}{1 - \alpha \sqrt{1-k_1} \sqrt{1-k_2} e^{\theta}} \] (7)

where the round trip phase shift(\( \theta \)) is expressed as \(-\omega n(\omega)L/c\) and \( L \) is the circumference of the closed loop. Dispersion in the fiber loop resulting from SBS can be considered by using the Taylor expansion \( n(\omega) \) as \( \omega_0 \) as \( n(\omega) = n_0 + (\omega - \omega_0)n_1 + (\omega - \omega_0)^3 n_3 \),
\[ n_i = \frac{dn}{d\omega} \bigg|_{\omega=\omega_0}, \quad n_3 = \left(\frac{1}{6}\right)\frac{dn^3}{d\omega^3} \bigg|_{\omega=\omega_0} \] where \( n_0 \) is the mean index of the fiber, and \( \omega_0 \) is the cavity resonance frequency.

Fig.2 displays \((|b_1|^2/|a_1|^2)\) and \((|a_1|^2/|a_1|^2)\) in the absence of dispersion (blue line, \( n_1=0, n_3=0 \)) and in the presence of negative dispersion (red line, \( n_1<0, n_3\neq0 \)). We consider \( |a_1|=1 \) so that are \( |b_1|^2 \) and \( |a_2|^2 \) are normalized to the input and assume that the internal loss is negligible \((\alpha=1)\). We choose \( n_1 \) carefully so that regardless of the frequency change the cavity resonance occurs i.e. ideal White Light Cavity (WLC) Condition. For our case that the length of the dispersive medium is equal to the cavity circumference, the ideal WLC condition is that \( n_3=0 \) where \( n_g \) is the group index of the dispersive fiber [7]. Next, \( n_3 \) is adjusted to reduce the infinite linewidth of the ideal WLC to finite value. For other parameters, we choose \( k_1=k_2=0.01, \quad \ell = l \)m, \( n_0=1.45 \) and \( L=10m \) where \( L \) is the circumference of the closed loop. For the ordinary ring resonator \((n(\omega)=n_0)\), at resonance the input completely transfer to the output. For the ring cavity with the WLC effect, the resonance spectral region is expanded so that full power of the input transfers to the output over broader frequency range than the ordinary resonator.

Next, we take into account Eq.(5) and Eq.(7) as the transfer functions of the configuration in Fig.1(b). \( H_{b1,a1} \) corresponds to Eq.(5). \( H_O \) and \( H_{WLC} \) are Eq.(7) with \( n(\omega)=n_0 \) and with \( n(\omega)=n_0+(\omega-\omega_0)n_1+(\omega-\omega_0)^3n_3 \), respectively. For \( H_{WLC} \), we choose \( n_1, n_3 \) to be equal to the values used in Fig.2. In previous Fabry-Perot (FP) analysis, we derived the group index \((n_g)\) for a pulse propagating through FP configuration in term of \( \angle H_{WLC} \). Now, we are interested in \( n_g \) associated with the configurations in Fig.1. \( \angle H(\omega) \) is the ring resonator contribution to the phase (the subscripts ‘i’ denotes O or WLC or b1,a1) and thus the phase shift associated with the propagation from the inputs to the outputs of the resonators is expressed by \( \angle (n_{eff}\ell)/c = (n_0\omega\ell)/c - \angle H_i \) in terms of the mean index of the fiber \( n_0 \) and the effective refractive index of the resonators \( n_{eff} \). Deriving \( n_{eff} \) as a function of \( \angle H_i \) and inserting \( n_{eff} \) into \( n_g = n_{eff} + \omega(\frac{dn_{eff}}{d\omega}) \), one can obtain \( n_g \) as

\[ n_g = n_0 - \frac{c}{\ell} \frac{d\angle H_i}{d\omega} \quad (8) \]

Pulse distortion is characterized by \( \Delta T \approx -\frac{d^2\angle H_i}{d\omega^2}\Delta \omega \) as done in FP analysis [1].

Fig.3(a) exhibits \( \angle H_{b1,a1}, \angle H_O \) and \( \angle H_{WLC} \). Fig.3(b) presents the pulse output after propagating distance \( \ell \) through free space(black) and the outputs of the systems associated with \( H_O \)(blue) and \( H_{WLC} \)(red). The cavity parameters are the same as used in Fig.2. For illustration, we choose the input to be Gaussian pulse in the form of \( S(t)=\exp(-t^2/t_0^2)\exp(j\omega_0 t) \) and the pulse bandwidth \( \Delta \nu_{pulse} (\approx 1/t_0) \) is 10 times the Full Width Half Maximum (FWHM) of the resonator \( \Delta \nu_{cavity} \). By convolution theorem, we obtain the output amplitude as \( S_{out}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S(\omega)\exp(-jk_0t)H_i(\omega)\exp(j\omega t)d\omega \) where \( k_0 = n_0\omega\ell/c \). Since the amplitude transmission factor \( \alpha_1=\sqrt{1-k_0^2} \) is equal to the transmission coefficient of the 1st coupler \( \sqrt{1-k_1} \), the system can be regarded as a critically coupled fiber ring cavity. Therefore, \( \angle H_{b1,a1} \) is discontinuous at resonance and leaps as much as \( \pi \). Such discontinuity and phase jump correspond to the case of a critically coupled microresonator as indicated in ref.9 (Since we define the phase shift as \( \phi = \omega t - \omega_0 n_{eff}\ell/c \) but ref.9 uses \( \phi = \omega n_{eff}\ell/c - \omega t \), there is sign difference.) Due to the phase shift \( \pi \) resulting from the second coupler, at resonance \( \angle H_O \) and \( \angle H_{WLC} \) jump \( \pi \) more than \( \angle H_{b1,a1} \) thereby the
phase leap being $2\pi$. $\angle H_0$ indicates $n_0 > n_0$ inside the dotted circle. Since the pulse carrier frequency $\omega_0$ corresponds to the cavity resonance, and $\Delta \nu_{\text{pulse}} = 10 \times \Delta \nu_{\text{cavity}}$, its spectrum completely covers such slow light zone $\{d\angle H_0/d\omega < 0, d^2\angle H_0/d\omega^2 \neq 0\}$ in $\angle H_0$ and mainly belongs to the spectral region of $|H_0| = 0$. The pulse output associated with $H_0$ is delayed and seriously distorted (attenuated) as displayed in Fig.3(b). $\angle H_{\text{WLC}}$ in Fig.3(a) suggests that due to the negative dispersion the system resonance $\{H_{\text{WLC}} = 1, d\angle H_{\text{WLC}} /d\omega = 0, d^2\angle H_{\text{WLC}} /d\omega^2 = 0\}$ occurs sufficiently broad to cover the pulse spectrum. The output (red) associated with $H_{\text{WLC}}$ in Fig 3(b) indicates the pulse propagates without time advancement or delay maintaining its original shape.

Now, we employ the fiber ring resonator investigated in Fig. 1~3 to design a fiber-based buffering system. In the previously proposed system, the FP based buffering system is composed of Right White Light Cavity (RWLC), intermediate delay zone and Left White Light Cavity (LWLC). Fig 4 displays in our fiber-based system. Two ring resonators serve as RWLC and LWLC. The ports at right side of the right ring resonator (R) are connected to the ports at left side of the left ring resonator (L) through the fiber spools to construct the closed loop delay zone. Let us assume that the bandwidth of the input pulse is much broader than FWHM of the resonators and consider the operating scheme to delay such wide pulse without distortion. At the stage of pulse loading to the intermediate delay loop, we turn on bi-frequency Brillouin pumps to activate WLC effect on the right resonator. The WLC bandwidth is sufficiently wide to resonate over the pulse spectrum and thus the pulse is successfully loaded remaining its original shape as indicated in Fig 3(b). After loading, we turn off the pumps so that the pulse is trapped and circulates inside the delay zone. At this trapping step, it is necessary to consider the constraint on the carrier frequency of the pulse. If the carrier frequency corresponds to the resonance ($\omega_0$) of the resonators without WLC effect, the major spectral components of pulse spectrum around $\omega_0$ are located inside the cavity transmission in Fig.2(b). At every passage through the couplers inside the delay loop, such components leak out through $\text{R}$ and $\text{L}$ causing pulse distortion. To avoid such spectrum loss, we shift the carrier frequency from the resonance by several times $\Delta \nu_{\text{cavity}}$ so that in the absence of WLC effect, the pulse circulates the loop with minor loss associated with the coupling coefficients $\sqrt{k_1}, \sqrt{k_2}$. Activating WLC effect on $\text{L}$, we can extract the data pulse from the delay loop. Let us assume that $\text{R}$ and $\text{L}$ has same physical parameters ($\sqrt{k_1}, \sqrt{k_2}$, $\alpha$ and $\Delta \nu_{\text{cavity}}$). The amplitude transfers through the resonators ($H_{1t} = a/a_1, H_{1t'} = a'/a'$) are given by Eq(7). Next, after N multiple round trip ar is related to $a'$ by

$$H_{1t'} = \left(\sqrt{1-k_1}, \sqrt{1-k_2}\right)^N \alpha^\frac{-jk_0^2N+1}{2}$$

(9)

where $L_2$ is the delay zone length, $k_0 = \omega_0n_0/c$. Fig. 5 displays data pulse outputs after propagating distance $\ell$ (reference), 5 round trips and 10 round trips. For illustration, we choose a Gaussian pulse with the shifted carrier frequency in the form of $S(\omega) = t_0/\sqrt{2\pi} \exp\left[(\omega - \omega_0 - \xi)t_0^2/4\right]$ where $\xi = 1.5 \times \Delta \nu_{\text{pulse}}$. For reference after propagating $\ell$, $S_{\text{out}}(t) = 1/\sqrt{2\pi} \int_{-\infty}^{\infty} \exp(-jk_0^2 \ell)S(\omega)\exp(j\omega t)\omega$. The system output is written as

$$S_{\text{out}}(t) = 1/\sqrt{2\pi} \int_{-\infty}^{\infty} \exp(-jk_0^2 \ell)H_{1t'}(\omega)H_{1t}(\omega)S(\omega)\exp(j\omega t)\omega$$

(10)

We use the cavity investigated in Fig.2 for $\text{R}$ and $\text{L}$ and choose $\ell = 1m, L_2 = 6.7Km$. The out pulses lag behind the reference as much as $1.78 \times 10^{-4}$ sec for N=5 and $3.40 \times 10^{-4}$ sec for
N=10 which corresponds to \[ n_0(2N+1)L_2 \]/c. One can observe the attenuation resulting from the power coupling to \( \mathcal{A}_rr \) and \( \mathcal{A}_rr \) during the propagation inside the delay zone.

The numerical simulations in Fig.2 suggest that a half of the WLC linewidth (\( \Delta \nu_{WLC} \)) corresponds to the dynamic range of our system over which a data pulse can be delayed without distortion. The WLC linewidth coincides with the spectral region where negative dispersion occurs. Since the negative dispersion is produced in the middle of the two gain lines, in gain doublet system, the achievable \( \Delta \nu_{WLC} \) was determined by the gain separation \( \delta \) and was estimated proportionally to \( \delta^{2/3} \) [7]. It is necessary that \( \delta \) is properly changed in the range larger than the gain bandwidth (\( \Delta \nu \)) to produce WLC effect. Since the two gains are more widely separable under broadband gain line condition, one can broaden \( \Delta \nu \) to increase \( \Delta \nu_{WLC} \). The Brillouin pump spectrum was broadened by superposing Gaussian white noise on laser diode dc current. Such Brillouin pump was widely used to produce broadband gain profile [19~22]. When Double Brillouin pumps were separated by \( 2\nu_B \) (\( \nu_B \): Brillouin frequency) and had equal power, gain width was expanded equal to \( 2\nu_B \) [22]. Now we adopt the Double pumps technique in ref.22 to produce broadband negative dispersion. We use two groups of the double equal power pumps. Pump1 and 1' (2 and 2') are separated by \( \nu_B \), and pump 1 and 2 (1' and 2') by \( \delta \). Due to the separation by \( \nu_B \) and the equal power, the loss induced by pump 1 (pump 2) is completely cancelled by the gain line of pump1' (pump2'). Therefore, we can eliminate the overlapping between the gain and the loss of pump1 (pump2) and thus avoid the gain spectrum distortion. We increase the gain bandwidth up (\( \Delta \nu_B \)) to \( 2\nu_B \) and stop before the gain of pump1 (pump2) meets the loss of pump1' (pump2'). In Fig.6 (c), finally only two broadened gain lines survive separated by \( \delta \). In the two groups of the double pumps configuration, however, Fig.6 (d) indicates that the loss spectrum of pump1' extends beyond the gain of pump2 (Note inside the dotted area). Such overlapping causes the gain distortion. To prevent it, we can infer from Fig.6 (d) that \( \delta \) is related to the width of the broadened gain width \( \nu' \) by \( \delta + \nu'/2 = 2\nu_B \), where \( \nu' \) is the width measured along the bottom of the gain. The single gain bandwidth can be expanded if we employ more pumps in individual group. More generally speaking, N pumps create a single gain with the maximum bandwidth of \( N\nu_B \) [22]. In the two group configuration, the achievable gain separation with the broadened gain-doublet of width \( \nu' \) is written as \( \delta + \nu'/2 = N\nu_B \). Since the Brillouin frequency of optical fibers is 8~12GHz, \( \delta \) in the range of GHz can be easily obtained. Negative dispersion occurs in such GHz area between the two gains and thus one can increase WLC linewidth up to GHz.

In conclusion, we proposed the theoretical model of a fiber-based data buffering system. Delay time was determined by the fiber length and the number of round trips inside the intermediate loop. We can lengthen the fiber to expand propagation distance in the loop so that the traveling time increases causing more delay time per one round trip. Hence, the number of the loop circulation required for a specific time delay is reduced. It means that the number of crossing through the couplers decreases to obtain same delay and thus a data pulse can be delayed with the coupling loss negligible. The bandwidth of a Gaussian pulse is 10 times FWHM of the ring resonator. However, in the presence of WLC effect, the pulse spectrum belongs to the region of the constant amplitude (|\( H_{11} \)|=1) for the white light fiber resonator and thus the input pulse is loaded into the loop without distortion. The delay of 2\( \mu \)s pulse was obtained by using the 6.7km fiber loop in the time scale of \( 10^{-4} \) remaining its original shape. We proposed the scheme to create negative dispersion over the spectral range of multiple times Brillouin frequency to increase achievable WLC linewidth. It is possible to improve the dynamic range of our system. Unlike earlier system [1], this fiber-based buffering system is realized by simple configuration with the expandable dynamic range.
Fig. 1 Schematics of (a) fiber ring resonator, (b) ring resonator coupled to a fiber

\[ |b_1|^2/|a_1|^2 \]

\[ (\omega-\omega_0)/2\pi \times 10\text{MHz} \]

(a)

\[ |a_2|^2/|a_1|^2 \]

\[ (\omega-\omega_0)/2\pi \times 10\text{MHz} \]

(b)

Fig. 2 Power transfer of the configuration in Fig. 1 (b).

(a) \[ |b_1|^2/|a_1|^2 \]
(b) \[ |a_2|^2/|a_1|^2 \] for non-dispersive fiber (blue) and for negative dispersion (red)

(\( n_1 = -1.192 \times 10^{-15} / \text{rad} \), \( n_3 = 1.223 \times 10^{-32} / \text{rad}^3 \))
Fig. 3 (a) Phases associated with the transfer functions of the system in Fig.1(b), \( \angle H_{bl,a1} = \text{arg}(b_1/a_1) \), \( \angle H_O = \text{arg}(a_r/a_1) \) in the absence of WLC effect, \( \angle H_{WLC} = \text{arg}(a_r/a_1) \) for \( n_1 = -1.192 \times 10^{-15} \text{ rad} \), \( n_2 = 1.223 \times 10^{-32} \text{ rad}^3 \) (b) Reference pulse after propagating \( \ell \) (black), system output associated with \( H_O \) (blue), \( H_{WLC} \) (red). Output pulse in the presence of WLC effect is overlapped with the reference.

Fig. 4 Schematics of the fiber-based buffering system.
Fig. 5 Output pulse sequence of the fiber-based buffering system (a) Reference pulse, (b) 5 (c) 10 times round trips inside the delay zone

Fig. 6 Scheme to create negative dispersion in the scale of several GHz. (a) Double pumps (red, green) in group I,II. The pumps in green provide gain spectrums to compensate for the loss spectrums of the pump in red. (b) Pump spectrum is broadened by Gaussian white noise to create the broadband gain equal to Brillouin shift ($\nu_B$). (c) Gain bandwidth of 2$\nu_B$ (d) Due to the overlapping of the gain in group II with the loss in group I, $\delta + \nu'/2 = 2\nu_B$ should be observed to avoid gain distortion resulting from such overlapping.
Reference


Dual-frequency and cascaded Brillouin lasing produced by resonant pumps in a fiber ring cavity

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Abstract: We demonstrate cascaded Brillouin laser and double-frequency Brillouin laser in single mode fiber. Based on the measured Brillouin gain characteristics, a short fiber ring cavity is built by using a variable optical coupler. As increasing the pump power, the 3rd order Brillouin stokes wave is observed at the pump of 100mW. For the applications to white light cavity (WLC) or data buffering system, the sidebands of Electro-Optic Modulator (EOM) outputs with the fundamental eliminated is used to provide bi-frequency pumps. The pumps are simultaneously resonant to the ring cavity so that two stokes waves are lasing with the beat note equal twice of the modulation frequency.

When Laser light propagates through optical fibers, the incident light is back-scattered by the refraction index modulated gratings associated a sound wave of frequency called Brillouin shift frequency ($\nu_B$)¹. This non-linear process, so-called stimulated Brillouin scattering (SBS) has been investigated in conventional silica fibers²,³ or fibers with enhanced Brillouin coefficient⁴,⁵. It leads to many applications such as Brillouin amplifier, lasing, and optical pulse delay⁶,⁷,⁸,⁹,¹⁰. Using Brillouin pump to induce group index change, the data pulse was delayed less than the pulse duration of 20~50ns¹⁰ above. White Light Cavity (WLC) in a fiber ring resonator was proposed to build a data buffering system to trap the data pulse in long fiber loop¹¹. WLC is a cavity that contains anomalously dispersive medium. Such dispersion compensates for the frequency-dependent wavelength change and thus WLC resonate without reduction of build up factor over broader frequency range than ordinary cavities. This proposed buffering system overcame the delay-bandwidth constraint imposed on current slow light technique.

This paper, we build a fiber ring cavity based on the Brillouin characteristics of fiber in use. With a resonantly enhanced single pump, 1st, 2nd, and 3rd order of Brillouin threshold are evaluated as 20, 60 and 100mW. For bi-frequency pumps, the pumps are simultaneously locked to different longitudinal modes. Two Brillouin lasing modes at frequencies corresponding to the dual gain peaks show equal intensity without any noticeable cross-talks between two resonant pumps. For the application to WLC in fiber resonators, our analysis of cavity modes, pump beams and dual gains offer proper gain separations as well as free spectral range (FSR) of the cavity.

Before constructing a fiber ring resonator, we review the theoretical model of the complex Brillouin gain factor produced by SBS process. We show the experimental demonstration to examine the Brillouin characteristics of a fiber. The information about the fiber will be useful to build a fiber ring resonator. Let us consider the coupling of the stokes field (probe) at frequency $\nu$ to the pump field via SBS. The stokes field propagates in the fiber with the length of L while the pump counterpropagates. Assuming that the pump is non-depleted, the slowly varying amplitudes of the stokes field at steady state after the interaction with the pump field is described by

$$E_S = E_{S0}e^{(\alpha+\beta)\frac{L}{2}} \quad (1)$$

where $E_S$ is the amplitude after the propagation along L, $E_{S0}$ is the input field amplitude (weak stokes seed), $\alpha$ and $\beta$ are expressed as

$$\alpha = \left( g_0 I_p \right) / \left[ 2(1 + 4(\nu - \nu_B)^2 / \Gamma_B^2) \right]$$

and $\beta = \left( 2 \Gamma_B / \nu_B \right)$.
\[ \beta = \left( g_0 I_p (v - v_B) / \Gamma_B \right) / \left( 1 + 4(v - v_B)^2 / \Gamma_B^2 \right) \] 

\[ \alpha = \frac{1}{2} \left[ \frac{g_0 I_{p1}}{1 + 4(v - v_B - \Delta)^2 / \Gamma_B^2} + \frac{g_0 I_{p2}}{1 + 4(v - v_B + \Delta)^2 / \Gamma_B^2} \right] \]

provided the pumps are symmetrically separated from \( v_B \) as amount of \( \Delta \). \( I_{p1}, I_{p2} \) are the intensities of the bi-frequency pumps. In such gain-doublet system, the gain separation corresponds to \( 2\Delta \). We define the parameter \( G \) as \( g_0 I_{pj} / 2 \) (j=1,2) associated with the amplitude gain peak.

Next, we measure Brillouin frequency, gain linewidth and \( G \) factor as increasing the pump power. Fig. 1 displays the schematics of the experimental set-up for Brillouin gain spectrum measurements. The light source is a 1550nm CW laser diode (LD), (linewidth<1MHz). The LD output is divided by a 90:10 coupler. A 3dB coupler is used to build a fiber loop mirror\(^{12} \) and the 10\% of LD’s light is inserted into the port of 1 m fiber pigtail of the coupler. Hence, a 1 m external cavity is implemented by the loop mirror so that it provides optical feedback to reduce the linewidth of LD. The 90\% output is divided again by another 3dB coupler. One is inserted into an Electro-optic modulator (EOM-1) and is augmented by an Er-doped fiber amplifier (EDFA) to serve as the Brillouin pump. The pump from EOM-1 without modulation is used to produce a single gain. For the generation of the double peaks, the D.C voltage on EOM-1 is adjusted to eliminate the fundamental and thus the gain separation corresponds to \( 2f_M \) (\( f_M \): modulation frequency). The modulation frequency of the EOM-2 is swept \( \Delta v \) around \( v_B \) to provide a probe.

Fig.1 Experimental set-up to measure Brillouin spectrum: FPC, fiber polarization controller; EDFA, Er-doped fiber amplifier, EOM, Electro optic modulator

Fig. 2(a), 3 illustrate the Brillouin spectrum of 88m Corning SMF-28e fiber. In Fig.2(a), the single gain is characterized by scanning around \( v_B \) as much as \( \Delta v=40\)MHz in the step of 0.5MHz. We increase the pump power until the gain is saturated. The Brillouin shift frequency and gain width are measured as 10.867GHz and \(~10\)MHz, respectively. \( v_B \) is 50 MHz higher and \( \Gamma_B \) is a
factor of 3 less than the previous measurement for SMF-28e. In ref.2, they used the long fiber (6.4km) and the low power pump in the scale of mW compared to ours. Hence, the Brillouin interaction length (L) for the given pump is long and thus the pump depletion occurs to broaden the linewidth more than our case. Fig. 2(b) illustrates the gain factor, G (/m) associated with the gain spectrum displayed in Fig. 2(a). Assuming the absence of saturation, we plot the linearly increasing graph (Purple line). The slope is calculated from the experimental data for the pump powers of 600 and 800mW. G for the higher power (1, 1.5, 2W) is plotted with the calculated slope. In the comparison to the experimental data displayed as the yellow graph, we can confirm that the gain factor, G (/m) starts to be saturated at the pump of 1.5W. In Fig.3, the double gain peaks are displayed for the case that the modulation frequencies of EOM-1 are $v_M=17.32$MHz, $25.98$MHz and $\Delta v=100$MHz. The anti-symmetry of the double gains results from unbalanced sideband output of EOM-1. The peak of the individual gain approximately corresponds to that of the single gain associated with the pump power which is equal to a half of the bi-frequency pump power. $\Gamma_B$ is measured as ~12MHz.

![Figure 2(a) Brillouin gain spectrum of a 88m single mode fiber, Y axis in voltage on photodiode (Volt. on PD)](image)

![Figure 2(b) G (Gain peak amplitude at resonance: /m) for linear increase without saturation and for real experimental data.](image)

![Figure 3 Dual Brillouin gain spectrum for the gain separation of 34.64MHz ($v_M=17.32$MHz) and 51.96MHz($v_M=25.98$MHz)](image)

![Figure 4 displays the schematics of the experimental setup to demonstrate Brillouin fiber laser.](image)
90% of LD output is divided by a 3dB coupler. One is inserted into a Fabry-perot (FP) with a FSR of 37GHz and a finesse of 30. It provides a reference spectrum which corresponds to a laser frequency ($\nu_L$). The other is modulated by EOM while suppressing the fundamental ($\nu_L$) and only two sidebands are amplified by EDFA to generate bi-frequency pump. For a single pump, we remove the EOM. The coupling ratio of a variable coupler (VC) is set to critical coupling condition. At such condition, a full width half maximum (FWHM) of our fiber resonator is measured as 0.23MHz. A lock-in amplifier (LIA) and A.C. servo lock the cavity length at the pump resonance. First, we observe cascaded Brillouin lasing while the single pump is resonant with the cavity. We both adjust the coupling ration of V.C. and increase pump power to attain the higher order Brillouin.

Fig. 4 Schematic of cascaded and bi-frequency Brillouin fiber lasers

Fig. 5 (a) 1st order (Pump power: P=20mW) (b) 1st and 2nd order (60mW) (c) 1st, 2nd and 3rd order (100mW) Brillouin laser
Fig. 5 shows Brillouin lasers monitored by the F.P. spectrum analyzer. Fig. 5(a) indicates that in the presence of 20mW pump, the smaller peak is a backreflection of the pump and the larger peak is the 1st order Brillouin. The 2nd order and 3rd order Brillouin lasers are observed at 60mW and 100mW, respectively as displayed in Fig. 5 (b),(c). The separation between adjacent orders of the Brillouin spectrums is equal to $v_B(10.867\text{GHz})$. The clockwise propagating 2nd order Brillouin produces small spectrum peak compared to the odd orders due to high isolation between the port 1 and 2 of V.C.

Based on the measured $v_B$ and $\Gamma_B$ in Fig. 2 and 3, a fiber ring resonator is designe to obtain $N \times \text{FSR} = v_B$ where $N$ and FSR are an arbitrary integer and the free spectral range of a cavity, respectively. The FSR of our cavity is 17.3262MHz which is broader than $\Gamma_B$. Due to $N \times \text{FSR} = v_B$, one longitudinal cavity modes is present close to the center of the gain. Provided FSR is broader than $\Gamma_B$, the Brillouin single gain supports only one longitudinal cavity mode at the peak of the gain profile and thus we can obtain a single mode Brillouin laser. Next, for the future applications such as white light cavity (WLC) demonstration, let us consider the case that we control the modulation frequency to make the bi-frequency pump resonant with the cavity. Fig. 6(a) and (c) illustrates that EOM outputs consist of two sidebands ($v_L \pm v_M$) and the fundamental ($v_L$). While a ramp signal drives a piezoelectric actuator (PZT) to scan the cavity length, the signal from EOM is monitored through port 3 of V.C. in the setup of Fig.4. The two sidebands share the equal cavity length to $L$ for $\text{FSR} = \lambda / L$ and $L^*$ for $\text{FSR} = \lambda / L^*$ where $k$ is 0 or positive integer (For $L^*$, $k=0$ implies that EOM provides the fundamental). However, the sidebands occupy the different modes. For example, $(N + 2)\lambda_1 = L, (N + 3)\lambda_2 = L$ where $\lambda_1$ and $\lambda_2$ are wavelength associated with $v_L \pm v_M$ and $v_L + v_M$ ($v_M = 8.6631\text{MHz}$) respectively, $N$ is an arbitrary positive integer. After eliminating $v_L$, the cavity is locked at L or L' so that $v_L \pm v_M$ simultaneously resonate in the cavity. Fig. 6(b) and (d) display the configurations of the cavity modes (thick bars), the double gains and pump spectrums (color arrows). The bi-frequency pumps create the gain-doublet which is represented by the same color as the pumps’ in use. For the cavity locking at L, fig. 6(b) illustrate that the even number of cavity modes appears between the two gains and thus no cavity mode exists in the middle of two gains. If the length is changed to $L^*$, the cavity modes exist displaced from the modes supported by $L$ as amount of $\delta = 1/2 \times \text{FSR}$ indicated in Fig. 6(d). Therefore, the center of the two gains coincides with the cavity mode corresponding to $v_L \pm v_B$. For WLC demonstration, the cavity mode requires being equal to $v_L \pm v_B$ or the deviation from $v_L \pm v_B$ is small in comparison to gain separation. The gain doublet is symmetrically arranged around $v_L \pm v_B$. As we can scan a probe around $v_L \pm v_B$ to observe WLC effect, the approach illustrated in Fig. 6(d) is more suitable. However, it can have the residual fundamental after suppression thereby the residual is possibly resonant with the cavity to distort the double gain profile. Since the sidebands are locked onto different position from the fundamental illustrated in Fig.6(b), the dual gains can be generated without distortion. In WLC demonstration, we choose the cavity mode nearest by the center of gain doublet and scan the probe frequency around it. Negative dispersion is created in the middle of the two gains. The gain separation should be wider compared to the difference (FSR/2) between the gain center and the neighboring cavity modes and thus the scanning range completely belongs to the dispersion region. Wavelength change due to the scanning is compensated by a negative dispersion slope corresponding to the group index close to zero for our case, so-called WLC effect. The presented techniques in Fig.6 force the gain profile to
support the cavity mode at the gain peak, allowing the single mode Brillouin-shifted wave to resonate and eventually be lasing. Since the probe can be embedded in the lasing signal, the Brillouin laser is the main source of background noises. For WLC operation below Brillouin threshold, we could propose alternative FSR to fulfill \((N + 1/2)\text{FSR} = \nu_B\). Fig. 7 illustrates the case that FSR is 17.4016MHz. Once the cavity length is fixed at \(L\) or \(L^*\), the resonant pumps produce the gain profiles downshifted from the pump frequencies as much as \(\nu_B\). Due to the given condition \((N + 1/2)\text{FSR} = \nu_B\), the gain peaks appear in the middle of the two neighboring cavity modes illustrated in Fig. 7. Since the Brillouin linewidth is less than the FSR, the resonant modes are present sufficiently apart from the gain peak. Hence, we can achieve higher gain peaks than the Brillouin resonating cavity, while staying below Brillouin threshold.

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**Fig 6.** (a) Pump resonance peaks from the fiber ring cavity. The pump is modulated at \(\nu_M = (2k + 1)/2 \times \text{FSR}\). (b) Bi-frequency pumps (color arrows) are resonant with the cavity modes (black bars). The pumps produce dual gains which is downshifted from the pump spectrums as much as \(a, b, c\) equal to \(\nu_B=10.867\text{GHz}\). The gains contain two modes at the center of each gain profile. For \(\nu_M = \text{FSR}/2\), the resonant pumps (red arrows) produce the two gains (red). As increasing \(\nu_M\), the gain separation are expanded to 3FSR for \(\nu_M = 3/2 \times \text{FSR}\) (green) and 5FSR for \(\nu_M = 5/2 \times \text{FSR}\) (blue), (c) \(\nu_M = k \times \text{FSR}\), (d) The gain separation is even number times FSR. For example, 2FSR for \(\nu_M = \text{FSR}\) and 4FSR for \(\nu_M = 2 \times \text{FSR}\). The single pump and gain are represented by the brown. The mode under the gain profile agrees with that of the 1st order Brillouin laser presented in Fig. 5(a).
Fig. 7 FSR is modified equal to 17.4016MHz so that $(N + 1/2)\text{FSR} = \nu_B$. (a) Pump resonance peaks for $\nu_m = (2k + 1)/2 \times \text{FSR}$, (b) Gain profiles, pump spectrums and cavity modes associated with the cavity locking on L, (c) $\nu_m = k \times \text{FSR}$, (d) Same diagram as (b) when $\nu_m = k \times \text{FSR}$.

To demonstrate dual frequency Brillouin laser, the pumps are modulated at $\nu_M = 7.9932\text{GHz}$ i.e. $\nu_m = (2k + 1)/2 \times \text{FSR}$, $k=461$. From the standpoint of the Brillouin threshold, the cavity condition presented in Fig 7 is useful for WLC application in principle. To evidently prove the generation of dual gains in the cavity, however, we use the cavity with FSR of 17.3262MHz and thus observing two different Brillouin lasing modes at the frequencies associated with dual gain peaks. Fig. 7(a) exhibits the EOM output observed by a photodetector (PD) at the port 3 of V.C. with scanning the ring cavity. Fig. 7(b) displays the dual frequencies Brillouin lasers. According to the investigation in Fig.6, the lasing frequencies correspond to the longitudinal cavity modes, $\nu_L - \nu_B = \nu_m$. From the equal intensity for the lasing peaks, one can determine that the two resonant pumps have same intensity and produce the gains without causing noticeable cross-talks.

In conclusion we have demonstrated a fiber ring cavity pumped by two frequencies locked to different longitudinal modes simultaneously. For the application to WLC demonstration, the cavity FSR was chosen to satisfy $(N + 1/2)\text{FSR} = \nu_B$ or $N \times \text{FSR} = \nu_B$. For both cases, various pump separation is considered to ensure that there is an available cavity mode at the center of two Brillouin gains. Also the cavity mode is downshifted as much as $\nu_B$ from the center of the two pumps spectrum. The corresponding dual gains were characterized indirectly by observing the production of two Brillouin lasing modes at frequencies corresponding to the gain peaks. The resulting two lasing peaks was observed to have essentially the equal intensity, thus indicating that the presence of two resonant pumps do not cause any noticeable cross-talks. When the input intensity of a single frequency pump was increased beyond the secondary threshold, we observed first second and third order Brillouin lasing simultaneously. We inspected the configurations of the modes, single gain and pump spectrums. It confirmed that the lasing signal was a single mode. The information obtained from these observations can be used to determine the optimal combination of power level in each of the two pumps as well as the proper choice of FSR for
operation of WLC while staying below the Brillouin lasing threshold.

Fig. 8 (a) Pump resonance peaks, (b) Dual frequency Brillouin lasing. The two lasing peaks are separated by $2v_M (=17.326\text{GHz})$. 
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Parameter constraints in realizing a White Light Cavity using Brillouin scattering in a fiber ring resonator

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Abstract: We investigate physical constraints on a fiber-based White Light Cavity (WLC). Brillouin absorption or Brillouin double gains via Stimulated Brillouin scattering (SBS) is employed to create negative dispersion in a fiber ring resonator. For Brillouin absorption or dual gains, very small fraction of the broadening is due to the WLC effect. In order to achieve sufficient negative slope for zero group index ($n_g=0$) for WLC condition, theoretical analysis and experimental results suggest that high Brillouin coefficient as well as low transmission loss is essentially required for fibers.

Techniques to create negative dispersion over a limited frequency range have been performed through non-linear interaction such as two-wave mixing in photorefractive crystal\textsuperscript{1} and stimulated Brillouin scattering (SBS)\textsuperscript{2,3,4}. In a so-called fast light demonstration, optical pulse propagates through materials with such an anomalous dispersion faster than free space. Fast light in optical fibers via SBS process have attracted much attentions, due to availability in current communication and potential applications. In particular, data buffering systems based on fast-light technique in fibers was proposed to exceed the delay-bandwidth constraint encountered in a typical data buffer by means of slow light\textsuperscript{5}. Also, it can be employed to enhance the rotational sensitivity of a passive resonator gyroscope\textsuperscript{6} and to realize Superluminal ring laser as a versatile hypersensitive sensor.

White Light Cavity (WLC) is fundamental to implement all these devices. WLC is a cavity that resonates over a wider frequency range than an ordinary cavity without a drop in the field build-up factor. It has been implemented in a cavity filled with rubidium vapor where dual Raman gains were produced around the cavity resonance to achieve the fast-light condition ($n_g<1$)\textsuperscript{7}. For WLC demonstration in fibers, primary analysis and experiments have been carried out to find out the proper free spectral range (FSR) as well as the optimal pump power level\textsuperscript{8}. FSR was adjusted to ensure that the cavity mode exists at the center of dual Brillouin gains. The Brillouin pump power was controlled to maintain WLC below Brillouin lasing threshold.

In this paper, we investigate physical constraints in realizing such a WLC via Brillouin scattering in conventional single mode fibers. In the experiment, Brillouin pump is resonantly enhanced in the cavity. To create negative dispersion, SBS of a single pump induces the probe depletion (Brillouin absorption) or bi-frequency pumps provide dual gains. In both cases, the cavity resonance was observed to be broadened by a very small amount due to WLC effect. Numerical simulation confirms that under given experimental conditions the group index change ($\Delta n_g = -0.01 \sim -0.03$) is small. As such, negative dispersion is not sufficient to induce large WLC effect. Theoretical analysis shows that major restraints on WLC based on conventional optical fibers are the requirements of extremely high pump power as well as the power damage threshold of optical elements. Key solution to these problems is to use fibers with both higher Brillouin coefficient and low transmission loss. To our knowledge, however, current fabrication technique can provide fibers with high Brillouin coefficient but high transmission loss in comparison to conventional silica fibers. We propose alternative fiber resonator to realize WLC with such given fibers.

First, let us consider the probe field propagating an optical fiber resonator. Fig.1 displays that a fiber ring resonator is build by a 2 by 2 coupler. Provided that the coupler is internally lossless, the complex field amplitudes is explained by the matrix
where \( K \) is the intensity coupling coefficient. The relationship between \( a_2, b_2 \) is expressed in terms of the internal loss of the resonator (\( \alpha \)) and the round trip phase shift (\( \theta \))

\[
a_2 = \alpha e^{\alpha \theta} b_2
\]

Using Eq(1), (2), we can derive the power transmission ratio \( \frac{|b_1/a_1|^2}{|b_2/a_1|^2} \) and the inserted power into the resonator \( |b_2/a_1|^2 \) with respect to the input \( |a_1|^2 \)

\[
\left| \frac{b_1}{a_1} \right|^2 = \frac{\alpha^2 + |t|^2 - 2\alpha |t| \cos \theta}{1 + \alpha^2 |t|^2 - 2\alpha |t| \cos \theta}
\]

\[
\left| \frac{b_2}{a_1} \right|^2 = \frac{1 - |t|^2}{1 - 2\alpha |t| \cos \theta + \alpha^2 |t|^2}
\]

where \( t \) is complex amplitude transmission coefficient (\( K + T = 1, T = |t|^2 \)). For simplicity, we will choose \( |a_1|^2 = 1 \). Next, consider \( \theta \) in the presence of bi-frequency Brillouin pumps. The probe at frequency \( \omega \) propagates the ring resonator whose circumference is \( L \). The phase shift resulting from the propagation is written as \( kL \). \( k \) is the wave vector amplitude of the probe. To observe WLC effect, the bi-frequency pumps produce dual gains via SBS process in the fiber resonator and negative dispersion is created between the gains. For our case that Brillouin gain medium length corresponds to \( L \), the group index, \( n_g = 0 \) is required to achieve WLC condition. The Brillouin dual gain \( \alpha_{Br} \) and the resultant phase \( \beta \) are given by

\[
\alpha_{Br} = \frac{1}{2} \left[ \frac{g_0 I_{p1}}{1 + 4(\nu - \nu_B - \Delta)^2 / \Gamma_B^2} + \frac{g_0 I_{p2}}{1 + 4(\nu - \nu_B + \Delta)^2 / \Gamma_B^2} \right]
\]

\[
\beta = \frac{g_0 I_{p1}(\nu - \nu_B - \Delta)/\Gamma_B}{1 + 4(\nu - \nu_B - \Delta)^2 / \Gamma_B^2} + \frac{g_0 I_{p2}(\nu - \nu_B + \Delta)/\Gamma_B}{1 + 4(\nu - \nu_B + \Delta)^2 / \Gamma_B^2}
\]

where \( \nu_B, g_0, \) and \( \Gamma_B \) are Brillouin frequency, gain coefficient, linewidth, respectively. \( I_{p1}, I_{p2} \) are pump intensities per unit area. We consider the equal intensity, i.e. \( I_{p1} = I_{p2} = I_p \) (W/m²). \( 2\Delta \) corresponds to gain separation. The probe propagates the cavity where negative dispersion is created by SBS. The phase shift after the one round trip can be written as \( kL = n_f c L + \beta L \). \( n_f \) and \( c \) is the mean index of the fiber and speed of light. \( \alpha \) defines the internal loss of the resonator in the absence of Brillouin gain and thus is rewritten as \( \alpha \exp(\alpha_{Br} L) \) after employing the dual gains in the resonator for WLC demonstration. Let us call \( \alpha \exp(\alpha_{Br} L) \) modified gain. \( k \) is expressed by Taylor expansion around the cavity resonance frequency, \( \omega_0 \) as

\[
k = k_0 + k_1(\omega - \omega_0) + k_2(\omega - \omega_0)^2 + k_3(\omega - \omega_0)^3
\]

where \( k_0 = n_f \omega_0 / c \),

\[
k_1 = n_f / c + d\beta/d\omega|_{\omega_0}, \quad k_2 = (1/6) d^2\beta/d\omega^2|_{\omega_0}, \quad k_3 = (1/6) d^3\beta/d\omega^3|_{\omega_0}
\]

Assuming \( \omega_0 = \nu_B \), the second order term is \( k_2 = 0 \) due to the antisymmetrical dispersion profile. Finally, we obtain the transmitted power spectrum of the fiber-based WLC by inserting \( kL \) of Taylor expansion and \( \alpha \exp(\alpha_{Br} L) \) into \( \theta \) and \( \alpha \) in Eq(3), respectively. In the experiment, we control the cavity length with a lock-in detection circuit so that the pumps resonate with the ring cavity. Eq.(3) at resonance (\( \theta = 2m\pi, m \) is integer) describes the relationship between the input pump intensity at port 1 and the resonant pump intensity inside the cavity. Suppose that the resonant pumps are non-
depleted, the Brillouin pumps in the cavity is \( I_j \left( 1 - |t|^2 \right) / \left( 1 - \alpha |t|^2 \right) \) where \( I_j \) (W) is the individual pump power at port 1, \((j=1,2\) denotes 1st and 2nd pump). This yields \( I_p \) using the effective area \( A_{\text{eff}} \) of a fiber. For the case that a single pump produce Brillouin absorption to create negative dispersion, all the expressions and the arguments given above apply provided we change \( \alpha_{\text{Br}} \rightarrow -1/2g_0I_p/\left( 1 + 4(\nu - \nu_B)^2 / \Gamma_B^2 \right) \), \( \beta \rightarrow -\left( g_0I_p(\nu - \nu_B) / \Gamma_B \right) / \left( 1 + 4(\nu - \nu_B)^2 / \Gamma_B^2 \right) \).

Let us briefly estimate the WLC linewidth in comparison to an ordinary cavity. The wave number for the probe varies in the dispersive fiber resonator. We define the dephasing parameter \( \varphi \) resulting from the change of the wave number \( \Delta k \) as \( \varphi = \Delta kLN \) where \( N \) is the number of round trip. For the ring resonator, \( \Delta k = n_g \Delta \omega / c \) in the dispersive medium can be easily derived from \( \Delta \omega = \frac{n_g}{c} / \gamma_0 \) (\( n_g \) is group index). We ignore additional dephasing resulting from the higher order dispersion, i.e. \( \left( d^2n/d\omega^2, n > 1 \right) \). Inserting \( \Delta k \) and \( N \) into \( \varphi \) and reminding that \( \varphi = 2\pi \) when \( \Delta \omega \) corresponds to the linewidth of the dispersive resonator, the relation between the WLC linewidth of \( \Delta \omega \) and the cavity life time \( \gamma_0 \) for normal ring resonator is \( \Delta \omega = \Delta \omega_{\text{cavity}} = \gamma_0 \).

First, we employ Brillouin absorption spectrum to observe WLC effect. Fig.2 displays the experimental set-up for WLC demonstration. We use the same fiber ring cavity made of Corning SMF-28e with a free spectral range (FSR) of 17.32MHz as used in the previous work. In ref. 8, the cavity length was adjusted so that one of the cavity modes corresponds to Brillouin frequency \( \nu_B \). A fiber loop mirror is used to stabilize a fast fluctuation of the laser diode (LD). 50:50 polarization maintaining coupler (PMC) divides the 90% LD output. One is amplified by Er-doped fiber amplifier (EDFA) to provide a Brillouin single pump. A probe is modulated by EOM at Brillouin frequency which was previously measured as 10.867GHz. The fundamental is suppressed by adjusting D.C. on EOM. The two sidebands are delivered to a Fabry-Perot (FP) spectrum filter. It is actively controlled by an A.C. servo and Lock-in-amplifier (LIA) to filter one of the sidebands. The upper sideband \( \nu_L + \nu_B \) is transmitted to interact Brillouin absorption for WLC demonstration. After FP filter is locked to \( \nu_L + \nu_B \), the modulation frequency of the probe is slowly swept around \( \nu_B \) as amount of \( \Delta \nu = 3 \text{MHz} \) to attain a resonance peak of the fiber cavity. We use the lower sideband \( \nu_L - \nu_B \) to observe a narrowing peak due to normal dispersion in a single gain and compare to the broadened cavity response with the upper sideband probe \( \nu_L + \nu_B \). Fig.3(a),(b) show experimental results and numerical simulations the cavity resonance for different pump power. To obtain the probe transmission (black) without SBS inside the cavity, we modulate the probe at 7.0145GHz which is sufficiently separated from Brillouin shift frequency \( \nu_B \). As such, it propagates the cavity without interacting the Brillouin pump while the cavity is locked to the pump resonance. The linewidth of the ordinary cavity is measured as 0.24MHz. We change the pump power below Brillouin threshold to avoid mixing the probe with Brillouin lasing noise. As increasing pump power, we observe the broadening (narrowing) of the resonance peak with the interaction with Brillouin absorption (gain). The probe transmission is lower (higher) than the reference in the presence of Brillouin absorption (gain) than the reference transmission. We attribute such transmission variation to the induced (reduced) cavity internal loss by Brillouin absorption (gain). Fig. 3(b) displays the simulations for the cavity response associated with the experimental results. In ref. 8, we defined G factor as \( -1/2g_0I_p \) for absorption \((1/2g_0I_p \) for gain). Note that in single absorption(gain) configuration, G implies absorption (gain) amplitude at Brillouin resonance \( \nu = \nu_B \) and is proportional to the input pump intensity. In simulation, we choose
\( \alpha = \sqrt{0.97}, t = \sqrt{0.945} \) and G’s to obtain the graphs which are consistent with the experimental data. The simulations confirm our interpretation of the transmission variation in the experiment. Following the transmission as a function of \( \alpha \) presented in ref.[11], the transmission characteristics of the cavity without the pump is over-coupled (\( \alpha > t \)) so that the internally circulating field coupled into the output of the resonator \( \sqrt{K_a}_2 \) is excessive compared to the transmitted field \( \sqrt{T_{a_1}} \). After applying Brillouin pump, the probe of \( \nu_{l} + \nu_{B} \) undergoes \( \alpha \exp(\alpha_{ph} L) \) such that \( \sqrt{K_a}_2 \) is reduces to be equal to \( \sqrt{T_{a_1}} \) at critical coupling point \( \exp(\alpha_{ph} L) \approx t \). \( \sqrt{T_{a_1}} \) completely interferes with \( \sqrt{K_a}_2 \) at 2mW pump \( |G| = 0.0005 \) and thus the probe transmission vanishes. As the pump power becomes higher \( (4mW, |G| = 0.0015, \text{and 6mW, } |G| = 0.0018) \), \( \sqrt{K_a}_2 \) decreases again due to the augmented Brillouin absorption loss i.e. \( \sqrt{T_{a_1}} > \sqrt{K_a}_2 \). Note, \( |b_1|^2 \) corresponds to resultant power after the interference. As the absorption is more induced, it is observed to increase in under-coupled region \( \exp(\alpha_{ph} L) < t \). When gain is introduced to the resonator, \( \sqrt{K_a}_2 \) is increased larger than the case in the absence of the pumps. As such, \( |b_1|^2 \) is raised after the inclusion of the Brillouin gain. The gain amplitude at \( v = v_{B} \) corresponds to \( |G| \) for the absorption of the equal pump power. However, we also notice that in the numerical simulation different \( |G| \) s are chosen to provide consistency with the experimental results for the given power values. For example, \( |G| = 0.0005 \) and 0.0015 are for the gain and for the absorption at the pump of 2mW, respectively. The power of the resonating pump with the cavity depends on the stability of the cavity length lock to fulfill the resonance. Since the achievable gain (absorption) amplitude at \( v = v_{B} \) is determined by such resonating power, the gain amplitude \( (|G| = 0.0015) \) is high compared to the absorption \( (|G| = 0.0005) \) due to the more stabilized lock. Fig.3(a),(b) indicate that due to small group index change, the resonance peak broadening(narrowing) with the allowed pump power below Brillouin lasing threshold is minute compared to the reference. Therefore it is hard to conclude if the broadening results from negative dispersion or the induced absorption. Since the lasing occurs at the cavity mode close to \( v_{l} - v_{B} \) within Brillouin gain profile, the lasing signal is possibly removed from the cavity resonance associated with Brillouin absorption probe \( (v_{l} + v_{B}) \) by using the optical filter elements such as fiber bragg grating (FBG). Adding FBG to port 3 of circulator 1 in Fig.2, we can increase the pump power without restraint to attain \( n_{g} = 0 \) for ideal WLC effect. Before proceeding to the experimental demonstration, we simulate the WLC response displayed in Fig.3(c). In the simulation, the WLC shows the finite linewidth due to the third order dephasing term \( k_3 \). Since \( \sqrt{K_a}_2 \) is reduced by the extra absorption, \( \sqrt{T_{a_1}} (T = 0.945) \) contributes to the transmitted power \( |b_1|^2 \) associated with the resonance peak without interfering with \( \sqrt{K_a}_2 \) and thus \( |b_1|^2 \) approaches to complete transmission \( (|b_1|^2 = \sqrt{T_{a_1}} = a_1) \) at WLC condition. For comparison, we simulate \( |b_1|^2 \) (green) for the normal cavity \( (n_{g} = n_{f}) \) with the loss corresponding to Brillouin absorption in the presence of WLC effect. Note that WLC response is broader compared to the normal cavity (black), however, the reduction in cavity buildup factor (probe transmission without the interference) due to additional Brillouin loss makes WLC effect undistinguishable from the non-dispersive cavity with equal loss (green).
Next, we used Brillouin dual gains to demonstrate WLC. The cavity length is controlled to resonate with bi-frequency pumps as presented in previous work. Fig. 4 displays the experimental setup. Light is modulated by EOM1 to provide the bi-frequency pumps with the elimination of the fundamental. The modulation frequency is chosen to be 17.32MHz. EOM2 modulates the probe at $\nu_B$ with the fundamental suppressed. The EOM2 output is inserted into the fiber ring cavity through an optical circulator without blocking the upper sideband ($\nu_L+\nu_B$). Due to strong Brillouin absorption at $\nu_L+\nu_B$ inside the cavity, we expect that the anti-stokes component ($\nu_L+\nu_B$) of the probe is depleted and negligibly affects on Brillouin interaction between the pumps and the stokes probe ($\nu_L-\nu_B$). Fig. 5(a) and (b) illustrate the experimental results and numerical simulations. In the experiment, we monitored the pump resonance peak from port 3 of the variable coupler (V.C.) and adjusted the coupling ration of V.C. to be the critical coupling point ($t=\alpha$) i.e. the transmission power is zero. To evaluate the cavity linewidth under no-gain conditions, we measured the full width half maximum (FWHM) of the pump resonance. It was 0.36MHz. The critically coupled cavity becomes black out at the resonance due to the perfect destructive interference between $\sqrt{T_{a_1}}$ and $\sqrt{K_{a_3}}$. In Fig 5(a), the probe transmission peak confirms that the cavity in the presence of the 2mW pumps is near by the critical point. Its linewidth is close to 0.36MHz. As increasing pump power, however, the probe experiences more gains such that the complete destruction of $\left|b_1\right|^2$ doesn’t occur due to the excessive $\sqrt{K_{a_3}}$. The peaks at resonance approaches to the D.C. level of $\sqrt{T_{a_1}}$. We increased the pump power until the lasing occurred. The simulations in Fig.5(b) confirm such a gain effect. To obtain agreement with the experimental data, we choose $t=\sqrt{0.93}, \alpha=\sqrt{0.93}$ and $G$ values. For the pump of 9.2 and 9.6mW, the transmission peaks exhibit the horizontal area concomitant with distortion around resonance. We attribute this effect to mode competition between the stokes probe and the residual anti-stokes wave. In the simulation, we raised $d^3\hat{\beta}/d\omega^3$ independently from $d\beta/d\omega$ to include the dephasing effect resulting from such a competition. Also, the experimental results show the broadened peaks compared to the simulations due to the depletion of the anti-stokes probe. Fig. 5(c) illustrates the configurations of the cavity mode, dual gains and pump spectrums associated with the experiment.

Again, we use the setup presented in Fig.2 to block the upper sidebands ($\nu_L+\nu_B$). In fig. 6(a)~(d), we compare the transmission peaks under the gain doublet condition to the cases of the single gain. Also, the reference transmission without Brillouin interaction is displayed. The gain separation is 69.28MHz(=4FSR) indicated in Fig.6(e). The reference cavity linewidth was determined first by scanning the probe around 7.0145GHz. In gain doublet, Brillouin gain compensates for the internal cavity loss to narrow resonance peak. Also the linewidth is broadened by WLC effect under dual gains. Since there is no experimental diagnostics to separate them, we observe the combination of these two effects. In Fig.6(a), for the case of 4mW bi-frequency pumps the transmission profile indicates that WLC effect is balanced by the linewidth narrowing due to the gain. As a result, its linewidth is almost equal to the reference peak. As increasing the pump power, WLC effect is small compared to the linewidth narrowing induced by the gains. With the dual pumps of 8 and 12 mW, we observed the transmission peaks narrower than the reference. Note, however that in the cases of the single pump, there is additional linewidth narrowing caused by normal dispersion. Therefore, it is noticeable that the probe transmissions with the 4 and 8mW dual pump are seen to be more broadened than the cases of the 1.5 and 2mW single pump, respectively. In the experiment, the resonances (minimum transmission) were observed at the same frequency
as the reference.

We revised the cavity length so that FSR obeys $\text{FSR}(N + 1/2) = \nu_B$. We can convince that the gain peaks don’t support any cavity mode. By this choice, we can avoid allowing the Brillouin-shifted wave (stokes wave) to resonate simultaneously. As such, Brillouin lasing threshold grows higher than the previous cases. The new FSR was measured 17.4016MHz. The gain separation was chosen to be 3xFSR(52.2MHz). We increased the pump power until Brillouin lasing occurs at modes closer to the gain peaks. However, we stopped increasing before the probe mode starts lasing. Again, we used the setup displayed in Fig.2. The optical chopper is added to separate the probe from the lasing signal of the gain peaks. To evaluate the cavity linewidth without SBS process, we scanned the probe around 8.7008GHz which is sufficiently separated from $\nu_B(10.867GHz)$. Fig.7 illustrates the experimental data and the configurations of cavity modes, pump spectrum, dual gain peaks. The cavity response at 14mW becomes broad due to the small amount of WLC effect.

From the simulations and experimental results so far, we found that the WLC effect under given experimental parameters is small due to the small group index change ($\Delta n_g = -0.01 \sim -0.03$, i.e. $n_g = n_i$). To create noticeable WLC effect, we investigate necessary parameters to induce $n_g = 0.1, 0.725$ and 1.1. By the relationship $\Delta \omega, n_i / n_g = \Delta \omega$, we can anticipate that the WLC linewidth will broaden approximately a factor of 14.5 for $n_g = 0.1$, 2 for $n_g = 0.725$ and 1.32 for $n_g = 1.1$ in comparison to the normal cavity. For the investigation, we choose FSR to yield $\text{FSR}(N + 1/2) = \nu_B$. The gain separation is chosen to correspond to FSR so that single longitudinal cavity mode is located at the center of the two gains. To observe WLC effect, we scan the probe frequency around such a mode. We inspect allowable FSR’s of WLC for the given $n_g$ to keep below Brillouin threshold at the probe scanning mode. First, let us consider gain that the probe experiences during one round trip in the ring cavity so-called gain per pass. The cavity resonance frequency $(\nu_0)$ in the middle of two gains is equal to $\nu_B$, due to the cavity design rule $\text{FSR}(N + 1/2) = \nu_B$ as well as the proper gain separation. In the double gains, Eq(5) indicates that the probe at the resonance $(\nu_0 = \nu_B)$ experiences the exponential Brillouin gain of $\alpha_{Br(\nu=\nu_B)} = 2G/(1 + 4\Delta^2/\Gamma_B^2)$. The probe is scanned around $\nu_B$ within the spectral region where negative dispersion occurs. To observe WLC effect, the scan range is required to be larger than FWHM of the reference cavity. We consider the cases that the cavity FSR is almost equal to the width measured at the bottom of the gain profile i.e. $\sim 2\Gamma_B$. Due to high Q of fiber cavities, FWHM is less than $\sim 2\Gamma_B$ a factor of 10~100. The minimum scan range is substantially less than $2\Gamma_B$ and thus most of such a scan region exhibits the constant Brillouin gain close to $\alpha_{Br(\nu=\nu_B)}$. Therefore, we can assume that the probe feels the constant gain $\alpha_{Br(\nu=\nu_B)}$ while scanning around $\nu_0$. Before proceeding to find gain per pass for the probe, let us discuss the effective cavity length ($L_{eff}$). In numerical simulation, we will introduce the significant internal cavity loss such that $L_{eff}$ should be taken into account. By relating $\alpha$ associated with the loss characteristics in our case, $L_{eff}$ in ref.[5~8] is redefined as $L_{eff} = -L/(1 - \alpha)/(\ln \alpha)$ in terms of the cavity length. It leads to the exponential gain per pass for the probe $\alpha_{Br(\nu=\nu_0)}L_{eff}$ \cite{12, 13, 14, 15}. Using $L_{eff}$, the modified gain is changed to $\alpha \exp(\alpha_{\nu_B}L_{eff})$. According to $a_2 = \alpha \exp(\alpha_{\nu_B}L_{eff}) a_0 b_2$, $\alpha \exp(\alpha_{\nu_B}L_{eff})$ represents the round trip loss factor in the presence of Brillouin pumps. Sufficient pump power can allow values of $\alpha \exp(\alpha_{\nu_B}L_{eff}) \geq 1$. At $\alpha \exp(\alpha_{\nu_B}L_{eff}) = 1$, the induced Brillouin gain completely compensates for the internal cavity loss and thus the cavity reaches at the lasing threshold. As we are interested in observing WLC effect under Brillouin threshold, the cavity gain factor is
\( \alpha \exp(\alpha_{\text{in}} L_{\text{eff}}) < 1 \). It leads to the condition below Brillouin threshold at the probe scanning mode i.e. \( \alpha_{\text{Br}(\nu=\nu_j)} L_{\text{eff}} < -\ln(\alpha) \). Define \( d\beta/d\omega |_{\omega_0} = 2G(d\gamma/d\omega |_{\omega_0}) \) where \( \gamma \) is the sum of two Lorentzians remaining after factoring \( G = g_p I_p/2 \) from \( \beta \). Using \( k = n_g/c + d\beta/d\omega |_{\omega_0} = n_g/c \) together with the new definition, we can express \( G = (n_g - n_j)/(2c\gamma) \) where \( \gamma = d\gamma/d\omega |_{\omega_0} \) is negative. It is instructive to graphically compare \(-\ln(\alpha)\) to \( \alpha_{\text{Br}(\nu=\nu_j)} L_{\text{eff}} \) for \( n_g = 0.1, 0.725, \) and \( 1.1 \). Fig. 7 illustrates the lasing threshold \(-\ln(\alpha)\) and the gain per pass \( \alpha_{\text{Br}(\nu=\nu_j)} L_{\text{eff}} \) as function of FSR. For illustration, we consider different internal cavity loss so that \( \alpha = 0.955, 0.911, 0.855 \). We choose \( t = 0.955 \) and \( \Gamma_B = 10 \text{MHz} \). Remind that the gain separation (2\( \Delta \)) is equal to FSR. Fig. displays \(-\ln(\alpha)\) and \( \alpha_{\text{Br}(\nu=\nu_j)} L_{\text{eff}} \) as function of FSR. Due to the low threshold in high Q resonator \( (\alpha=1) \), \(-\ln(\alpha)\) reduces as we withdraw the cavity loss i.e. \( \alpha \to 1 \). To understand the behavior of \( \alpha_{\text{Br}(\nu=\nu_j)} L_{\text{eff}} \), note that \( G \) should be essentially increased to induce more group index change \( (\Delta n_g) \). The gain \( \alpha_{\text{Br}(\nu=\nu_j)} \) seen by the probe increases proportionally to \( G \) and thus small \( n_g \) (large \( \Delta n_g \)) requires higher \( \alpha_{\text{Br}(\nu=\nu_j)} L_{\text{eff}} \). Since the inclusion of the cavity loss reduces \( L_{\text{eff}} \), \( \alpha_{\text{Br}(\nu=\nu_j)} L_{\text{eff}} \) for the same \( n_g \) increases as \( \alpha \to 1 \). The cavity mode for the probe should stay below lasing threshold. As such, for WLC demonstration, we can employ the cavity FSR relevant to the condition \( \alpha_{\text{Br}(\nu=\nu_j)} L_{\text{eff}} < -\ln(\alpha) \), the so-called operational FSR. For example, the operational FSR is FSR>176MHz for \( n_g = 0.1, \alpha = 0.855 \). Since the \( n_g \) reduction entails additional \( \alpha_{\text{Br}(\nu=\nu_j)} L_{\text{eff}} \), note that for \( n_g = 0.1 \) with \( \alpha = 0.955, 0.911 \) \( \alpha_{\text{Br}(\nu=\nu_j)} L_{\text{eff}} \) is higher than the lasing threshold in all FSR’s of interest. Fig.8 illustrates \( G \) for \( n_g = 0.1, 0.725, \) and \( 1.1 \). To understand the behavior of \( G \), we discuss about \( n_g \) together with the negative dispersion slope as well as gain separation. Negative dispersion slope becomes large to reduce the group index. Either larger \( G \) for constant gain separation or narrow gain separation for constant \( G \) leads to the steeper negative dispersion relevant to small \( n_g \) indicated in Fig. 8. Fig.9 displays the input pump power \( I_p \) for the cavities with different internal loss. Using Brillouin coefficient \( g_0 = 0.7682 \times 10^{-11} \text{m/W} \) [1, 7], we first find the pump intensity per unit area \( I_p \) inside cavity. Consider briefly the polarization property of the fiber. Taking the polarization into account, \( G \) can be rewritten as \( 1/2 g_0 I_p \kappa \) where \( \kappa = 1 \) if the polarization is maintained or otherwise \( 0 \). While the probe propagates inside the cavity, its polarization direction is properly adjusted to correspond to the pumps. Hence, the previous definition \( G = 1/2 g_0 I_p \) (i.e. \( \kappa = 1 \)) is suitable for our interesting cases. To demonstrate WLC, we control the cavity length to force Brillouin pumps to be resonant with the cavity. According to the pump build-up factor \( \eta = (1 - |t|^2)/(1 - \alpha |t|^2) \), \( I_p \) is related to \( I_j \) by \( I_j = I_p A_{\text{eff}} / \eta \). We choose \( A_{\text{eff}} = 5 \times 10^{-11} \text{m}^2 \) for a silica single mode fiber. Consider \( I_j \) within the operational FSR. To achieve the given \( n_g \), \( I_j \) is essentially needed higher than the maximum power achievable from commercial EDFAs as well as the power limitation imposed on the fiber optic elements such as circulators and couplers. Even though we can experimentally increase \( I_j \) such a high level, \( G \) stops increasing due to Brillouin gain saturation. In the previous Brillouin characteristics measurement, the fiber used for our cavity showed it was saturated at \( G = 0.04 \). Inserting \( I_j \) into \( G \), however, note \( G = (1/2 g_0) (I_p \eta/A_{\text{eff}}) \). As such, the problem with the saturation is overcome by using fibers of high Brillouin...
coefficient and small effective area. Tellurite fiber ($g_0=1.6986\times10^{-10}$ m/W, $A_{\text{eff}}= 0.6967\times10^{-11}$ m$^2$) or As$_2$Se$_3$ chalcogenide fiber ($g_0=6.08\times10^{-9}$ m/W, $A_{\text{eff}}= 3.9\times10^{-11}$ m$^2$) has 20–800 times larger $g_0$ but also smaller $A_{\text{eff}}$ than conventional fibers. However, their transmission loss is major drawback when the cavity is controlled to achieve the pump resonance. We built a 6m ring resonator which was composed of 4m tellurite fiber and 2m SMF28e. While scanning the cavity, the pumps were coupled into the cavity to obtain the resonance peak output. Due to the loss, the coupled light $\sqrt{\kappa a_2}$ was completely absorbed before traveling one round trip such that we observed the only transmitted field $\sqrt{\tau a_1}$. Tellurite fiber was reported to exhibit large loss at low input power$^{15}$. Alternatively, we propose a WLC illustrated in Fig.11. A fiber ring resonator is built by a coupler spliced with the high Brillouin coefficient fibers. Low loss optical circulator is used to insert Brillouin pumps into the ring$^{16,17}$. The pumps propagate without resonating with the cavity and thus the transmission loss problem is avoided due to no requirement of the cavity length control. However, we need more input power than the pump resonant cavity. The high power EDFA compensates it. Furthermore, high $g_0$ fibers allow us to drop the input power level which is required to achieve sufficient $\Delta n_g$ for large WLC effect.

In conclusion, we have investigated the feasibility of realizing WLC using SBS, under two different configurations. In one case, we used a single frequency Brillouin pump resonantly enhanced in the cavity. The counter-propagating, single frequency probe was tuned to be resonant at a different longitudinal mode of the same cavity. The cavity FSR were chosen to have values to ensure that the probe experience depletion due to the SBS process of the pump. The probe was modulated at 7.0145GHz which is sufficiently separated from Brillouin shift frequency ($\nu_B=10.867$GHz). It was scanned to characterize the cavity linewidth without Brillouin interaction in the presence of the pump. When the probe was scanned around $\nu_B$, the observed linewidth was broadened. The broadening in this case was due to a combination of the WLC effect, and the pump induced depletion. Experimentally, it is impossible to separate out these two effects. We developed an analytical model, and used it to simulate numerically the probe transmission profile under conditions that match the experiment closely. The simulation shows that only a very small fraction of the broadening is due to the WLC effect. An analysis of the theoretical model also reveals that this is a fundamental constraint for such a configuration. In the second case, we used a pump with two frequency components, each locked to a cavity mode. The counter-propagating, single frequency probe was also resonant, at a different cavity mode. The various gain separations as well as FSR were chosen to ensure that the probe frequency was tuned to the center of the two gains. During the probe scanning around $\nu_B$, the probe transmission profile was seen to be broadened by a very small amount, due to WLC effect. Analytical model and simulation confirmed the conclusion that WLC effect under the experimental parameters is very small, and consistent with our experimental observation. In order to identify optimal conditions for observing a large WLC effect, we extended our model to allow for an arbitrary combination of bare cavity loss, free spectral range, pump power, and Brillouin coefficient. The results show that, given the properties of the fiber we are using, the key limitations are due to the damage thresholds of the various components (such as couplers and circulators), as well as the maximum power available from commercial EDFA. Alternatively, these constraints can be overcome by using a fiber with a Brillouin coefficient that is significantly larger than that of the fiber we are using, without a significant increase in the transmission loss. Future efforts at realizing a high bandwidth WLC using a fiber resonator therefore have to be directed towards overcoming these constraints.
Fig. 1 Schematics of fiber ring resonator

Fig. 2 Schematics of experimental setup to observe WLC effect: LIA, lock-in amplifier; EOM, electro-optic modulator; FPC, fiber polarization coupler; PMC, polarization maintaining coupler; VC, variable coupler. The optical chopper combined with LIA will be used to capture the data presented in Fig. 7.
Fig. 3 (a) Experimental data of the cavity response for Brillouin absorption and gain with varying pump power (b) Simulation results indicating close agreement with the experimental results. The colors of the peaks correspond to the same color of the peaks as indicated in Fig. 3(a). G used in simulation and the calculated $n_g$ for the give G are that $G=0.0002$, $n_g=1.4538$ for yellow, $G=0.0003$, $n_g=1.4557$ for cyan, $G=0.0005$, $n_g=1.4595$ for magenta, $G=-0.0005$, $n_g=1.4405$ for blue, $G=-0.0015$, $n_g=1.4214$ for green, $G=-0.0018$, $n_g=1.4156$ for red. The negative (positive) G represent absorption (gain). (c) Black is a reference resonance peak without SBS. Red represents the cavity response at WLC condition ($n_g=0$, $G=0.0759$). Green is simulation result where we assume that the cavity fiber is non-dispersive (i.e. $n_g=n_f$, $n_f=1.45$ for SMF-28e) but take into account the loss induced by Brillouin absorption in the calculation.

Fig. 4 Schematics of experimental setup for WLC with dual gains
Fig. 5 (a) Experimental data with varying the pump power, Gain separation is 34.64MHz equal to 2×FSR, (b) Numerical simulations done with the parameters that G=0.001(n_g=1.4499) for 2mW, G=0.013(n_g=1.4338) for 8mW, G=0.0014(n_g=1.4326) for 8.5mW, G=0.015(n_g=1.4314) for 9.2mW and G=0.0155(n_g=1.4499) for 9.6mW. The group indexes are calculated from G values. The colors of the peaks represent the power values indicated in Fig.3(a), (c) Cavity modes(black bars), pump spectrums(red arrows) and gain lines(red peaks), a, b and c correspond to Brillouin shift frequency (10.867GHz). FSR is 17.32MHz.
Fig .6 (a) Experimental data comparing the ring cavity response for gain doublets to the cases for a single gain with varying pump power. Black is the response without Brillouin interaction. The anti-stokes component ($ν_L+ν_B$) of the probe is blocked by the F.P. spectrum filter. (b) Numerical simulations showing the consistency with the experimental data. The colors correspond to those of the graphs in (a). Blue dot represents the case of 4mW pump. The parameters are $G=1.9\times10^{-4}$, $n_g=1.4518$ (1.5mW), $G=3.8\times10^{-4}$, $n_g=1.4536$ (2mW) for single gain and $G=0.5\times10^{-4}$, $n_g=1.4481$ (4mW) for dual gains, (c) Experimental data showing the cavity response with different pump power from (a), (d) Numerical simulations with parameters that $G=3.8\times10^{-4}$, $n_g=1.4536$ (2mW), $G=1.0\times10^{-3}$, $n_g=1.4595$ (4mW) for a single gain and $G=0.8\times10^{-2}$, $n_g=1.447$ (8mW), $G=1.8\times10^{-2}$, $n_g=1.4433$ (12mW) (e) Configurations of cavity modes (black bars), pump resonances (red arrows) and gains (red peaks) a,b,c are Brillouin shift frequency (10.867GHz). For the comparison of linewidth, we moved the data to align the rising portions of the peaks in (a),(c)
Fig. 7 (a) Probe transmission profiles showing a small WLC effect. It is separated by the optical chopper from the lasing signal at the modes close to the gain peaks. When the pump power reaches 18mW, the probe mode also starts lasing. (b) FSR(=17.4016MHz) is adjusted to be larger than the gain bandwidth ($\Gamma_B$). The gain peaks are shifted from the pump spectrums as amount of $\nu_B$ such that according to $\text{FSR}(N + 1/2) = \nu_B$, the peaks is located at the center of the two neighboring cavity modes. a, b, c are $\nu_B (10.867\,\text{GHz})$

Fig. 8 For $n_g$’s, the curves represent $\alpha_{Br(v=\nu_0)} L_{eff}$ associated with different $\alpha$ values. The straight lines corresponds to $-\ln(\alpha)$. The operational FSR is FSR$>176\,\text{MHz}$ for $n_g=0.1$, $\alpha=0.855$, FSR$>95\,\text{MHz}$ for $n_g=0.725$, $\alpha=0.855$, FSR$>160\,\text{MHz}$ for $n_g=0.725$, $\alpha=0.911$, FSR$>48\,\text{MHz}$ for $n_g=1.1$, $\alpha=0.855$, FSR$>80\,\text{MHz}$ for $n_g=1.1$, $\alpha=0.911$, and FSR$>180\,\text{MHz}$ for $n_g=1.1$, $\alpha=0.955$. For $n_g=0.1$, $\alpha=0.955$ and $n_g=0.1$, $\alpha=0.911$, there is no available operational FSR.
Fig. 9 Exponential Brillouin gain peak amplitudes for different $n_g$ values

Fig. 10 Pump power being required to obtain $n_g$. Note the power within the operational FSR mentioned in Fig. 9.
Fig 11. Schematics of proposed setup for WLC demonstration
Reference

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REFERENCE FOR APPENDIX

VITA

Honam Yum grew up in Korea. He received his Bachelor of Arts degree in Ceramic Engineering from Yonsei University in 2000. In the fall of 2001, he came to Texas A&M University and started his master’s in Electrical and Computer Engineering (ECE). His research work during M.S. was in the area of holography which leads to three journal publications. After finishing his M.S. in 2004, he started working for Liquid Crystal Display (LCD) business in Samsung Electronics. He obtained thin film fabrication techniques and optical system design. He published 5 patents related to those works. After working for approximately a year and half and gaining valuable experiences in the industrial field, he returned to Texas A&M. In the fall of 2006, he began his Ph.D in ECE Program. His research projects during Ph.D focused on pulse propagation in dispersive medium, fast-light intracavity medium and its applications to gyroscope, data buffering. After completion his Ph.D, he is looking forward to continuing research works as post-doctor. He may be reached at the Department of Electrical and Computer Engineering, Texas A&M University, 3128 TAMU, College Station, TX 77843-3128, USA. His email is hn_yum@yahoo.com.